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# Advanced Fixed-Point Results for New Type Contractions via Simulation Functions in $b$-Metric Spaces with an Application to Nonlinear Integral

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**Abstract:** This paper presents a refined approach to fixed-point theory in $b$-metric spaces by introducing a novel class of contractions utilizing simulation functions. The proposed framework generalizes and strengthens existing results, providing deeper insights into the underlying structure of $b$-metric spaces. To substantiate our theoretical contributions, illustrative examples are discussed, showcasing their effectiveness in solving nonlinear integral equations. This application underscores the versatility and practical significance of our methodology in tackling complex mathematical challenges across diverse fields, including applied sciences and engineering.

**Keywords:** *b-metric space; simulation function; fixed point; integral equation.*

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## 1 Introduction

Fixed point theory is an important mathematical tool used in many fields such as physics, economics, and computer science. Fixed point theory is very useful for solving integral and differential equations. This makes it very important for application in mathematics and science. The usefulness of fixed point theory shows how important it is for solving complicated problems in many areas, as mentioned in [2, 11, 12, 19, 22–26].

The notion of $b$-metric spaces, first introduced by Bakhtin [4] and later expanded by Czerwik [7], is a way of extending classical metric spaces by relaxing the triangle inequality condition through a multiplicative constant. Unlike standard metric spaces,

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