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Estimation of Pitch Angle and Heave Position of Remotely Operated Vehicle Using Linear Quadratic Gaussian

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Abstract: In a Remotely Operated Vehicle (ROV), there are six degrees of freedom: surge, sway, heave, roll, pitch, and yaw. In linear motions, there are surge, sway and heave. In angular rotations, there are roll, pitch and yaw. In practice, there are disturbances and noise in the linear motion and angular rotation in their measurements. Therefore, in this research, the estimation of pitch angle and heave position of a ROV will be carried out by Linear Quadratic Gaussian (LQG). LQG is used for optimal control when there are disturbance input and measurement noise in the plant model. From the simulation, the estimation of the state solution and optimal control with various noise can be compared by LQG.

Keywords: *ROV; pitch angle; heave position; LQG.*

Mathematics Subject Classification (2020): 37N35, 49N10.

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1 Introduction

Seventy percent of Indonesia is covered by sea, so Indonesia has significant marine resource. For exploring marine resource, a Remotely Operated Vehicle (ROV) with its control is required. In a ROV, there are six degrees of freedom: surge, sway, heave, roll, pitch, and yaw. In linear motions, there are surge, sway and heave. Surge, sway, and heave are linear motions on the x-axis, y-axis, and z-axis, respectively. In angular rotations, there are roll, pitch and yaw. Roll, pitch, and yaw are angular rotations on the x-axis, y-axis, and z-axis, respectively [1].

In practice, there are disturbances and noise in the linear motion and angular rotation in their measurements. Therefore, in this research, the estimation of pitch angle and heave position of ROV will be carried out by Linear Quadratic Gaussian (LQG). Linear Quadratic Gaussian (LQG) control refers to an optimal control problem where the plant model is linear with a white noise disturbance input and white measurement noise. LQG is used for optimal control when there are a disturbance input and measurement noise in the plant model. Generally, optimal control is used to find the control minimizing the performance index and result in the trajectory of state solution [2].

From the previous research, an AUV has been developed by the method of Linear Quadratic Regulator (LQR) minimizing the control with no reference [3], Linear Quadratic Tracking (LQT) minimizing the control included to the tracking path [4], PID control for determining the response [5]. The estimation method is also established by the Kalman Filter and H-infinity method [6]. Besides the AUV, the LQR and LQT are also applied in the inverted pendulum [7], missile and projectile [8], mobile robot [9] and human arm models [10].

The dynamical model of LQG is almost similar to the LQR, however in the LQG model, there are distubance input and measurement noise. To determine the estimation of the state solution, we need a Riccati solution and feedback gain from the LQR and Kalman Gain. LQG is also used for optimizing the given performance index. To run the simulation, the calculation of LQR should be made before. Then the Riccati solution and feeback gain of LQR are brought to the LQT as variables. From the simulation, the estimation of state solution of pitch and heave and optimal control with various noise can be compared by LQG with small mean square error (MSE).

2 Mathematical Model of ROV

There are some motions and rotations in a ROV which will be constructed as a mathematical model. In linear motions such as surge, sway, and heave, there are motions on the x-axis, y-axis, and z-axis, respectively. In angular rotations such as roll, pitch, and yaw are rotations on the x-axis, y-axis, and z-axis, respectively. The mathematical model of ROV is shown in Figure 1. According to Newton's Law, the mathematical model of ROV [5] is

$$X = m\left[i - vr + wq - x_G\left(q^2 + r^2\right) + y_G\left(pq - \dot{r}\right) + z_G\left(pr + \dot{q}\right)\right], \tag{1}$$

$$Y = m\left[\dot{v} + ur - wp + x_G\left(pq + \dot{r}\right) - y_G\left(p^2 + r^2\right) + z_G\left(qr - \dot{p}\right)\right], \tag{2}$$

$$Z = m\left[\dot{w} - uq + vp + x_G(vp - \dot{q}) + y_G(qr + \dot{p}) - z_G(p^2 + q^2)\right], \tag{3}$$

$$\begin{aligned} K = {} & I_x\dot{p} + (I_x - I_y)qr + I_xy(pr - \dot{q}) - I_yz(q^2 - r^2) - I_xz(pq + \dot{r}) \\ & + m\left(y_G(\dot{w} - uq + vp) - z_G(\dot{v} + ur - wp)\right), \end{aligned} \tag{4}$$

$$M = I_y\dot{q} + (I_x - I_z)pr - I_x y(qr - \dot{p}) - I_y z(p1 - \dot{r}) - I_x z(p^2 - r^2) \\ - m\left(x_G(\dot{w} - uq + vp) - z_G(\dot{u} - vr + w1)\right), \tag{5}$$

$$N = I_z\dot{r} + (I_y - I_x)pq - I_x y(p^2 - q^2) - I_y z(pr - \dot{q}) - I_x z(qr - \dot{p}) \\ - m\left(x_G(\dot{v} + ur - wp) - y_G(\dot{u} - vr + wq)\right). \tag{6}$$

The model parameters are: m is the ROV mass, I_x, I_y, I_z are the inertia moments on the x-axis, y-axis, and z-axis, respectively, x_G, y_G, z_G are the positions of gravity center on the x-axis, y-axis, and z-axis, respectively. The other notations are:

X : surge force	x : surge position	u : surge velocity
Y : sway force	y : sway position	v : sway velocity
Z : heave force	z : heave position	w : heave velocity
K : roll moment	ϕ : roll angle	p : roll rate
M : pitch moment	θ : pitch angle	q : pitch rate
N : yaw moment	ψ : yaw angle	r : yaw rate

The problem is only limited for linear motion and angular rotation. By removing surge, sway, roll and yaw, the variables are $v = r = p = \varphi = \psi = y = 0$. The state solutions are the pitch angle θ, pitch velocity q, heave position z, and heave velocity w.

The simplicity of the model of ROV can be explained as follows.

As the pitch moment equation, take equation (5): $M = I_y\dot{q} + (I_x - I_z)pr - I_x y(qr + \dot{p}) - I_y z(pq - \dot{r}) - I_x z(p^2 - r^2) - m\left(x_G(\dot{w} - uq + vp) - z_G(\dot{u} - vr + wq)\right)$.

By removing surge, sway, roll and yaw, the variables are $v = r = p = \varphi = \psi = y = 0$. The pitch moment M can be elaborated as $M = M_q\dot{q} + M_q q + M_{\dot{w}}\dot{w} + M_w w$, then the pitch moment equation is

$$M_{\dot{q}}\dot{q} + M_q q + M_{\dot{w}}\dot{w} + M_w w = I_y\dot{q} - mx_G\dot{w} + mx_G uq + mz_G wq, \tag{7}$$

where $M_{\dot{q}}$ is the added inertia mass moment related to the pitch rate, $M_{\dot{w}}$ is the added inertia mass moment related to the heave velocity, M_q is the pitch moment coefficient induced by the pitch rate, M_w is the pitch moment coefficient induced by the heave velocity.

As the heave force equation, take equation (3): $Z = m\left[\dot{w} - uq + vp + x_G(vp - \dot{q}) + y_G(qr + \dot{p}) - z_G(p^2 + q^2)\right]$. By removing surge, sway, roll and yaw, the variables are $v = r = p = \varphi = \psi = y = 0$. The heave force Z can be elaborated as $Z = Z_{\dot{q}\dot{q} + Z_q q + Z_{\dot{w}}\dot{w} + Z_w w}$, then the heave force equation is

$$Z_{\dot{q}}\dot{q} + Z_q q + Z_{\dot{w}}\dot{w} + Z_w w = m\dot{w} - muq - mx_G\dot{q} - mz_G q^2, \tag{8}$$

where $Z_{\dot{q}}$ is the added mass related to the pitch rate, $Z_{\dot{w}}$ is the added mass related to the heave velocity, Z_q is the heave force coefficient induced by pitch rate, Z_w is heave force coefficient induced by the heave velocity.

From equations (7) and (8), the nonlinear system of ROV is

$$\dot{\theta} = q, \tag{9}$$

$$(M_{\dot{q}} - I_y)\dot{q} + (M_{\dot{w}} + mx_G)\dot{w} = (z_G W - z_B)\theta + mx_G uq + mz_G wq - M_q q - M_w w + M_\delta \delta, \tag{10}$$

$$\dot{z} = w\cos\theta - u\sin\theta, \tag{11}$$

$$(Z_{\dot{q}} + mx_G)\dot{q} + (Z_{\dot{w}} - m)\dot{w} = -muq - mz_G q^2 - Z_q q - Z_w w - Z_\delta \delta. \tag{12}$$

By the linearization $\sin\theta \approx, \cos\theta \approx 1$ and the Jacobian around the equilibrium point $(\theta^*, q^*, z^*, w^*) = (0, 0, 0, 0)$, the linear form is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (M_{\dot{q}} & 0 & (M_{\dot{w}} + mx_G) \\ 0 & 0 & 1 & 0 \\ 0 & (Z_{\dot{q}} + mx_G) & 0 & (Z_{\dot{w}} - m) \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{q} \\ \dot{z} \\ \dot{w} \end{bmatrix} =$$
$$\begin{bmatrix} 0 & (z_G W - z_B) & -u & 0 \\ 1 & -(M_q - mx_G u) & 0 & -M_w \\ -u & 0 & 0 & 1 \\ 0 & -(Z_q + mu) & 0 & -Z_w \end{bmatrix} \begin{bmatrix} \theta \\ q \\ z \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ -Z_\delta \\ M_\delta \\ 0 \end{bmatrix} \delta. \tag{13}$$

3 Methods

3.1 Linear Quadratic Regulator (LQR)

The Linear Quadratic Regulator is used to find the control minimizing the performance index and results in the trajectory of state solution. Some steps of the LQR are computing the solution of the Riccati Equation $P(t)$ in equation (14) and the feedback gain $K(t)$ in equation (15). From the result of the feedback gain, we can compute the solution of state space and optimal control [11].

$$\dot{P} = -P(t)A - A^T P(t) - Q + P(t) B_u R^{-1} B_u^T P(t), P(t_f) = H, \tag{14}$$

$$K(t) = R^{-1} B_u^T P(t). \tag{15}$$

3.2 Linear Quadratic Gaussian (LQG)

3.2.1 State space model

The state space model of LQG is similar to the LQR model, however there is additional noise so that the state space model of LQG [12] is

$$\dot{x}(t) = Ax(t) + B_u u(t) + B_w w(t), \tag{16}$$

$$m(t) = C_m x(t) + v(t), \tag{17}$$

$$E[w(t) w^T(t+\tau)] = S_w \delta(\tau), \tag{18}$$

$$E[v(t) v^T(t+\tau)] = S_v \delta(\tau), \tag{19}$$

$$E[v(t) w^T(t+\tau)] = 0. \tag{20}$$

Here, equation (16) is the plant model, equation (17) is the measurement and equations (18) to (20) are the noise model of plant and measurement. The performance index which will be optimized is

$$J = \frac{1}{2} E\left[x^T(t_f) H x(t_f) + \int_0^{t_f} \left(x^T(t) Q x(t) + u^T(t) R u(t) \right) \right], \tag{21}$$

where H and Q are positive semidefinite and R is positive definite.

3.2.2 The development of the Riccati equation

The development of the Riccati equation can be computed in equation (22) with the initial condition $F_e(0) = 0$. After the solutions are obtained, the solution of the Riccati equation can be applied in computing the Kalman Gain in equation (23).

$$\dot{F}_e(t) = AF_e(t) + F_e(t)A^T + B_w S_w B_w^T - F_e(t)C_m^T S_v^{-1} C_m F_e(t), F_e(0) = 0, \tag{22}$$

$$G(t) = F_e(t)C_m^T S_v^{-1}. \tag{23}$$

3.2.3 Kalman filter equation

To estimate the state solution, we use the Kalman Filter equation in equations (24) and (25) using the feedback gain from the LQR and Kalman Gain in equation (26).

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B_u u(t) + G(t)(m(t) - C_m \hat{x}(t)), \tag{24}$$

$$\dot{\hat{x}}(t) = [A - G(t)C_m - B_u K(t)]\hat{x}(t) + G(t)m(t), \tag{25}$$

$$u(t) = -K(t)\hat{x}(t). \tag{26}$$

4 Results and Discussion

4.1 Model of ROV

The model of ROV used is as in equations (27) and (28), where θ is the pitch angle, w is the heave velocity, q is the pitch rate, and z is the heave position. The simulations will be run with various rates of S_w and S_v. The solution of state space and its estimation, i.e., the pitch angle and heave position, and optimal control will be obtained as comparison.

$$\begin{bmatrix} \dot{\theta} \\ \dot{w} \\ \dot{q} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.0175 & -1.273 & -3.559 & 0 \\ -0.052 & 1.273 & -2.661 & 0 \\ -5 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ w \\ q \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0.085 \\ 21.79 \\ 0 \end{bmatrix} \delta, \tag{27}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ w \\ q \\ z \end{bmatrix}. \tag{28}$$

4.2 Simulation 1

In this simulation, the values of $S_w = 0.01$ and $S_v = 0.01$ will be used and the result of feedback gain can be seen in Figure 1, the Kalman gain can be seen in Figure 2. The state solution and its estimation, i.e., the pitch angle and heave position, can ben seen in Figure 3 and optimal control can be seen in Figure 4.

Figure 1 describes the solution of the feedback gain from the LQR with given final conditions being zero. The feedback gain is obtained from the Riccati solution which is computed backwardly. Figure 2 describes the solution of the Kalman gain with given initial condition being zero.

The Kalman gain is obtained from development of the Riccati solution which is computed forwardly. Figure 3 shows the state solution and its estimation of the pitch angle

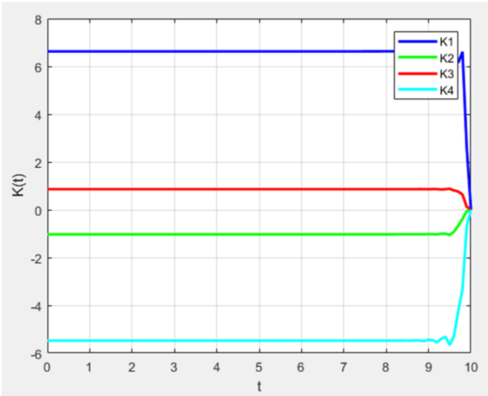


Figure 1: The Solution of the Feedback Gain.

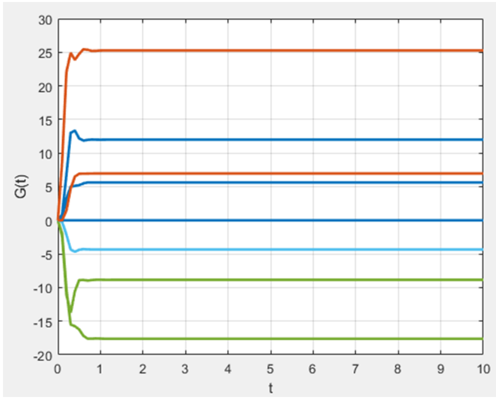


Figure 2: The Solution of the Kalman Gain.

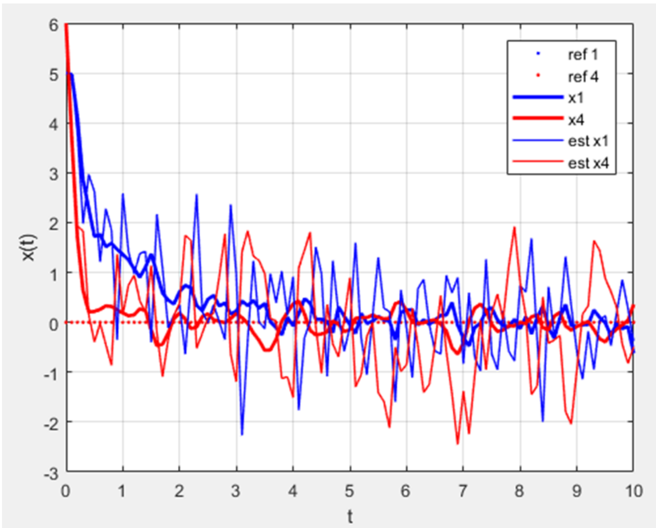


Figure 3: The State Solution and Estimation with Large Noise.

denoted by x_1 and the heave position denoted by x_4. The RMSE of the estimation is 0.9268 for the pitch angle estimation and 1.0646 for the heave position estimation. Figure 4 represents the optimal control and its solution with RMSE being 16.0780.

4.3 Simulation 2

In this simulation, the values of $S_w = 0.001$ and $S_v = 0.001$ will be used and the result of the feedback gain can be seen in Figure 5, the Kalman gain can be seen in Figure 6. The state solution and its estimation, i.e., the pitch angle and heave position, can be seen in Figure 7 and optimal control can be seen in Figure 8.

Figure 5 describes the solution of the feedback gain from the LQR with given final conditions being zero. The feedback gain is obtained from the Riccati solution which is

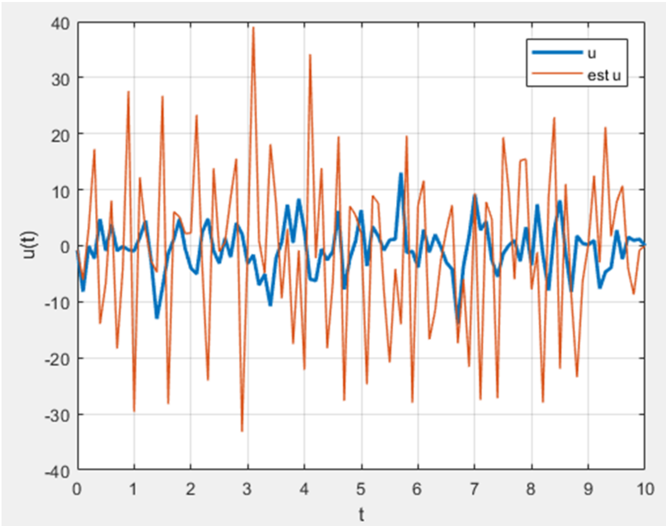


Figure 4: The Optimal Control and Estimation with Large Noise.

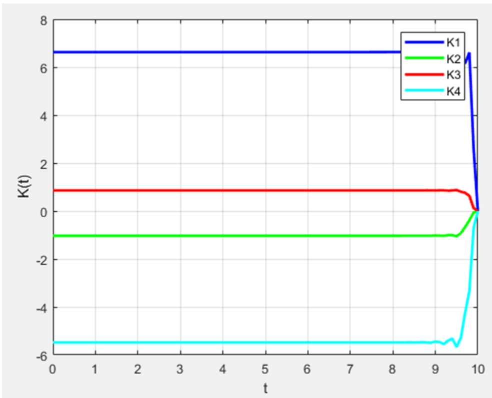


Figure 5: The Solution of the Feedback Gain.

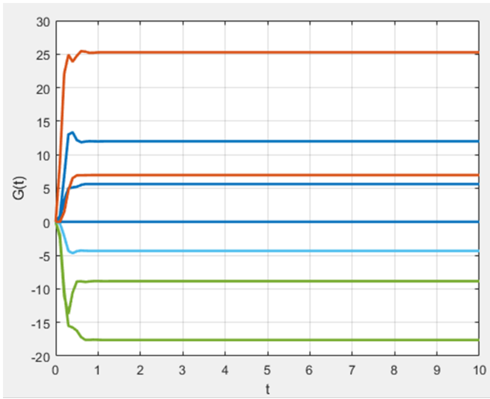


Figure 6: The Solution of the Kalman Gain.

computed backwardly. Figure 6 describes the solution of the Kalman gain with given initial condition being zero. The Kalman gain is obtained from development of the Riccati solution which is computed forwardly. Figure 7 shows state solution and its estimation of the pitch angle denoted by x_1 and the heave position denoted by x_4. The RMSE of the estimation is 0.2473 for the pitch angle estimation and 0.2652 for the heave position estimation. Figure 8 represents the optimal control and its solution with RMSE being 4.6209.

4.4 Simulation 3

In this simulation, the values of $S_w = 0.0001$ and $S_v = 0.0001$ will be used and the result of the feedback gain can be seen in Figure 9, the Kalman gain can be seen in Figure 10.

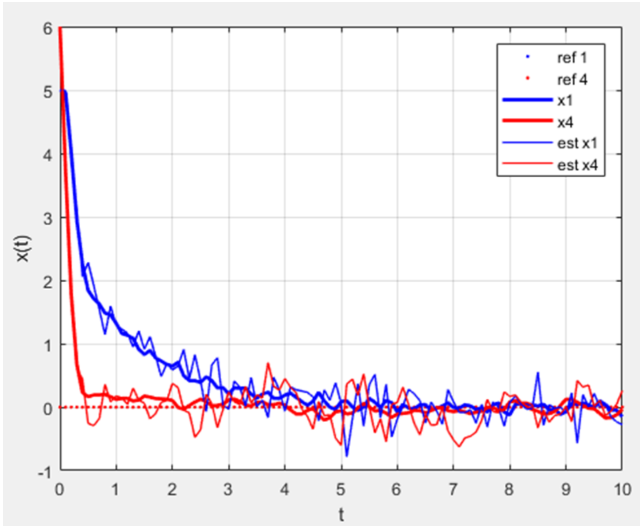


Figure 7: The State Solution and Estimation with Moderate Noise.

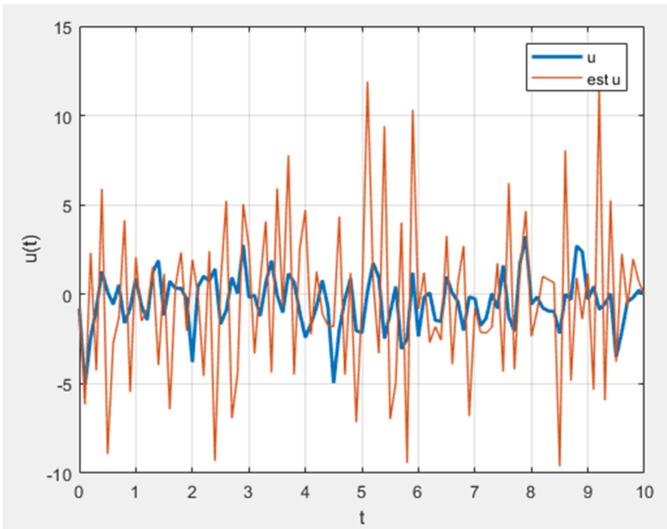
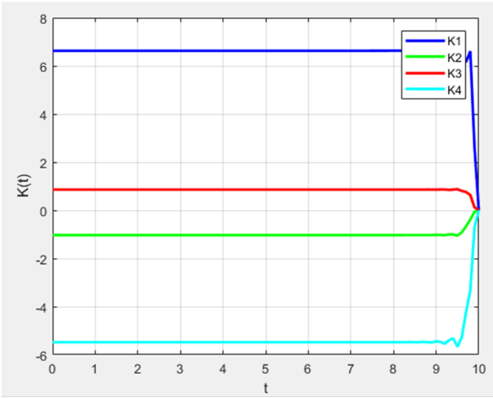


Figure 8: The Optimal Control and Estimation with Moderate Noise.

The state solution and its estimation, i.e., the pitch angle and heave position, can be seen in Figure 11 and optimal control can be seen in Figure 12.

Figure 9 describes the solution of the feedback gain from the LQR with given final conditions being zero. The feedback gain is obtained from the Riccati solution which is computed backwardly. Figure 10 describes the solution of the Kalman gain with given initial condition being zero. The Kalman gain is obtained from development of the Riccati solution which is computed forwardly. Figure 11 shows the state solution and



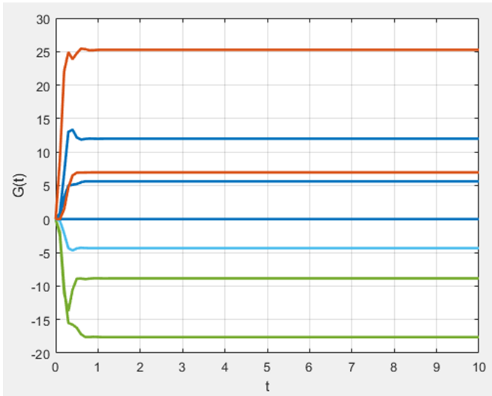


Figure 9: The Solution of the Feedback Gain. **Figure 10**: The Solution of the Kalman Gain.

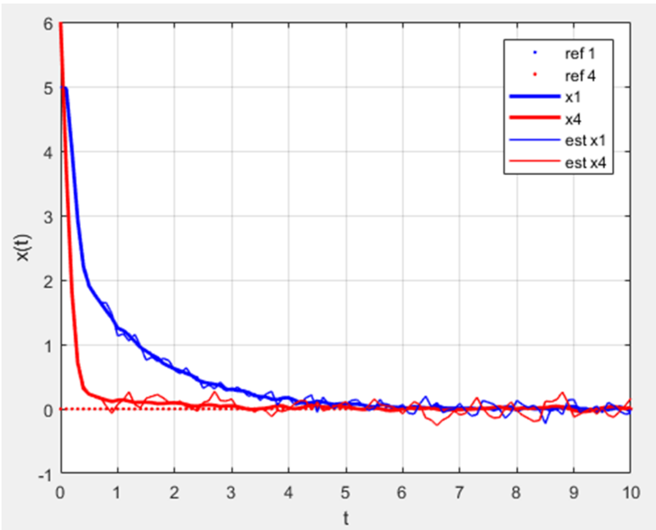


Figure 11: The State Solution and Estimation with Small Noise.

its estimation of the pitch angle denoted by x_1 and the heave position denoted by x_4. The RMSE of the estimation is 0.0762 for the pitch angle estimation and 0.0956 for the heave position estimation. Figure 12 represents the optimal control and its solution with RMSE being 1.3031.

5 Conclusion

The estimation of the pitch angle and heave position of a ROV is carried out by Linear Quadratic Gaussian (LQG). From the simulation, the estimation of the state solution and optimal control with various noise can be compared by LQG. From the results of three simulations, the small values of S_w and S_v can give better estimation with a small value of RMSE in both state solution and optimal control.

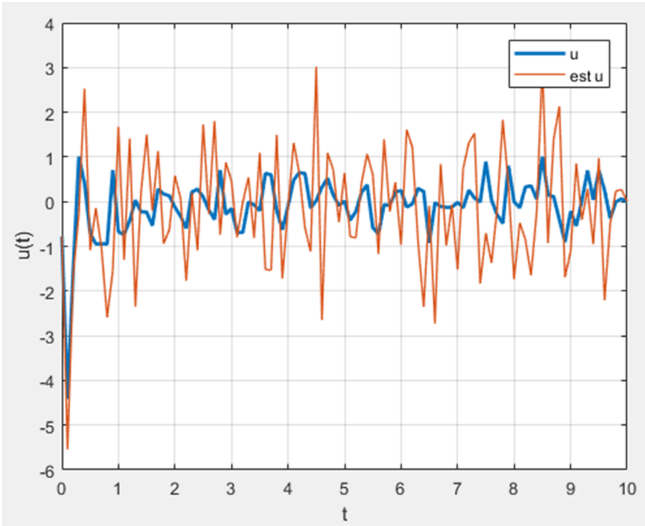


Figure 12: The Optimal Control and Estimation with Small Noise.

The developments of this research are the application of LQG to different objects besides the ROV. Moreover, the optimization of weight matrices in the performance index can be done by available methods.

6 Acknowledgement

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