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A Reliable Service Provider System-Mathematical Model and Dynamical Behavior

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Abstract: In this paper, a two-layered service provider system is considered in which one layer directly interacts with a client system and each member of the first layer is supported by a sub-group of members in the second layer, in completing a task put forth by the client. The mathematical model presented is a modification of an existing model studied by the authors. Since different groups are working on the task, a consensus is to be achieved by the system. In terms of mathematics, sufficient conditions are established on the system parameters so that the solutions remain asymptotically close to each other, and upon restricting the inputs, the solutions become bounded. This means that all the solutions approach a bounded solution of the system, implying that a consensus is formed. Once it is established that all members of the system are working together, it is necessary to meet the requirements of the client. Mathematically, it is achieved in terms of asymptotic stability of the desired solution through the Lyapunov functional method. For this, a set of sufficient conditions is obtained for the parameters and the functional relations of the system. Numerical examples are provided to verify the results and are supported by simulations. On the whole, our study provides a reliable service provider system that understands the requirements of the client, makes the assessment of its own capacities, gets back to the client for proper inputs and finally, delivers the output desired by the client.

Keywords: *focal and non-focal parts; client and server; time delays; variable inputs; desired solution.*

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1 Introduction

The world is longing for services - either to provide or to receive. Thus, the service sector is exhibiting growth enormously. Customer satisfaction is the utmost concern for any service provider system for its survival, and it is its most important performance indicator [9, 14]. Service providers can be broadly classified into two categories, namely, (i) product-specific, where the service provider fixes to certain products and offers only relevant services and (ii) client-specific, where the service provider caters to any needs of the client. For example, services to automobiles, electronic goods, etc., may be regarded as product-specific, while services of architects, lawyers, software developers, etc., are client-specific. In this paper, we are going to consider a client-specific service provider system. Usually, such systems maintain different layers. They may have one main layer (consisting of CEOs, Managers, Marketing team, etc., for example) that directly interacts with the client, understands the requirements and defines the problem. The other layers (may consist of the Technical team, Finance team, etc., for example) are equipped with various skills to work on issues of the client as posed by the first layer and report back to the first layer. The system arrives at a final solution due to a combined effort of the two layers. At the same time, clients can be of two types: those who continuously interact and participate in the development of their product along with the server, and those who simply provide necessary inputs, specify what is required and wait for a solution from the server, as this could save their time and resources. We prefer, in this study, clients of the second category.

The service provider system we are going to consider in this work consists of two layers, which may be called the 'outer layer' and the 'inner layer' of the system. The outer layer (may also be called 'focal layer') directly receives inputs from external sources (client), processes the information among its constituents, passes on instructions to inner layer units, and waits for their responses. The inner layer (may be called 'non-focal layer') forms a closed subsystem of the entire system in the sense that it receives and reacts to instructions from the outer layer only. Accordingly, they carry out activities internally. The overall performance of the system is judged by the outcome given by the outer layer in coordination with its inner counterparts. Our aim here is to propose and understand the behavior of such a dynamical system in mathematical terms. Usually, client-server models are understood as queuing models in which customers wait in queue lines for being served. Various performance measures and methods to evaluate them are also available in the literature of queuing theory [1, 2, 6, 8]. The model we are presenting is not a queuing model with a client waiting in a queue, but a system that requires coordination and cooperation among constituent groups for success or survival [2, 3]. The only question considered here is whether the requirements of the client are eventually met by the server system or not. How the parameters and functional relations in the system are to be chosen or restricted to make the system to perform for the satisfaction of the client's needs. This in itself tests the suitability of the model presented.

In general, the performance of a dynamical mathematical model is measured in terms of a qualitative study of its solutions, namely, the existence and uniqueness of solutions (i.e., the availability of non-conflicting solutions), the existence of equilibria (i.e., solutions of known or fixed behavior), boundedness (i.e., controllable solutions) and finally, the stability of an equilibrium solution (i.e., the behavior of other solutions when staying near a known, fixed solution showing that the entire system is controllable and therefore predictable) [12, 13]. Besides this, an important contention of this study is to answer the question: When all the qualities are present in the system, under what extra conditions the system will approach a pre-specified solution required by the client? Our aim in this paper is to provide reasonable answers to the above questions in terms of mathematical results. The model we are considering here is motivated by the study in [12], wherein the authors considered a two-layered network to understand the dynamics in a cooperative supportive neural network. We modify the model to suit our needs here and finally see that the results presented here are applicable to more general situations and include some of the earlier works on similar models in [12]. Furthermore, making use of the techniques from [13], we wish to obtain concrete results to reach the solution/output desired by the client.

The paper is organized as follows. In Section 2, the model is described in detail, and the basic properties of the dynamical system are discussed. In Section 3, the qualitative properties of solutions, namely, their boundedness and asymptotic closeness, are discussed. In Section 4, sufficient conditions on system parameters are established to ensure client satisfaction under suitable inputs from the client. Theoretical numerical examples with simulations are provided to illustrate the effectiveness of the results. A discussion is presented in Section 5, followed by concluding remarks with some open problems in Section 6.

2 The Model and Basic Properties

The following picture (Figure 1) is a formal representation of our assumptions. We

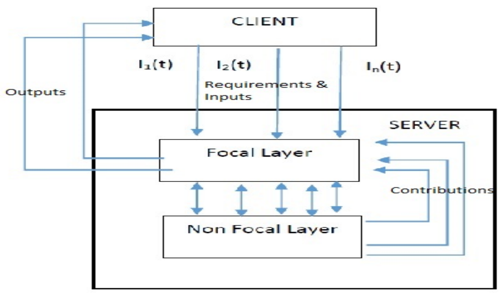

Figure 1: A schematic diagram of the proposed client-service provider system.

provide our mathematical model describing the dynamics of the server system. We consider the following system of equations:

$$x_i' = -a_i x_i + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j)) + \sum_{k=1}^{r_i} c_{ii_k} g_{i_k}(x_i, y_{i_k}) + I_i(t),$$

$$y_{i_k}' = -c_{i_k} y_{i_k} + \sum_{l=1}^{r_i} d_{i_l} h_{i_l}(y_{i_l}) + J_{i_k}(x_i(t-\delta_i)), \tag{1}$$

$i = 1, 2, ..., n, k = 1, 2, ..., r_i, 1 \leq r_i \leq n.$

In (1), $x_i(t)$ denotes the state of a typical focal member at any time t and $y_{i_k}(t)$ denotes the state of a typical non-focal member at time t and k is the number of y_{i_k}'s attached to each specific x_i. Here $' = \frac{d}{dt}$ denotes the time derivative of the variables.

The positive constants a_i, c_{i_k} respectively denote the rates at which the members x_i, y_{i_k} come back to resting states in the absence of any activity. They are usually called resting potentials or passive decay rates in the literature. The functional relations f_j, $j = 1, 2, ..., n$, denote the interactions among the focal members. The constant b_{ij} denotes the connection strength between the i^{th} and j^{th} focal members and is the rate at which focal parts communicate with each other. g_{i_k} shows how focal and non-focal members communicate with each other. This function plays a vital role in the dynamics of x_i's, the response of x_i's to the client, and is the only channel through which y_{i_k}'s contribute to focal parts. c_{ii_k} is the constant rate at which the x_i's receive information from y_{i_k}. $I_i(t)$ is the input at any time t from the client system to an x_i. The parameter d_{i_l} is the constant rate at which a typical member y_{i_l} of the non-focal subgroup corresponds with its co-member y_{i_k}. The function h_{i_l} is the functional response of y_{i_l} towards y_{i_k}. $J_{i_k}(x_i)$ is the functional input or instruction to y_{i_k} from x_i. Further, τ_i and δ_i respectively denote the time delays in the processing of information in the focal part and transmission of information from focal to non-focal parts.

We assume that the nonlinear response functions f_j, g_{i_k} and h_{i_l} satisfy the following local Lipschitz conditions, which are commonly used in the literature to establish the existence of solutions to any dynamical system:

$$\begin{aligned}
&|f_j(x_j) - f_j(\overline{x}_j)| \leq p_j|x_j - \overline{x}_j|,\\
&\|g_{i_k}(x_i, y_{i_k}) - g_{i_k}(\overline{x}_i, \overline{y}_{i_k})\| \leq M_{1i_k}|y_{i_k} - \overline{y}_{i_k}| + M_{2i_k}|x_i - \overline{x}_i|,\\
&|h_{i_k}(y_{i_k}) - h_{i_k}(\overline{y}_{i_k})| \leq q_{i_k}|y_{i_k} - \overline{y}_{i_k}|,\ |J_{i_k}(x_i) - J_{i_k}(\overline{x}_i)| \leq N_{i_k}|x_i - \overline{x}_i|. \qquad (2)
\end{aligned}$$

Here, $p_j, M_{1i_k}, M_{2i_k}, q_{i_k}$ and N_{i_k} are the corresponding Lipschitz constants. For the examples of functions commonly used in the literature, one can refer to [12, 13].

Since the inputs from the client should not disturb the aim of the task by deviating too much from what is initially presented to the server, we may assume that

(A). The input functions I_i for $i = 1, 2, ..., n$ are bounded continuous functions of time-variable $t \in [0, \infty]$.

Under the conditions (2) on the response functions J_{i_k} and assumption (A) on the inputs, we may easily establish that the system possesses unique solutions that are continuable in their maximal intervals of existence with suitably chosen initial conditions (see, e.g., [11]). Hence, we assume that these conditions hold throughout the text and proceed with the analysis of the behavior of solutions of the system (1).

3 Behavior of Solutions

Since different members are working on the task/project to meet the requirements of the client, a variety of solutions are expected corresponding to appropriate initial conditions. But there should be an affinity among all such solutions in the sense that they are close enough, all meeting the requirements of the client. In other words, all solutions of the system should be close enough eventually. Further, no solution should be beyond the purview of the system requirements. That is, any solution should be manageable. Mathematically, this requires all solutions to be bounded. This makes the system controllable. We shall see how we need to restrict the system parameter spaces and functional relations suitably to achieve this. First, we begin with the asymptotic nearness of solutions to see how close they are to each other, and no single solution is out of range.

Definition. Two solutions $u(t)$ and $v(t)$ of the system $z'(t) = F(t, z(t))$ are said to be asymptotically near or close to each other if $\lim_{t\to\infty} |u(t) - v(t)| = 0$.

Theorem 3.1 *For any pair of solutions (x_i, y_{i_k}) and $(\overline{x}_i, \overline{y}_{i_k})$ of (1), we have*

$\lim_{t\to\infty} |(x_i, y_{i_k}) - (\overline{x}_i, \overline{y}_{i_k})| = 0$, *provided the response functions satisfy (2) and the parameters satisfy* $\overline{A} = min\{A, B\} > 0$, *where for* $i = 1, 2, ..., n$ *and* $k = 1, 2, ..., r_i$,

$$A = min\Big\{a_i - \sum_{j=1}^{n} |b_{ji}|p_i - \sum_{k=1}^{r_i} \Big(|c_{ii_k}|M_{2i_k} + N_{i_k}\Big)\Big\},$$

$$B = min\Big\{c_{i_k} - \sum_{l=1}^{r_i} |d_{i_l}|q_{i_l} - |c_{ii_k}|M_{1i_k}\Big\}. \tag{3}$$

Proof. We consider the functional $V(t) = \sum_{i=1}^{n}\Bigg[|x_i - \overline{x}_i| + \sum_{j=1}^{n} |b_{ij}| \int_{t-\tau_j}^{t} |f_j(x_j(z)) - f_j(\overline{x}_j(z))|dz + \sum_{k=1}^{r_i} \Big[|y_{i_k} - \overline{y}_{i_k}| + \int_{t-\delta_i}^{t} |J_{i_k}(x_i(z)) - J_{i_k}(\overline{x}_i(z))|dz\Big]\Bigg]$ and use Lyapunov stability to prove the result.

Remark 3.1 It is clear from Theorem 3.1 that all the solutions of system (1) are together, that is, do not behave differently and do not deviate much from each other under the conditions specified. We shall now obtain conditions for the solutions of the system to be bounded. Then, under the conditions of Theorem 3.1, all the solutions stay near a bounded solution. Thus, we may predict the behavior of the system and try to control it.

Theorem 3.2 *Assume that the parameters satisfy the condition (3) and let the response functions satisfy, besides (2), the conditions* $f_j(0) = 0$, $g_{i_k}(0, 0) = 0$, $h_{i_l}(0) = 0$ *and* $J_{i_k}(0) = 0$ *for* $i = j = 1, 2, ..., n$, $k = l = 1, 2, ..., r_i$, *where* $1 \le r_i \le n$. *Further, if the inputs satisfy* $\displaystyle\int_0^\infty \sum_{i=1}^{n} |I_i(s)|ds < \infty$, *then all the solutions of (1) are bounded.*

Proof. To prove the statement, we employ the functional $V(t) = \sum_{i=1}^{n}\Bigg[|x_i(t)| + \sum_{j=1}^{n} |b_{ij}| \int_{t-\tau_j}^{t} |f_j(x_j(z))|dz + \sum_{k=1}^{r_i} \Big[|y_{i_k}(t)| + \int_{t-\delta_i}^{t} |J_{i_k}(x_i(z))|dz\Big]\Bigg]$.

Now we shall establish the conditions under which the solutions of (1) approach each other exponentially. We may expect further restrictions on parameters for such faster convergence. We use the following lemma for our next result.

Lemma 3.1 *([13]) If* $\varphi : [t_0, \infty)$ *is continuous such that* $\varphi^{'}(t) \le -p\varphi(t) + q \sup_{t-t_0 \le s \le t} \{\varphi(s)\}$ *for* $t \ge t_0$ *and if* $p > q > 0$, *then there exist positive constants* l, r *such that* $\varphi(t) \le le^{-rt}$ *for* $t \ge t_0$.

Theorem 3.3 *For any set of input functions, any pairs of solutions* (x_i, y_{i_k}) *and* $(\overline{x}_i, \overline{y}_{i_k})$ *of (1) are asymptotically exponentially close, provided the parameters satisfy the condition* $\alpha > \beta$, *where*

$$\alpha \equiv \min_{1\le i\le n, 1\le k\le r_i} \Big\{a_i - \sum_{k=1}^{r_i} \Big(|c_{ii_k}|M_{2i_k}\Big), c_{i_k} - \sum_{k=1}^{r_i} |d_{i_k}|q_{i_k} - |c_{ii_k}|M_{1i_k}\Big\},$$

$$\beta \equiv \sum_{i=1}^{n} \Big[\sum_{j=1}^{n} |b_{ji}|p_i + \sum_{k=1}^{r_i} N_{i_k}\Big]. \tag{4}$$

Proof. Using Lemma 3.1 and by considering the functional $V(t) = \sum_{i=1}^{n} \Big[|x_i - \overline{x}_i| + \sum_{k=1}^{r_i} |y_{i_k} - \overline{y}_{i_k}| \Big]$, we prove the result.

Remark 3.2 One may notice that the conditions on parameters in (4) are weaker than those in (3). That is, parameters have to take more strain for the solutions to be exponentially close. When we need a faster convergence depending on the client demands, then by straining the parameters more as in Theorem 3.3, any two solutions of the system come close to each other exponentially.

4 Performance of the System

The client system, being the end-user, should know what it needs to provide the server system to get what it requires. This is essential because there should be a correlation between the input provided by the client and the output produced by the server system. First, the client needs to spell out what is required. The focal parts of the server then study the request, understand it, provide the non-focal system with the necessary inputs, obtain its contribution and finally, see how the requirements of the client are to be met. In other words, the client requirements are to be discussed by constituent units and the system is to be checked for its abilities and limitations. It is where the interactions among units (functional relations) and their connection strengths (represented by parameters) play a vital role in deciding whether the client requirements are within the reach of the server system. Once it is identified that the solution is within the scope of the server system, it may ask for the information needed from the client to put forth the desired solution.

This may be consolidated as follows. The client puts forth its required solution. The server system tests itself whether such a solution is possible through its dynamics, functional relations and parametric spaces. The server identifies the suitable information needed from the client. Once the inputs provided by the client are within the range prescribed, the server provides the required solution.
Before going further, we rewrite the system (1) as

$$\begin{aligned} x^{'}(t) &= -ax(t) + bf(x(t-\tau)) + c_1 g(x(t), y(t)) + I(t), \\ y^{'}(t) &= -cy(t) + dh(y(t)) + J(x(t-\delta)). \end{aligned} \tag{5}$$

The present method is motivated by [13]. The technique is different from the usual feedback control techniques [5, 7] used in the literature. As described above, we assume that (i) The solution required by the client is α; (ii) An input of $J(\alpha)$ is provided by focal part x to non-focal part y; (iii) Corresponding to this input, β is the contribution of y with some strain on parameters and functions; (iv) The server then estimates the inputs $I(t)$ required from the client; (v) Once the requisite inputs are provided, the focal part x produces the output α with some strain on parameters and functional relations.
Mathematically, the solutions represented by (x, y) of (5) should reach a pre-specified output (α, β). This means an output of α is expected by the client from the focal part x of the system corresponding to an input of $I(t)$, with β denoting the expected contribution of the non-focal part y.

Theorem 4.1 *Assume that the parameters of (5) satisfy the conditions*

$$0 < min\{a - bp - c_1 l - N, c - dq - c_1 m\} < \infty. \tag{6}$$

Assume also that the response functions and J satisfy conditions (2). Then for any arbitrarily chosen output (α, β) satisfying

$$J(\alpha) = c\beta - dh(\beta), \tag{7}$$

all solutions $(x,\ y)$ of system (5) converge to (α, β), provided the input to the focal part satisfies the condition

$$\int_0^\infty \Big[|I(s) - a\alpha + bf(\alpha) + c_1 g(\alpha, \beta)|\Big] ds < \infty. \tag{8}$$

Proof. We employ the functional $V(t) = |x(t) - \alpha| + |y(t) - \beta| + b \int_{t-\tau}^{t} |f(x(s)) - f(\alpha)| ds + \int_{t-\delta}^{t} |J(x(s)) - J(\alpha)| ds$ and prove the result.

Remark 4.1 In Theorem 4.1, there are three requirements on the functions and parameters of the system. By the condition $J(\alpha) = c\beta - dh(\beta)$ (i.e., equation (7)), we mean that the contribution of y, that is, β, should satisfy the input requirement from x. This may provide a number of solutions $(\alpha,\ \beta)$ for the system over a space of parameters for each choice of J, as may be seen in examples provided below. For a unique solution, we may prefer further conditions such as the monotonicity of h, the interaction function among non-focal members. The condition (8) in Theorem 4.1 indicates that all inputs should eventually stay around the output-based server system requirements (i.e., near $a\alpha - bf(\alpha) - c_1 g(\alpha, \beta)$). This clearly states that the client cannot demand a suitable solution without providing proper inputs. Also, note that the restriction on I depends only on the contributions α and β of x and y, respectively, but not on J, the way how x instructs y. Finally, the sufficient conditions (6) on parameters specify how the server system restricts itself to arrive at the desired solution of the client.

We shall now illustrate Theorem 4.1 through a numerical but theoretical example. We fix up the parameters $(\alpha,\ \beta)$ satisfying $J(\alpha) - c\beta + dh(\beta) = 0$. Using MATLAB, the convergence of solutions of the system to the desired state is confirmed through simulations that are already established by Theorem 4.1. In the example provided here, the Lipschitz constants of (2) are given by $p = l = N = q = m = 1$.

Example 4.1 Consider the system

$$\begin{aligned} x^{'} &= -11x(t) + 3tanh(x(t-\tau)) + 2(x(t) + y(t)) + I(t), \\ y^{'} &= -9y(t) + 4tanh(y(t)) + J(x(t-\delta)). \end{aligned}$$

Corresponding to an expected output of $(\alpha,\ \beta)$, here the external input is chosen as $I(t) = (11\alpha - 3tanh(\alpha) - 2(\alpha + \beta))(1 + te^{-t})$ to satisfy condition (8) of Theorem 4.1.

(i) We let $J(x) = tanhx$. Then the desired output (α, β) is restricted to the region $D = \Big\{(\alpha, \beta)/9\beta - 4tanh(\beta) - tanh(\alpha) = 0, \Big\}$, from (7). We let $\alpha = 23 + 7i$. Then for this value of α, we have $\beta = 0.198$, satisfying the above equation. Clearly, all the conditions of Theorem 4.1 are satisfied, and hence, $(x, y) \to (\alpha,\ \beta) = (23 + 7i, 0.198)$ for large t by

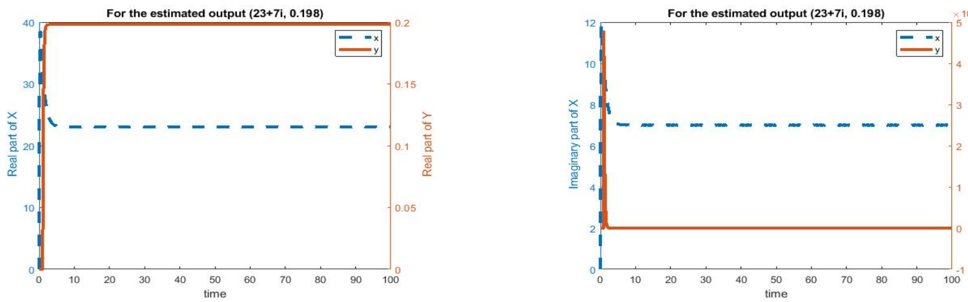

Figure 2: Input-output correlation through the system in Example 4.1.

virtue of Theorem 4.1 with $I(t)$ chosen above. The solutions approaching the a priori output (α, β) are depicted in Figures 2.

(ii) If we take $J(x) = \frac{x}{1+x}$, $x \neq -1$, the desired output (α, β) is restricted to the region $D = \left\{(\alpha,\beta)/9\beta - 4tanh(\beta) - \frac{\alpha}{1+\alpha} = 0, \right\}$. If we let $\alpha = 4 + 5i$, then we obtain $\beta = 0.178 + 0.019i$. For the input as specified above, all the other parametric conditions of Theorem 4.1 are satisfied. Hence, by Theorem 4.1, we get $(x, y) \to (\alpha, \beta) = (4 + 5i, 0.178 + 0.019i)$ as $t \to \infty$. The same is represented pictorially in Figure 3.

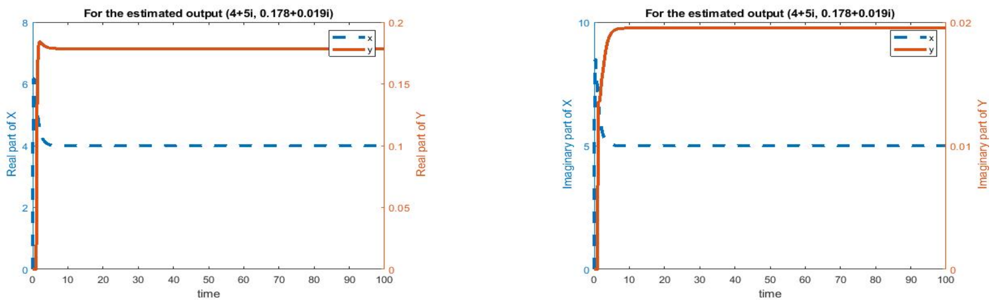

Figure 3: System eventually reaching a state desired by the client. Real and imaginary parts are shown separately.

Remark 4.2 Theorem 4.1 requires that the inputs are restricted by conditions specified by the server system as in (8) and the client should be in a position to provide them. Further, in the absence of any inputs (that is, $I(t) \equiv 0$, $J_{i_k}(x_i) \equiv 0$), it is not difficult to see [10] that $x_i \to 0$, $y_{i_k} \to 0$ as $t \to \infty$ under conditions (6), which shows that the system is going to a resting state in this case. On the other hand, when the client approaches it with suitable inputs, Theorem 4.1 establishes that the system is providing the desired solution for the client, which clearly indicates that the service provider system is client-specific only.

Remark 4.3 The very purpose of this study is to understand a real-world situation in terms of a mathematical model rather than to compare results. However, as we have selected an existing model and modified it to suit our requirements, a question of comparison naturally arises. Mathematically, system (1) may be regarded as a general form or time delay version of (VI) from [12], which is an open problem. We notice that all the models in [12] become special cases of (1). To be specific, it may also be noted that, for $\tau_j = \delta_i = 0$, the system (4.5) from [12] is a special case of (1) for $J_{i_k}(x_i) = J_{i_k}$, system (4.7) from [12] is a special case of (1) for $I_i(t) = I_i$ and system (1) reduces to system (2.1) from [12] when $J_{i_k}(x_i) = J_{i_k}$ and $I_i(t) = I_i$ (both are mere constants). Hence, the results presented here are also applicable to systems (2.1), (4.5), and (4.7) from [12] as well. In this context, when $I(t) \equiv I$ is a fixed constant and (α, β) is an equilibrium solution of (5) satisfying $I = a\alpha - bf(\alpha) - c_1 g(\alpha, \beta)$, then it may be noticed that Theorem 4.4 from [12] becomes a special case of Theorem 4.1 here for $\delta = 0$ and for $\delta > 0$, respectively.

Since we are specifying the inputs from the client for its desired solution and making no changes in the structure of the system, the technique is not for a re-engineering of the system (e.g., [4]) but for a client-friendly system, yet rigid in its own sense. It may also be noticed that the two layers are working in coordination with each other. Thus, there is a synchronization of efforts of two layers of the network, but not a synchronization of networks (two different networks approaching a complete or almost identical state) as studied in [10].

5 Discussion

In this paper, we have considered a two-layered mathematical model to represent the dynamical interactions between the focal and non-focal parts (regarded as an outer layer and inner layer, respectively) of the server system that caters to the needs of a client system. The client system has the flexibility to vary its inputs. It is assumed that each member in the focal part is directly connected to the client system, whereas the non-focal part receives its instructions from focal members, processes them and gets back to the focal member only. They are in no way connected directly to the external client system. Basic qualitative properties of such a dynamical system, namely, the existence and uniqueness of solutions, boundedness, and the asymptotic closeness of solutions, are discussed. Since things should not get lost in carrying out such activities, the boundedness and closeness of solutions are important. Theorems 3.1, 3.2, and 3.3 take care of such situations. An expected output for the client from the focal part is defined as α, which in turn, expects a contribution of β from the non-focal part. Relations are developed, and it is established that by suitably restricting the parameters of the system, the desired solution may be attained by the focal part when the client system provides suitable inputs. This is the conclusion of Theorem 4.1. The results provided are independent of the length of time delays. Numerical examples illustrate theoretical situations. Simulations are provided to establish pictorially that the solutions of the system are reaching the desired state as envisaged by Theorem 4.1. Comparisons are made with earlier works in mathematical terms wherever possible.

Accepting a pre-specified solution from the client, working out strategies to obtain the solution, getting back to the client for suitable inputs and producing the desired solution to the client once the suitable information is received - all show that the server system is reliable. Further, the inputs need not be simple, fixed constants, but may be time-varying functions also. Only a highly proficient client can fix the inputs once for all for a possible desired solution from the service provider. Thus, our server system can withstand variations in inputs and is flexible with the client.

References

[1] B. Dragovic, Nam Kyu Park, N. D. Zrnic and R. Mestrovic. Mathematical Models of Multi Server Queuing System for Dynamic Performance Evaluation in Port. *Mathematical Problems in Engineering* **2012** (3) (2012) 1–19.

[2] Bandana Priya, Ganesh Kumar Thakur, Sudesh Kumar Garg and M. Syed Ali. Robust Stability of Markovian Jumping Neural Networks with Time-Varying Delays. *Nonlinear Dynamics and Systems Theory* **20** (5) (2020) 535–541.

[3] Bo Xu, Ying Wang, Yu Han, Yuchang He and Ziwei Wang. Interaction patterns and coordination in two population groups: A dynamic perspective. *Chaos, Solitons and Fractals* **142** (2021) 110380.

[4] Chung-Horng Lung and Qiang Zhao. Pattern-oriented reengineering of a network system. *Journal of Systemics, Cybernetics and Informatics* **2** (4) (2004) 19–23.

[5] D. Bendel and M. Haviv. Cooperation and sharing costs in a tandem queuing network. *European Journal of Operations Research* **271** (3) (2018) 926–933.

[6] Debnath Bhattacharya, Soumita Seth and Tai-Loon Kim. Social network analysis to detect inherent communities based on constraints. *Applied Mathematics* & *Information sciences* **8** (1L) (2014) 385–396.

[7] Firman, Syamsuddin Toaha, Muh Nur. Asymptotic Stability of Some Class of Affine Nonlinear Control Systems through Partial Feedback Linearization. *Nonlinear Dynamics and Systems Theory* **21** (3) (2021) 238–245.

[8] Kan Li and Yin Pang. A unified community detection algorithm in complex network. *Neurocomputing* **130** (2014) 36–43.

[9] M. Y. Anshori, I. H. Santoso, T. Herlambang, M. Tafrikan, M. Adinugroho, K. Oktafianto and A. A. Firdaus. Analysis of Customer Satisfaction Survey on E-Wallets Using Simple Additive Weighting and TOPSIS. *Nonlinear Dynamics and System Theory* **24** (1) (2024) 28–40.

[10] Ruiyuan Zhu, Yingxin Guo and Fei Wang. Quasi-synchronization of heterogeneous neural networks with distributed and proportional delays via impulsive control. *Chaos, Solitons and Fractals* **141** (2020), 110322.

[11] V. B. Kolmanovskii and V. R. Nosov. *Stability of Functional Differential Equations*. Academic Press, London, 1986.

[12] V. Sree Hari Rao and P. Raja Sekhara Rao. Cooperative and Supportive Neural Network. *Physics Letters A* **371** (1) (2007) 101–110.

[13] V. Sree Hari Rao and P. Raja Sekhara Rao. Time Varying Simulations in Simple Neural Networks and Convergence to Desired Outputs. *Differential Equations and Dynamical Systems* **26** (2018) 81–104.

[14] R. Nunkoo, A. Sharma, K. K. F. So, H. Hu and A. F. Alrasheedi. Two decades of research on customer satisfaction: future research agenda and questions. *International Journal of Contemporary Hospitality Management* **37** (5) (2025) 1465–1496.