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Some New 4D Fractional-Order Hyperchaotic Rabinovich Systems: Dynamical Analysis and Control

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Abstract: This paper proposes some new 4D fractional-order hyperchaotic Rabinovich systems with Caputo derivatives. Using an exhaustive computer search, 20 simple hyperchaotic systems are found according to system parameters, initial conditions, and fractional-order derivatives of the 4D fractional-order Rabinovich system. These 20 systems are listed in this paper, and based on the Lyapunov exponents, the basic dynamical behaviors of these 20 systems have been investigated. By designing linear and nonlinear feedback controllers, the problems of local and global asymptotic stabilization are investigated for the first system of the proposed hyperchaotic systems. Furthermore, the numerical simulations show the feasibility and the effectiveness of the results.

Keywords: *fractional-order derivative; hyperchaotic Rabinovich system; Lyapunov exponents; feedback controller.*

Mathematics Subject Classification (2020): 34C28, 37D45, 37M22, 70K20, 93-04.

1 Introduction

Hyperchaos, characterized as a chaotic system with at least two positive Lyapunov exponents, has attracted many researchers due to its potential applications in science and engineering, such as neural networks, generation, control, synchronization, secure communications, lasers, image encryption [15], and so on. The hyperchaotic system is a higher dimensional chaotic system; its dynamics are richer and more extended in the phase plane, it has more complex behavior and abundant dynamics than chaotic systems.

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Historically, hyperchaos was first described by Rössler in 1979 as a model of a particular chemical reaction. After that, many hyperchaotic systems have been studied, including the Lorenz system, Rabinovich system, and Chua system [3, 4], among others.

Nonlinear dynamical systems exhibit unpredictable characteristics due to the disproportionate interactions that exist among their variables (inputs and outputs). The exploration of nonlinear dynamics contributes significantly to a more profound understanding, modeling, prediction, and control of the behaviors exhibited by complex systems that manifest in both natural and engineered systems. Nonlinear dynamics with their interdisciplinary applications can be found in almost every area of scientific research, such as physics, economics, and biology [8]. Chaos, as a dynamical behavior, is one of the most extensively studied topics in nonlinear dynamics in recent years. Comprehending chaos and nonlinear dynamics not only improves knowledge but also provides tools to find creative solutions to real-world phenomena.

Fractional-order calculus is a branch of applied mathematics that deals with differentiation and integration under an arbitrary order of these operations. In recent decades, fractional calculus has been a fruitful field of research, and it has been applied to almost every field of mathematics, science, engineering, and technology. During the last few years, fractional calculus and its applications have been undergoing rapid development, with more and more convincing applications in the real world. In fact, many applications of fractional calculus can be found in physics, signal processing, science and engineering, and many other domains (see e.g. [9] as well as the references therein). Fractional-order systems are closer to real-world phenomena due to their efficacy in modeling the memory and heredity properties of some phenomena [1].

A variety of fractional-order chaotic systems have been extensively investigated in the literature, for example, the fractional-order Lorenz, Chua, Chen, and Rabinovich systems, as well as systems exhibiting hidden dynamics, hyperchaotic behavior, and multi-scroll attractors [4, 14].

Many scientists and engineers have discovered the valuable applications of fractional-order chaos in many fields of science such as engineering, physics, mathematical biology, and psychology [12].

The Rabinovich differential system was first introduced as an important physical system in [10]. It is a nonlinear dynamical system recognized for displaying complex behaviors, including chaos and hyperchaos. Fractional-order chaotic Rabinovich systems have demonstrated significant potential across various advanced technological domains, including secure communication and cryptography, signal processing, artificial intelligence and machine learning, as well as the modeling of complex systems exhibiting memory-dependent dynamics [2]. Liu et al. [7] proposed a new 4D hyperchaotic system, which is generated via adding a controller to the 3D Rabinovich system, and studied its fundamental dynamics.

He and Chen in [4] proposed a 4D fractional-order hyperchaotic system and investigated its stability, hyperchaos, control of chaos, and synchronization for fixed system parameters ($a = 4,\ b = 1,\ h = 6.75,\ d = 1,\ k = 2$) and fixed initial conditions $(5.5, -1.25, 8.4, 2.75)$.

This paper proposes new 4D fractional-order hyperchaotic Rabinovich systems according to varied system parameters, initial conditions, and fractional-order derivatives. We also develop linear and nonlinear feedback controllers to address the problems of local and global asymptotic stability for the first system of these proposed systems.

The rest of this paper is organized as follows. Section 2 presents some preliminaries such as Caputo's fractional-order derivative and the theory of fractional-order nonlinear dynamical systems. Section 3 describes the proposed fractional-order hyperchaotic Rabinovich systems. Section 4 analyzes the stability and hyperchaos of these proposed systems. Section 5 investigates the local and global asymptotic stabilization problems for the first system of the proposed hyperchaotic systems by designing linear and nonlinear feedback controllers, respectively. In the end, the conclusions are drawn in Section 6.

2 Preliminaries

2.1 Caputo fractional derivative

Among several fractional-order derivative approaches considered in the literature, Caputo's approach has the feature that the initial conditions for fractional differential equations take on the same form as for integer-order differential equations. Due to this feature, the Caputo fractional derivative is perfect for use in real applications. The reader can refer to [11] for more details.

Definition 2.1 The α-th order Caputo fractional derivative of function $f(t)$ with respect to t and the terminal 0 is given by

$$ {}_0D_t^\alpha f = \frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau, $$

where m is an integer such that $m-1 \le \alpha < m$, and Γ is a well-known Gamma function.

2.2 Stability of the fractional-order nonlinear dynamical system

We define the Caputo fractional-order nonlinear dynamical system as follows:

$$ {}^C D^\alpha x_i = f_i(x_1, x_2, ..., x_n), \left(i = \overline{1,n}, 0 < \alpha < 1\right). \tag{1} $$

We obtain the equilibrium points of the system (1) via solving the system $f_i(x_1, x_2, ..., x_n) = 0, \left(i = \overline{1,n}, 0 < \alpha < 1\right)$.

To study the stability of the system (1), the following theorem is crucial.

Theorem 2.1 *[5] System* (1) *is asymptotically stable if all the eigenvalues* $\lambda_i, i = \overline{1,n}$, *of the Jacobian matrix* J *calculated at the equilibrium point of the system* (1) *satisfy the condition*

$$ |\arg(\lambda_i)| > \alpha.\frac{\pi}{2}, \left(i = \overline{1,n}, 0 < \alpha < 1\right), $$

where the Jacobian matrix is defined as

$$ J = \frac{\partial f}{\partial x},\ f = (f_1, f_2, ..., f_n)^T,\ x = (x_1, x_2, ..., x_n)^T. $$

Theorem 2.2 *[5] Let* $x(t) \in R^n$ *be a vector of continuous and differentiable function, then the inequality*
$\frac{1}{2}{}_a^C D_t^q x^T(t)\, x(t) \le x^T(t)\, D_t^q x(t)$ *holds.*

Theorem 2.3 *[5] The equilibrium point of system* (1) *is stable if for each* x, $x(t)^T\ f(x(t)) \le 0$, *and it is asymptotically stable if* $\forall x \ne 0,\ x(t)^T\ f(x(t)) < 0$.

3 New Fractional-Order Hyperchaotic Rabinovich Systems

He and Chen [4] proposed a 4D fractional-order hyperchaotic Rabinovich system as follows:

$$\begin{cases} D^{\alpha_1}x = hy - ax + yz, \\ D^{\alpha_2}y = hx - by - xz + w, \\ D^{\alpha_3}z = -dz + xy, \\ D^{\alpha_4}w = -ky, \end{cases} \tag{2}$$

where D^{α_i} denotes the derivatives of order α_i, $(0 < \alpha_i < 1, i = \overline{1,n})$ in the sense of Caputo, with fixed system parameter $(a, b, d, h, k) = (4, 1, 1, 6.75, 2)$.

The stability and hyperchaos of the system (2) with the fixed system parameter were analyzed by He and Chen in [4]. They showed that the origin is the only equilibrium point and it is unstable. Also, they demonstrated that there is a hyperchaotic behavior for the system (2) with varying fractional order α.

The authors propose new 4D fractional-order hyperchaotic Rabinovich systems according to different system parameters and different initial conditions.

Twenty 4D fractional-order hyperchaotic Rabinovich systems are introduced and listed in Table 1 as $FE_1 - FE_{20}$.

4 Stability and Hyperchaotic Behaviors of Proposed Systems

This section studies the dynamical behaviors of the new hyperchaotic proposed systems $FE_1 - FE_{20}$, including equilibria, stability, and hyperchaos.

4.1 Equlibria

In order to obtain the equlibrium points of systems $FE_1 - FE_{20}$, we must have

$$\begin{cases} hy - ax + yz \quad = 0, \\ hx - by - xz + w = 0, \\ -dz + xy = 0, \\ -ky = 0. \end{cases}$$

By calculation, we get that $\left(\overset{*}{x}, \overset{*}{y}, \overset{*}{z}, \overset{*}{w}\right) = (0, 0, 0, 0)$ is the only equilibrium point for all systems $FE_1 - FE_{20}$.

4.2 Stability

The Jacobian matrix of the system (2) is given by

$$J = \begin{bmatrix} -a & h+z & y & 0 \\ h-z & -b & -x & 1 \\ y & x & -d & 0 \\ 0 & -k & 0 & 0 \end{bmatrix}.$$

The characteristic equation $|J - \lambda I| = 0$ at the equilibrium point $(0, 0, 0, 0)$ is

$$(\lambda + d)\left[\lambda^3 + (a+b)\lambda^2 + \left(ab + k - h^2\right)\lambda + ak\right] = 0.$$

For all systems $FE_1 - FE_{20}$ (see Table 1), we have

$$\min_{1\leq i\leq 4} |\arg(\lambda_i)| = |\arg(\lambda_1)| = |\arg(\lambda_2)| = 0 < \alpha.\frac{\pi}{2} \qquad (0 < \alpha < 1).$$

According to Theorem 2.1, the equilibrium points $(0,0,0,0)$ are unstable for all systems $FE_1 - FE_{20}$.

4.3 Hyperchaos of the new proposed systems

We have used an exhaustive computer search by Matlab tools, considering combinations of the system parameters, fractional orders, and initial conditions to find the cases in which the Rabinovich system (2) shows hyperchaotic dynamics. Twenty simple examples $FE_1 - FE_{20}$ are found in this way.

All the cases have the origin as the only equilibrium point. For each case that is found, there are two positive Lyapunov exponents (≥ 0.001), which implies that this cases are hyperchaotic systems.

The Kaplan-Yorke dimension [6] is an estimate of the fractal dimension based on the spectrum of Lyapunov exponents of a chaotic system. Arrange the Lyapunov exponents in order from largest to smallest $LE_1 \geq LE_2 \geq ... \geq LE_n$, let j be the largest index for which $\sum_{i=1}^{j} LE_i \geq 0$ and $\sum_{i=1}^{j+1} LE_i < 0$, then the Kaplan-Yorke (Lyapunov) dimension is defined as

$$D_{KY} = j + \frac{1}{|LE_{j+1}|}\sum_{i=1}^{j} LE_i.$$

The Lyapunov spectra (which are calculated by Wolf's method [13]) and Kaplan-York dimensions are listed in Table 1 along with system parameters, fractional orders, and initial conditions.

The attractor dimensions of all cases $FE_1 - FE_{20}$ are just over 3.0, and the largest of them is FE_{16} with $D_{KY} = 3.0847$. In addition, Lyapunov exponents according to the time of some proposed systems are shown in Fig.1.

The projections of the strange attractors in the $xyz-$space for the cases $FE_1 - FE_{20}$ are shown in Fig.2.

Case	Paremeters	Eigenvalues	(x_0, y_0, z_0, w_0)	Fractional order	LEs	D_{KY}
FE_1	$a = 4, b = 1, d = 1,$ $h = 6.75, k = 2$	$\lambda_1 = 4.1236$ $\lambda_2 = 0.2079$ $\lambda_3 = -9.3315$ $\lambda_4 = -1$	$(0.1, 0, 1, 0.1)$	0.9	0.4843 0.01015 -0.000 -9.7903	3.0505
FE_2				0.85	0.5784 0.1133 0.0009 -12.0538	3.0574
FE_3				0.7	0.7732 0.0849 -0.0019 -21.8128	3.0392
FE_4	$a = 4, b = 1, d = 1,$ $h = 6, k = 3$	$\lambda_1 = 3.1019$ $\lambda_2 = 0.4522$ $\lambda_3 = -8.5541$ $\lambda_4 = -1$	$(1, 0, 1, 1)$	0.99	0.2352 0.0237 -0.0014 -6.5205	3.0395
FE_5				0.95	0.3149 0.0125 0.000 -7.7631	3.0422
FE_6				0.9	0.3566 0.0033 -0.0061 -9.5511	3.0370
FE_7	$a = 2, b = 1, d = 1,$ $h = 5, k = 2$	$\lambda_1 = 3.2577$ $\lambda_2 = 0.0966$ $\lambda_3 = -6.3543$ $\lambda_4 = -1$	$(-0.5, 0.5, 1, 0)$	0.97	0.2331 0.0025 -0.0588 -4.7335	3.0373

Table1. New 4D fractional−order hyperchaotic Rabinovich systems.

Case	Paremeters	Eigenvalues	(x_0, y_0, z_0, w_0)	Fractional order	LEs	D_{KY}
FE_8	$a = 4, b = 1, d = 1,$ $h = 5, k = 1$	$\lambda_1 = 2.4624$ $\lambda_2 = 0.2117$ $\lambda_3 = -7.6741$ $\lambda_4 = -1$	$(0.1, 0, 0.1, 1)$	0.99	0.1511 0.0594 0.0021 −6.4748	3.0328
FE_9				0.95	0.1827 0.0524 0.0025 −7.6661	3.0310
FE_{10}				0.9	0.2173 0.0683 0.0014 −9.4593	3.0303
FE_{11}				0.75	0.3198 0.0546 0.0006 −17.2426	3.0217
FE_{12}	$a = 4, b = 1, d = 2,$ $h = 7, k = 2$	$\lambda_1 = 4.3867$ $\lambda_2 = 0.1904$ $\lambda_3 = -9.5771$ $\lambda_4 = -2$	$(0.1, 0.1, 1, 0.1)$	0.99	0.3367 0.0269 −0.0006 −7.6700	3.0473
FE_{13}				0.97	0.3784 0.0285 −0.0000 −8.3725	3.0486
FE_{14}				0.95	0.4129 0.0285 −0.000 −9.1221	3.0484
FE_{15}				0.85	0.6263 0.0054 −0.0003 −13.8996	3.0454

Table 1. (**Continued**.)

Case	Paremeters	Eigenvalues	(x_0, y_0, z_0, w_0)	Fractional order	LEs	$\mathrm{D_{KY}}$
$\mathrm{FE_{16}}$	$a = 4, b = 1, d = 1,$ $h = 8, k = 1$	$\lambda_1 = 5.5329$ $\lambda_2 = 0.0682$ $\lambda_3 = -10.6011$ $\lambda_4 = -1$	$(0.1, 0, 1, 0.5)$	0.99	0.5358 0.0423 0.0013 -6.8434	3.0847
$\mathrm{FE_{17}}$				0.95	0.6251 0.0530 -0.0016 -8.1228	3.0833
$\mathrm{FE_{18}}$				9	0.7695 0.0639 -0.0012 -10.0625	3.0827
$\mathrm{FE_{19}}$				0.8	1.1437 0.0692 -0.0001 15.3143	3.0792
$\mathrm{FE_{20}}$				0.7	0.6733 0.0109 -0.0490 -22.38	3.0284

Table 1. (**Continued**.)

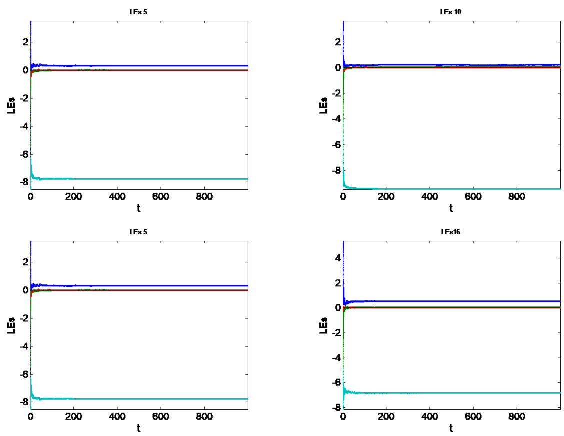

Figure 1: Lyapunov exponents with respect to time for some proposed 4D fractional-order hyperchaotic systems.

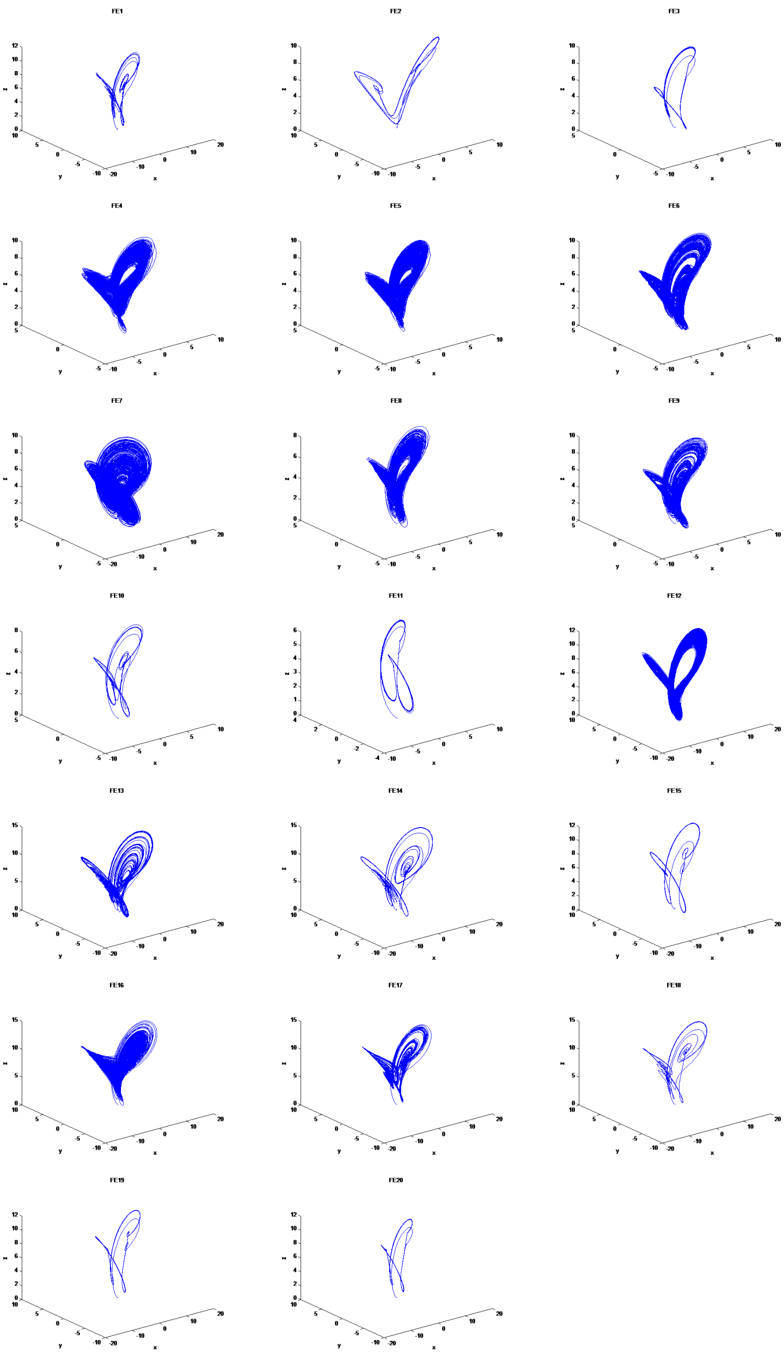

Figure 2: Attractors for the proposed 4D fractional order Rabinovich systems in the xyz-plane with initial conditions given in Table 1.

5 Chaotic Control of the Fractional-Order Rabinovich System FE_1

5.1 Linear feedback control

In this section, we study the chaotic control of system FE_1 by using the linear feedback control.

The linear controlled fractional-order Rabinovich system is as follows:

$$\begin{aligned} {}^c_aD^\beta_t x &= hy - ax + yz - k_1(x - x^*), \\ {}^c_aD^\beta_t y &= hx - by - xz + w - k_2(y - y^*), \\ {}^c_aD^\beta_t z &= -dz + xy - k_3(z - z^*), \\ {}^c_aD^\beta_t w &= -ky - k_4(w - w^*), \end{aligned} \tag{3}$$

with the fractional order $\beta = 0.9$, and the parameters $a = 4$, $b = 1$, $h = 6.75$, $d = 1$, $k = 2$. $(x^*, y^*, z^*, w^*) = (0; 0; 0; 0)$ is the equilibrium point of system FE_1 and k_1, k_2, k_3, k_4 are four linear control parameters.

The Jacobian matrix J of system (3) at the equilibrium point (x^*, y^*, z^*, w^*) is

$$J\left(x^*, y^*, z^*, w^*\right) = \begin{bmatrix} -a - k_1 & h + z^* & y^* & 0 \\ h - z^* & -b - k_2 & -x^* & 1 \\ y^* & x^* & -d - k_3 & 0 \\ 0 & -k & 0 & -k_4 \end{bmatrix}$$

and the corresponding characteristic equation for the parameters $a = 4$, $b = 1$, $h = 6.75$, $d = 1$, $k = 2$ is

$$f(\lambda) = \begin{vmatrix} \lambda + 4 + k_1 & -6.75 & 0 & 0 \\ -6.75 & \lambda + 1 + k_2 & 0 & -1 \\ 0 & 0 & \lambda + d + k_3 & 0 \\ 0 & 2 & 0 & \lambda + k_4 \end{vmatrix} = 0. \tag{4}$$

We selected the control parameters k_1, k_2, k_3 and k_4 in such a way that all eigenvalus λ of equation (4) should satisfy the condition $|\arg(\lambda_i)| > \pi > \beta.\pi/2$ to stabilize system (2).

5.1.1 Stabilization of the equilibrium point E

Lemma 5.1 *System* (3) *is locally asymptotically stable at the equilibrium point* $(0; 0; 0; 0)$ *with the control parameters* $k_1 = 2;\ k_2 = 10;\ k_3 = 1;\ k_4 = 10$.

Proof. Substitute the value of $k_1;\ k_2;\ k_3$ and k_4 into the characteristic equation (5), we can get

$$f(t) = \begin{vmatrix} \lambda + 6 & -6.75 & 0 & 0 \\ -6.75 & \lambda + 11 & 0 & -1 \\ 0 & 0 & \lambda + 2 & 0 \\ 0 & 2 & 0 & \lambda + 10 \end{vmatrix} \tag{5}$$
$$= \lambda^4 + 29\lambda^3 + 246.4375\lambda^2 + 601.25\lambda + 432.75 = 0,$$

and the characteristic roots of equation (5) are $\lambda_1 \simeq -15.4531$, $\lambda_2 \simeq -10.1701$, $\lambda_3 \simeq -2$, $\lambda_4 \simeq -1.3767$.

By Theorem 2.1, $|\arg(\lambda_i)| = \pi > (0.9).\pi/2$, $i = 1, 2, 3, 4$, so the controlled fractional-order Rabinovich system (3) is locally asymptotically stable at $(0; 0; 0; 0)$.

As depicted in Fig.3(a), the controllers are activated at $t = 20s$. The fractional order Rabinovich system FE_1 is hyperchaotic when $t < 20s$, and after activating the controllers ($t \geq 20$), the state $x(t)$, $y(t)$, $z(t)$, and $w(t)$ approaches the equilibium point gradually; when $t = 25$, the system is controled entirely to the equilibrium point $(x^*, y^*, z^*, w^*) = (0; 0; 0; 0)$.

If the controllers are activated at $t = 30$, Fig.3(b) shows that the fractional-order Rabinovich system FE_1 is hyperchaotic when $t < 30s$, and it is controlled entirely to the equilibrium point when $t = 35$.

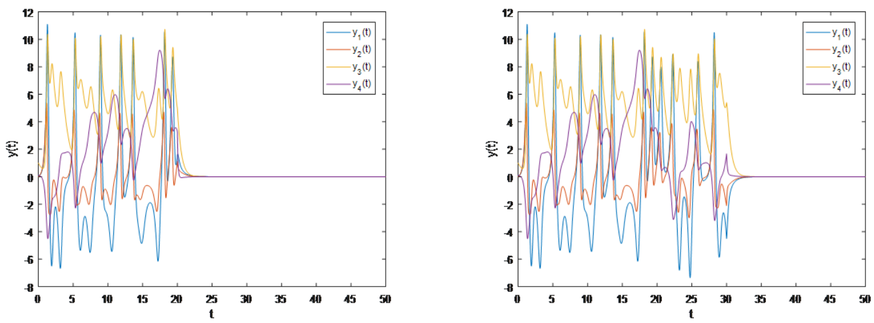

Figure 3: Stabilization of the fractional-order hyperchaotic Rabinovich system FE_1 for $k_1 = 2$, $k_2 = 10$, $k_3 = 1$ and $k_4 = 10$.

5.2 Nonlinear feedback control

By using the nonlinear feedback control method, we will enable the system (2) to achieve the global asymptotical stability.

The nonlinear controlled fractional-order Rabinovich system is

$$\begin{aligned} {}_a^cD_t^\alpha x &= hy - ax + yz - U_1, \\ {}_a^cD_t^\alpha y &= hx - by - xz + w + U_2, \\ {}_a^cD_t^\alpha z &= -dz + xy + U_3, \\ {}_a^cD_t^\alpha w &= -ky + U_4, \end{aligned} \tag{6}$$

where U_1; U_2; U_3; U_4 are the nonlinear controllers to be designed later.

Lemma 5.2 *System* (2) *can achieve the global asymptotical stability under the controllers*

$$U_1 = -k_1x; \; U_2 = -k_2y; \; U_3 = -k_3 - z - xy; \; U_4 = -k_4w - y \tag{7}$$

if and only if the conditions $k_1 > h^2 - a$; $k_2 > \frac{3}{2} - b$; $k_3 > -d$; $k_4 > \frac{k^2}{2}$ *hold, where* k_1; k_2; k_3; k_4 *are control parameters.*

Proof. Choosing a positive definite Lyapunov function

$$V = \frac{1}{2}\left(x^2 + y^2 + z^2 + w^2\right) \tag{8}$$

and applying Theorem 2.2 and the controllers (7), the fractional derivative of the Lyapunov function (8) of the nonlinear controlled system (6) is obtained as

$$\begin{aligned}
{}_a^c D_t^q V &\leq {}_a^c D_t^{\alpha 1} x + {}_a^c D_t^{\alpha 2} y + {}_a^c D_t^{\alpha 3} z + {}_a^c D_t^{\alpha 4} w \\
&= x\left(hy - ax + yz - U_1\right) + y\left(hx - by - xz + w + U_2\right) \\
&+ z\left(-dz + xy + U_3\right) + w\left(-ky + U_4\right) \\
&= x\left(hy - ax + yz - k_1 x_1\right) + y\left(hx - by - xz + w - k_2 y\right) \\
&+ z\left(-dz + xy - k_3 z - xy\right) + w\left(-ky - k_4 w - y\right) \\
&= -\left(hx - y\right)^2 - \left(a - h^2 + k_1\right) x^2 - \left(b - \frac{3}{2} + k_2\right) y^2 \\
&- \left(d + k_3\right) z^2 - \frac{1}{2}\left(y + kw\right)^2 - \frac{1}{2}\left(-k^2 + 2k_4\right) w^2,
\end{aligned}$$

where $k_1 > h^2 - a;\ k_2 > \frac{3}{2} - b;\ k_3 > -d;\ k_4 > \frac{k^2}{2}$ hold, the fractional derivative of the Lyapunov function is negative definite (${}_a^c D_t^q V < 0$), according to Theorem 2.3, so the controlled system (6) is globaly asymptotically stable.

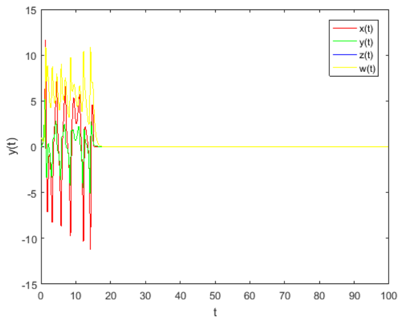

Figure 4: Stabilization of the fractional-order hyperchaotic Rabinovich system FE_1 for $k_1 = 45$, $k_2 = 1$, $k_3 = 0$ and $k_4 = 10$.

Fig.4 shows that with the parameters $a = 4$, $b = 1$, $h = 6.75$, $d = 1$, $k = 2$, $q = 0.9$ and the control parameters $k_1 = 45,\ k_2 = 1,\ k_3 = 0,\ k_4 = 10$, the Rabinovich system FE_1 is hyperchaotic if $t < 20$. After activating the controlers at $t = 20$, the system achieve global stability gradually.

6 Conclusion

The paper proposed 20 new four-dimensional fractional-order hyperchaotic Rabinovich systems with Caputo derivatives. Each system possessed a single equilibrium point located at the origin, and the stability at this point was analyzed. Based on the computation of Lyapunov exponents, it was demonstrated that the proposed systems exhibited hyperchaotic behavior under certain combinations of system parameters, initial conditions, and fractional-order derivatives. The attractors, their dimensions, and the Lyapunov exponents of selected systems were presented. Furthermore, the local and global asymptotic stabilization problems for the first of the proposed hyperchaotic systems were addressed through the design of linear and nonlinear feedback controllers, respectively.

This study contributes to the development of theoretical research in chaos and control theory by introducing novel fractional-order hyperchaotic Rabinovich systems and proposing an effective control strategy for system stabilization. Additionally, the results suggest potential practical applications in areas such as secure communication systems, modeling of memory-dependent materials (e.g., viscoelastic materials), and improved modeling of anomalous diffusion processes in plasma dynamics. Future research may further explore the dynamical properties and behaviors of these newly proposed hyperchaotic systems

References

[1] D. Baleanu, S. S. Sajjadi, J. H. Asad, A. Jajarmi and E. Estiri. Hyperchaotic behaviours, optimal control, and synchronization of a nonautonomous cardiac conduction system. *Advances in Difference Equation* **2021** (1) (2021) 1–24.

[2] C. T. Deressa. On the chaotic nature of the Rabinovich system through Caputo and Atangana–Baleanu–Caputo fractional derivatives. *Advances in Continuous and Discrete Models* **2022** (1) (2022), article number 66.

[3] Y. Ghettout, L. Meddour, T. Hamaizia, and R. Ouahabi. Dynamic Analysis of a New Hyperchaotic System with Infinite Equilibria and Its Synchronization. *Nonlinear Dynamics and Systems Theory* **24**(2) (2024) 147–158.

[4] J. He and F. Chen. A new fractional order hyperchaotic Rabinovich system and its dynamical behaviors. International Journal of Non-Linear Mechanics **95** (2017) 73-81.

[5] J. He and F. Chen. Dynamical analysis of a new fractional-order Rabinovich system and its fractional matrix projective synchronization. *Chinese Journal of Physics* **56** (5) (2018) 2627–2637.

[6] J. Kaplan and J. York. Functional differential equations and approximations of fixed points. Lecture Notes in Mathematics **730** (1979) 204–227.

[7] Y. J. Liu, Q. G. Yang and G. P. Pang. A hyperchaotic system from the Rabinovich system. Journal of Computational and Applied Mathematics **234** (1) (2010) 101–113.

[8] E. Mosekilde. Topics in Nonlinear Dynamics: Applications to Physics, Biology and Economic Systems. World Scientific, Singapore, (1996).

[9] S. Patnaik, J. P. Hollkamp and F. Semperlotti. Applications of variable-order fractional operators: a review. *Proceedings of the Royal Society A* **476** (2234) (2020) 20190498.

[10] A. S. Pikovski, M. I. Rabinovich, and V. Y. Trakhtengerts. On set of stochasticity in decay confinement of parametric instability. *Soviet Journal of Experimental and Theoretical Physics* **47** (4) (1978) 715–719.

[11] I. Podlubny. *Fractional Differential Equations.* Academic Press, San Diego, 1999.

[12] M. S. Tavazoei. Fractional order chaotic systems: history, achievements, applications, and future challenges. *The European Physical Journal Special Topics* **229** (2020) 887–904.

[13] A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastano. Determining Lyapunov exponents from a time series. *Physica* **16** (3) (1985) 285–317.

[14] F. Yu, S. Xu, Y. Lin, T. He, C. Wu and H. Lin. Design and analysis of a novel fractional-order system with hidden dynamics, hyperchaotic behavior and multi-scroll attractors. *Mathematics* **12** (14) (2024) 1–22.

[15] G. Zhao, H. Zhao, Y. Zhang and X. An. A new memristive system with extreme multistability and hidden chaotic attractors and with application to image encryption. *International Journal of Bifurcation and Chaos* **34** (01) (2024) 2450010.