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Advanced Kalman Filter Implementation for Estimating Yaw Coefficient in Amphibious Plane Motion Experiments

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Abstract: Indonesia is a maritime country with a larger ocean area than land. This issue must be addressed with proper supporting transportation that serves as both a method of mobilization and defence. Amphibious planes are versatile modes of transportation that are suitable for usage in Indonesian coastal areas. Amphibious aircraft are equipped with a navigation system and technology to move both in the air and on water. The amphibious aircraft's mobility is also engineered to allow it to take off and land on its intended trajectory. Several approaches have been developed to calculate the amphibious aircraft's trajectory. These approaches are continuously refined to achieve the desired level of accuracy. The commonly used estimation calculation methods are the Ensemble Kalman Filter and the Kalman Filter. The Ensemble Kalman Filter method is a development of the Kalman Filter which can be used to estimate linear and non-linear system models. The Kalman Filter is the forerunner of the Ensemble Kalman Filter method which can only be used to estimate linear dynamic system models. The Ensemble Kalman Filter method successfully obtained the best prediction error value with an RMSE value of 0.0876 with a total of 400 ensembles. Meanwhile, the Kalman Filter method successfully obtained the best prediction error value with an RMSE value of 0.000237.

Keywords: *amphibious plane; trajectory estimation; ensemble Kalman filter; Kalman filter.*

Mathematics Subject Classification (2020): 93E11, 93C41, 60G35.

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1 Introduction

Indonesia is the biggest country in Southeast Asia region. In addition, Indonesia is also the world's largest archipelagic country, with a vast maritime area [1, 2]. This circumstance positioned the Indonesian sea as the gateway to the Indo-Pacific region, a strategic fishing area, a hub for trade and commerce, and the largest country in Southeast Asia [2]. Given the scattered nature of the islands, many of which are isolated by sea, a reliable and multifunctional transportation mode is required. Furthermore, it can be used to help the country maintain its sovereignty.

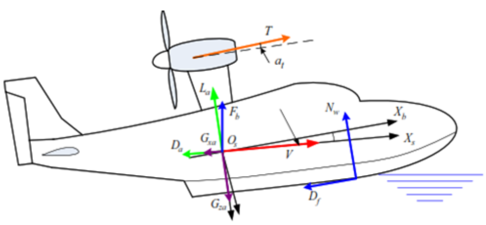
Amphibious planes are one type of supporting mode of transportation that may operate on both water and land. Amphibious planes are mostly employed as support vehicles for search and rescue operations, delivering products to isolated locations and underdeveloped areas, assisting with firefighting efforts, and so on [3]. Several experiments and research are being conducted to optimize the design and performance of amphibious aircraft. The main goal is to reduce the resistance force to allow for a faster takeoff with less engine power, resulting in lower fuel consumption [4].

Due to the variety of functions, careful consideration must be given to the design planning and acceleration calculations during takeoff and landing. Moreover, vertical acceleration is a more significant factor, which not only causes mental problems for the crew, but can cause structural damage to the fuselage when touching water. More attention should be given to the load characteristics of seaplanes when landing on water [5]. Therefore, for amphibious aircraft to move in accordance with their trajectory, sophisticated navigation technology and guidance systems are required.

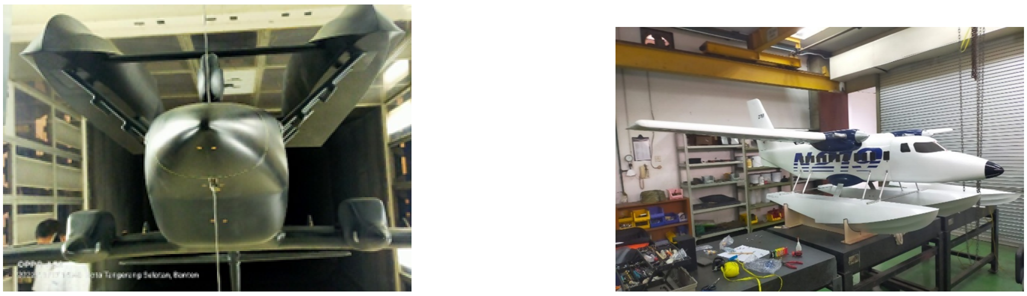
The calibration in this study was conducted using the Kalman Filter Ensemble (EnKF) and Kalman Filter techniques. The Kalman Filter Ensemble approach (EnKF) is a modified estimation approach of the Kalman Filter algorithm that may be used to estimate linear and nonlinear system models by producing a number of ensembles for their error co-variants during the prediction stage [6]. The Kalman Filter is a widely used estimating tool in control and navigation systems [8]. In the previous research, the Ensemble Kalman Filter (EnKF) approach was used to estimate the position of autonomous vehicles [9]. Meanwhile, the Kalman Filter (KF) approach is used to determine the orientation and translation of UAV motions [10].

2 Data and Mathematical Modeling

The forces acting on unmanned seaplanes can be divided into four categories: aircraft mass, hydrodynamic force, aerodynamic force, and engine thrust, as seen in Figure 1. The following equation represents a nonlinear unmanned seaplane's longitudinal dynamic motion.



(a)



(b) (c)

Figure 1: (a) and (b) The aerodynamic model of amphibious aircraft during the wind tunnel test [12].

It employs the Earth coordinate system $X_e; Y_e; Z_e$, then plane body coordinate system $X_b; Y_b; Z_b$ and the steady-translation coordinate system $X_s; Y_s; Z_s$.

$$\begin{aligned}
m\dot{V} &= T\cos(\alpha + \alpha_t) - D_a - N_w \sin\alpha - D_f \cos\alpha + G_{xa},\\
mV\dot{\alpha} &= mVq - T\sin(\alpha + \alpha_t) - L_a - N_w \cos\alpha + D_f \sin\alpha + G_{za},\\
I_y\dot{q} &= M_a + M_w + M_T,\\
\dot{\theta} &= q,\\
\dot{x}_g &= u\cos\theta + w\sin\theta,\\
\dot{z}_g &= -u\sin\theta + w\cos\theta.
\end{aligned}$$

The dataset used in this study is based on the research findings (primary data). The data in this study was analyzed using the Python programming language with a dataset similar to that in Table 2 below.

Symbol	Description	Symbol	Description
A	Angle of attack	m	Seaplane mass
Ø	Angle of Pitch	D_a	Aerodynamic drag force
V	Speed	G_{xa}	gravity along Xs
α_t	Angle between engine force to Xb	G_{za}	gravity along Zs
M_a	Total pitching moments from the air	X_g	position of aircraft along the Xe
M_w	Total pitching moments from the air	Z_g	aircraft positions along Ze
M_T	Total Pitching Moments from the Engine	u	aircraft speed components along Xb
T	Engine thrust	w	aircraft speed components along Zb
N_w	Normal directional water pressure at the bottom of the aircraft	q	Angular rate pitch (increase/ decrease of pitch angle
D_f	Water friction along the bottom of the aircraft	I_y	Seaplane moment of inertia against Yb
L_a	Aerodynamic lift force	$\dot{\theta}$	Increase/decrease of the angle of pitch
$\dot{V}$	Speed gain/decrease	$\dot{\alpha}$	Increase/decrease of the angle of attack

Table 1: Description of the amphibious aircraft model parameters.

2.1 Mathematical model of Ensemble Kalman Filter (EnKF)

$$x_{k+1} = f(k, x_k) + w_k, \tag{1}$$

$$z_k = Hx_k + v_k, \tag{2}$$

$$x_0 \sim N(\bar{x}_0, P_{x_0}),\ w_k \sim N(0, Q_k),\ v_k \sim N(0, R_k), \tag{3}$$

$$X_{0,i} = [x_{0,1}\ \ x_{0,2}\ \ x_{0,3}\ \ \cdots\ \ x_{0,N_e}], \tag{4}$$

$$\hat{x}_k^* = \frac{1}{N_e}\sum_{i=1}^{N_e} x_{k,i}, \tag{5}$$

$$P_k = \frac{1}{N_e - 1}\sum_{i=1}^{N_e}(\hat{x}_{k,i} - \hat{x}_k)(\hat{x}_{k,i} - \hat{x}_k)^T. \tag{6}$$

RUN	ALFA	BETA	CL	CY
13	-10,4	0	-0,567133333	-0,005233333
13	-9,393333333	0	-0,5438	-0,003833333
13	-8,35	0	-0,498733333	-0,005666667
13	-7,303333333	0	-0,440433333	-0,006466667
13	-6,24	0	-0,3466	-0,006866667
13	-5,18	0	-0,2584	-0,0046
13	-4,1	0	-0,150066667	-0,004233333
13	-3,03	0	-0,034	-0,003533333
13	-1,93	0	0,0798	-0,0025
13	-0,86	0	0,197133333	-0,0018
13	0,22	0	0,315566667	-0,001566667
13	1,29	0	0,438533333	-0,002833333
13	2,39	0	0,560533333	-0,0028
13	3,466666667	0	0,6812	-0,0023
13	4,56	0	0,802433333	-0,0015
13	5,65	0	0,924866667	-0,0012
13	6,73	0	1,0427	-0,001633333
13	7,81	0	1,160233333	-0,0017
13	8,89	0	1,279533333	-0,001366667
13	9,96	0	1,3949	-0,001366667
13	11,35	0	1,537083333	-0,000583333
13	12,395	0	1,636716667	-0,000116667
13	13,465	0	1,732683333	0,000416667
...	...	...	...	...
20	16,345	0	2,284183	-0,025583333

Table 2: Dataset.

2.2 Mathematical model of Kalman Filter (KF)

Kalman Filter (KF) is a method for solving state estimation problems. This method is commonly known as the Linear Quadratic Estimator (LQE) since it minimizes the quadratic function of estimate error in a linear dynamic system with white measurement and disturbance noise [8]. The Kalman Filter technique has the following mathematical function.

A discrete linear stochastic dynamic system is generally given in the form

$$x_{k+1} = A_k x_k + B_k u_k + G_k w_k, \tag{7}$$

$$z_k = H_k x_k + v_k, \tag{8}$$

$$x_0 \sim N(\bar{x}_0, P_{x_0}),\ w_k \sim N(0, Q_k),\ v_k \sim N(0, R_k), \tag{9}$$

where: x_0 : initialization of the system,
x_{k+1} : state variable at time $k+1$ and dimension $n \times 1$,
x_k : state variable at time k with initial estimated value $\bar{x}_0$ and early covariants P_{x_0}, $x_k \in \mathbb{R}^n$,
u_k : deterministic input vector at time k, $u_k \in \mathbb{R}^m$,

w_k : noise on the system with mean $\bar{w}_k = 0$ and covariants Q_k,
z_k : measurement variables, $z_k \in \mathbb{R}^p$,
v_k : noise on the measurement with mean $\bar{v}_k = 0$ covariants R_k,
A_k, B_k, G_k, H_k: the matrices with the values of their elements being the coefficients of their respective variables.

In the Kalman Filter, estimation is carried out in two stages: the prediction stage (time update) predicts state variables based on a dynamic system, and the correction stage (measurement update) corrects the measurement data to improve estimation results.

The prediction stage is influenced by the dynamics of the system by predicting state variables using the state variable estimation equation and the accuracy level is calculated using the error covariance equation. In the correction stage, the results of the estimation of the state variables obtained at the prediction stage are corrected using a measurement model. One part of this stage is to determine the Kalman Gain matrix that is used to minimize error covariance. The prediction and correction stages are carried out recursively by minimizing the covariance of estimation errors $(x_k - x\hat{k})$, x_k is a variable of actual circumstances and $x\hat{k}$ is an estimate of the variables of the situation.

3 Simulation Results

From the results of simulation using the Ensemble Kalman Filter method (EnKF) and Kalman Filter (KF), the visualization results in the form of prediction plot are shown in Figure 2 below.

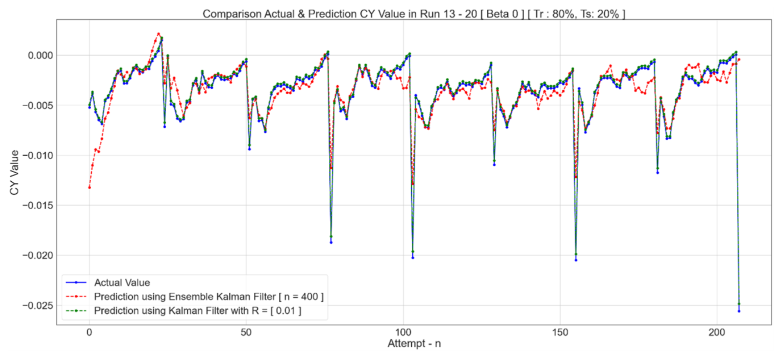


Figure 2: CY Value Prediction Plot using the EnKF method of 400 ensembles with observation models of 0.8 and KF.

The first simulation was carried out using the Kalman Filter Ensemble (EnKF) method with a total of 400 ensembles and Kalman Filter. At this stage, the observation model of the Ensemble Kalman Filter (EnKF) is set with a value of 0.8 with a total of 400 ensembles. The prediction results of the Ensemble Kalman Filter (EnKF) method produced an RMSE value of 0.0876. Then, for the Kalman Filter method with an R value of 0.01, it produced an RMSE value of 0.00023. The plot visualization of the Ensemble Kalman Filter (EnKF) method, indicated by the red line, looks close to the actual value with dynamic conditions. As for the plot visualization of the Kalman Filter method indicated by the green line, it looks close to the actual value with a small margin of error.

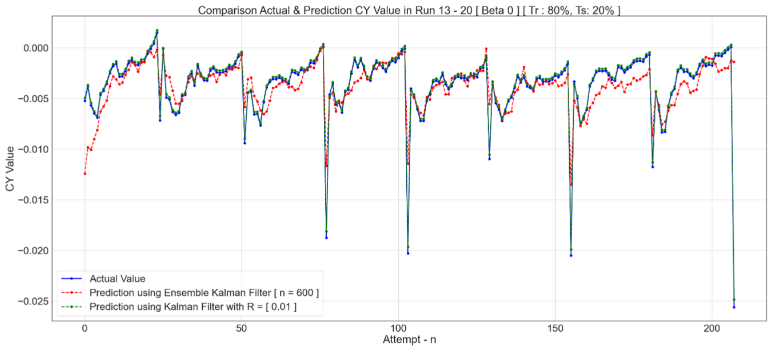


Figure 3: CY Value Prediction Plot using the EnKF method of 600 ensembles with observation models of 0.8 and KF.

The second simulation was carried out using the Kalman Filter Ensemble (EnKF) method with a total of 600 ensembles and Kalman Filter. At this stage, the observation model of the Kalman Filter Ensemble (EnKF) is set with a value of 0.8 with a total of 600 ensembles. The prediction results of the Ensemble Kalman Filter (EnKF) method produced an RMSE value of 0.0865. Then, for the Kalman Filter method with an R value of 0.01, it produced an RMSE value of 0.00023. The plot visualization of the Ensemble Kalman Filter (EnKF) method, indicated by the red line, looks close to the actual value with dynamic conditions. As for the plot visualization of the Kalman Filter method shown by the green line, it looks close to the actual value with a small margin of error. In this second simulation, the RMSE value of the Ensemble Kalman Filter (EnKF) method decreased by 0.0011.

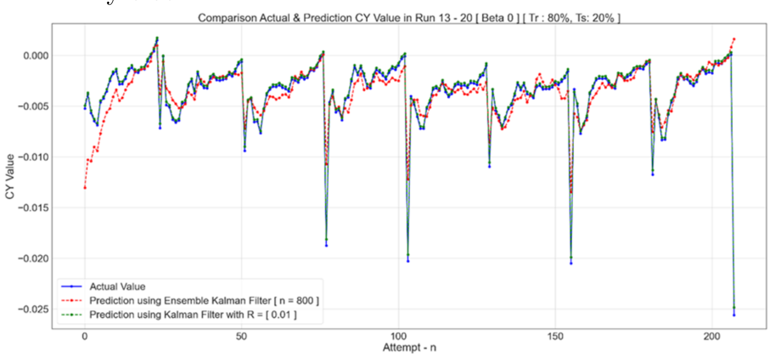


Figure 4: CY Value Prediction Plot using the EnKF method of 800 ensembles with observation models of 0.8 and KF.

The third simulation was carried out using the Kalman Filter Ensemble (EnKF) method with a total of 800 ensembles and Kalman Filter. At this stage, the observation model of the Ensemble Kalman Filter (EnKF) is set with a value of 0.8 with a total of 800 ensembles. The prediction results of the Ensemble Kalman Filter (EnKF) method produced an RMSE value of 0.0909. Then, for the Kalman Filter method with an R value of 0.01, it produced an RMSE value of 0.00023. The plot visualization of the Ensemble Kalman Filter (EnKF) method, indicated by the red line, looks close to the actual value with dynamic conditions. As for the plot visualization of the Kalman Filter method shown by the green line, it looks close to the actual value with a small margin of error. In this third simulation, the RMSE value of the Ensemble Kalman Filter (EnKF) method increased by 0.0044.

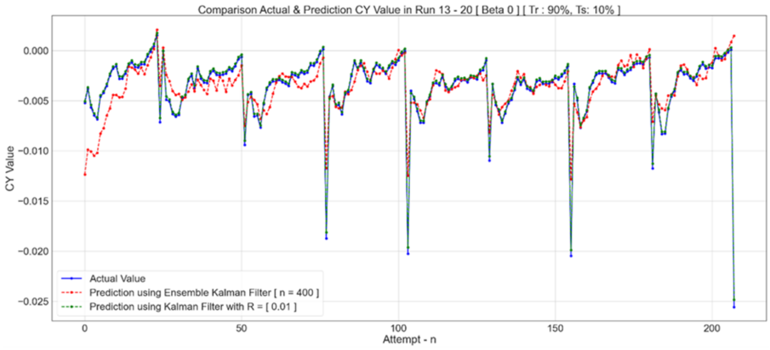


Figure 5: CY Value Prediction Plot using the EnKF method of 400 ensembles with observation models of 0.9 and KF.

The fourth simulation was carried out using the Kalman Filter Ensemble (EnKF) method with a total of 400 ensembles and Kalman Filter. At this stage, the observation model of the Kalman Filter Ensemble (EnKF) is set with a value of 0.9 with a total of 400 ensembles. The prediction results of the Ensemble Kalman Filter (EnKF) method produced an RMSE value of 0.0927. Then, for the Kalman Filter method with an R value of 0.01, it produced an RMSE value of 0.00023. The plot visualization of the Ensemble Kalman Filter (EnKF) method, indicated by the red line, looks close to the actual value with dynamic conditions. As for the plot visualization of the Kalman Filter method shown by the green line, it looks close to the actual value with a small margin of error. In this fourth simulation, the Kalman Filter method did not experience an increase or decrease in the error value (RMSE).

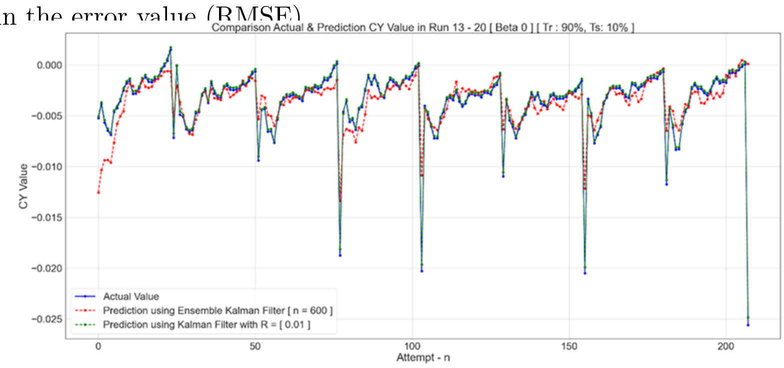


Figure 6: CY Value Prediction Plot using the EnKF method of 600 ensembles with observation models of 0.9 and KF.

The fifth simulation was carried out using the Kalman Filter Ensemble (EnKF) method with a total of 600 ensembles and Kalman Filter. At this stage, the observation model of the Kalman Filter Ensemble (EnKF) is set with a value of 0.9 with a total of 600 ensembles. The prediction results of the Ensemble Kalman Filter (EnKF) method produced an RMSE value of 0.0889. Then, for the Kalman Filter method with an R value of 0.01, it produced an RMSE value of 0.00023. The plot visualization of the Ensemble Kalman Filter (EnKF) method, indicated by the red line, looks close to the actual value with dynamic conditions. As for the plot visualization of the Kalman Filter method shown by the green line, it looks close to the actual value with a small margin of error. In this fourth simulation, the Kalman Filter method did not experience an increase or decrease in the error value, and the Kalman Filter Ensemble method experienced a decrease in the error value (RMSE) of 0.0038.

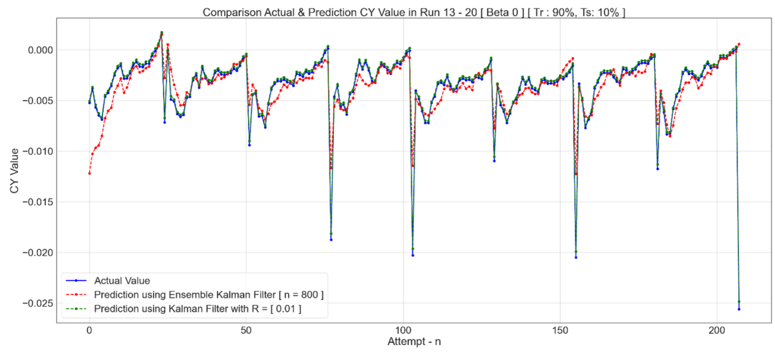


Figure 7: CY Value Prediction Plot using the EnKF method of 800 ensembles with observation models of 0.9 and KF.

The sixth simulation was carried out using the Kalman Filter Ensemble (EnKF) method with a total of 800 ensembles and Kalman Filter. At this stage, the observation model of the Kalman Filter Ensemble (EnKF) is set with a value of 0.9 with a total of 600 ensembles. The prediction results of the Ensemble Kalman Filter (EnKF) method produced an RMSE value of 0.0884. Then, for the Kalman Filter method with an R value of 0.01, it produced an RMSE value of 0.00023. The plot visualization of the Ensemble Kalman Filter (EnKF) method, indicated by the red line, looks close to the actual value with dynamic conditions. As for the plot visualization of the Kalman Filter method shown by the green line, it looks close to the actual value with a small margin of error. In this fourth simulation, the Kalman Filter method did not experience an increase or decrease in error value, and the Kalman Filter Ensemble method experienced a decrease in error value (RMSE) of 0.0005.

The simulation results of the Ensemble Kalman Filter (EnKF) method and the Kalman Filter method can be seen in Table 3 below.

Nilai Observation Model	Jumlah Nilai RMSE	Nilai RMSE Ensemble Kalman Filter	Nilai RMSE Kalman Filter
0,8	400	0,0876	0,00023
	600	0,0865	
	800	0,0909	
0,9	400	0,0927	
	600	0,0889	
	800	0,0884	

Table 3: Comparison of RMSE values.

Table 3 displays the entire series of simulations using the Ensemble Kalman Filter (EnKF) and Kalman Filter methods. The dynamic simulation by the Ensemble Kalman Filter (EnKF) method showed the best simulation results in the second simulation stage with an RMSE value of 0.0865. Meanwhile, the simulation by the Kalman Filter method showed constant results from the first simulation to the end.

4 Conclusion

Based on the simulation results, it can be concluded that the second simulation, which used the Ensemble Kalman Filter (EnKF) method, yielded the best prediction error (RMSE) of 0.0865. Furthermore, using the Kalman Filter method with R = 0.01 yielded the best prediction error (RMSE) of 0.00023 across all simulations. The results show that the Ensemble Kalman Filter (EnKF) and Kalman Filter methods produce good predictions with a 1% error rate and are consistent, thus they can be recommended for further research.

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