## Optimal Control of Nonlinear Uncertain Systems over an Infinite Horizon via Finite-Horizon Approximations

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Abstract: It is well-known that the Hamilton-Jacobi-Isaacs (HJI) equation associated with a nonlinear  $H^{\infty}$ -optimal control problem on an infinite-time horizon generally admits nonunique, and in fact infinitely many, viscosity solutions. This makes it difficult to pick the relevant viscosity solution for the problem at hand, particularly when it is computed numerically. For the finitehorizon version of the problem, however, there is generally a unique viscosity solution (under appropriate conditions), which brings up the question of obtaining the viscosity solution relevant to the infinite-horizon problem as the limit of the unique solution of the finite-horizon one. This paper addresses this question for nonlinear systems affine in the control and the disturbance, and with a cost function quadratic in the control, where the control is not restricted to lie in a compact set. It establishes the existence of a well-defined limit, and also obtains a result on global asymptotic stability of closed-loop system under the  $H^{\infty}$  controller and the corresponding worst-case disturbance.

**Keywords:** Nonlinear  $H^{\infty}$  control; Isaacs equation; viscosity solutions; global stability.

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