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Corrigendum

Set Differential Equations and Monotone Flows

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Remark 3.1

(1) In Theorem 3.1, if $G(t,Y) \equiv 0$, then we get a result when F is nondecreasing.

(2) In (1) above, suppose that F is not nondecreasing but $\widetilde{F}(t,X) = F(t,X) + MX$ is nondecreasing in X for each $t \in J$, for some $M > 0$. Then one can consider the IVP $D_H U + MU = \widetilde{F}(t,U)$, $U(0) = U_0$, to obtain the same conclusion as in (1). To see this, use the transformation $\widetilde{U} = Ue^{Mt}$. Assuming that $D_H\widetilde{U}$ exists, we have

$$D_H\widetilde{U} = [D_H U + MU]e^{Mt} = \widetilde{F}(t,\widetilde{U}e^{-Mt})e^{Mt} \equiv F_0(t,\widetilde{U}).$$

Thus the IVP is

$$D_H\widetilde{U} = F_0(t,\widetilde{U}), \quad \widetilde{U}(0) = U_0. \tag{3.17}$$

Then $\widetilde{V} = Ve^{Mt}$ is a lower solution and $\widetilde{W} = We^{Mt}$ is an upper solution for (3.17) and now we have the same conclusion as in (1).

(3) If $F(t,X) = 0$ in Theorem 3.1, then we obtain the result for G nonincreasing.

(4) If in (3) above, G is not monotone but there exists two functions MU and $\widetilde{G}(t,U)$ such that the Hukuhara difference $G(t,U) = MU + \widetilde{G}(t,U)$ exists and $\widetilde{G}(t,U)$ is nonincreasing in U for each $t \in J$. Then setting $U = \widetilde{U}e^{Mt}$, we obtain

$$D_H\widetilde{U} = G_0(t,\widetilde{U}), \quad \widetilde{U}(0) = U_0, \tag{3.18}$$

where $G_0(t,\widetilde{U}) = \widetilde{G}(t,\widetilde{U}e^{Mt})e^{-Mt}$. In this case, we need to assume that (3.18) has coupled lower and upper solutions to get the same conclusion as in (3).

(5) Suppose that in Theorem 3.1, $G(t,Y)$ is nonincreasing in Y and $F(t,X)$ is not monotone but $\widetilde{F}(t,X) = F(t,X) + MX$, $M > 0$ is nondecreasing in X. Then we consider the IVP

$$D_H U + MU = \widetilde{F}(t,U) + G(t,U), \quad U(0) = U_0. \tag{3.19}$$

The transformation in (2) yields the conclusion by Theorem 3.1 in this case as well.

(6) If in Theorem 3.1, F is nondecreasing and G is not monotone then we suppose that there exists two functions MU and $\widetilde{G}(t,U)$ as in (4) and consider the IVP

$$D_H\widetilde{U} = F_0(t,\widetilde{U}) + G_0(t,\widetilde{U}), \quad U(0) = U_0, \tag{3.20}$$

where $F_0(t,\widetilde{U}) = F(t,\widetilde{U}e^{Mt})e^{-Mt}$ and $G_0(t,\widetilde{U}) = \widetilde{G}(t,\widetilde{U}e^{Mt})e^{-Mt}$.

(7) If both F and G are not monotone in Theorem 3.1, then suppose that there are functions $\widetilde{F}(t,U)$, $\widetilde{G}(t,U)$ and MU for some constant $M > 0$ such that the Hukuhara difference $F(t,U) + G(t,U) = \widetilde{F}(t,U) + \widetilde{G}(t,U) + MU$ exists and $\widetilde{F}(t,U)$ is nondecreasing in U and $\widetilde{G}(t,U)$ is nonincreasing in U. Now the transformation $U = \widetilde{U}e^{Mt}$ gives,

$$D_H\widetilde{U} = F_0(t,\widetilde{U}) + G_0(t,\widetilde{U}), \quad U(0) = U_0, \tag{3.20*}$$

where $F_0(t,\widetilde{U}) = \widetilde{F}(t,\widetilde{U}e^{Mt})e^{-Mt}$, $G_0(t,\widetilde{U}) = \widetilde{G}(t,\widetilde{U}e^{Mt})e^{-Mt}$. Assuming that (3.20*) has coupled lower and upper solutions of type I, one gets the same conclusion by Theorem 3.1.

Also note that assumption (A2) in Theorem 3.1 is modified as follows:

(A2) $F, G \in C[J \times K_c(\mathbb{R}^n), K_c(\mathbb{R}^n)]$, $F(t,X)$ is nondecreasing in X and $G(t,Y)$ is nonincreasing in Y, for each $t \geq 0$, and F, G map bounded sets to bounded sets in $K_c(\mathbb{R}^n)$.