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Robust $\mathcal{H}_\infty$ Fuzzy Control Design for Time Delay Nonlinear Markovian Jump Systems: An LMI Approach

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Abstract: This paper considers the problem of designing a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller for a class of time delay nonlinear Markovian jump systems. The proposed controller guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value. Solutions to the problem are provided in terms of linear matrix inequalities. To illustrate the effectiveness of the design developed in this paper, a numerical example is also provided.

Keywords: $\mathcal{H}_\infty$ *fuzzy control; Takagi–Sugeno (TS) fuzzy model; linear matrix inequalities (LMIs); Markovian jump parameters; time-varying delay.*

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1 Introduction

Markovian jump systems are also called hybrid systems, that is, the state space of a system contains both continuous (differential equation) and discrete states (Markov process). The Markovian jump system has been widely used to describe a physical system that changes abruptly from one mode to another mode. These abrupt changes may be caused by environmental disturbances, component and interconnection failures, parameters shifting, tracking, and fast variations in the operating point of the system. Over the past few decades, the Markovian jump system has been extensively studied by many researchers (see [1 – 7]).

It is a well known fact that engineering processes frequently contain time delays. Stability and control synthesis for time delay systems have been one of the most significant issues in control engineering applications. Linear systems with Markovian jumps and time delays have been addressed by a number of researchers (see, for example, [9 − 11]). In [11], the delay-dependent robust stability and the $\mathcal{H}_\infty$ control of time delay linear Markovian jump systems have been investigated. Although many researchers have studied the control design for time delay linear Markovian jump systems for many years, the control design for time delay nonlinear Markovian jump systems remains as an open area.

In the past two decades, the $\mathcal{H}_\infty$ control design for a class of nonlinear systems described by a Takagi-Sugeno (TS) fuzzy model has been studied by a number of researchers (see [12 − 25]). In this TS fuzzy model, local dynamics in different state space regions are represented by local linear systems. The overall model of the system is obtained by "blending" of these linear models through nonlinear membership functions. In other words, a TS fuzzy model is essentially a multi-model approach in which simple sub-models are combined to represent the global behavior of the system. Recently, the design of fuzzy $\mathcal{H}_\infty$ control for a class of nonlinear systems without delays has been significantly considered and many results have been reported (e.g., [12 − 14]). Furthermore, there have been also some attempts in [18 − 23] in which robust fuzzy control analysis and synthesis for nonlinear time-delay systems have been examined. To the best of our knowledge, the global robust $\mathcal{H}_\infty$ fuzzy state-feedback control problem for a class of uncertain nonlinear Markovian jump systems with time-varying delay via an LMI approach has not yet been considered in the literature.

The main contribution of this paper is to design an $\mathcal{H}_\infty$ fuzzy state-feedback controller for a class of time delay nonlinear Markovian jump systems described by a Takagi-Sugeno (TS) fuzzy model. Based on an LMI approach, we develop a state-feedback controller that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than a prescribed value. The solutions are given in terms of a family of linear matrix inequalities.

This paper is organized as follows. In Section 2, system description and definition are presented. In Section 3, based on an LMI approach we develop a technique for designing a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than a prescribed value. The validity of this approach is demonstrated by an example from the literature in Section 4. Finally in Section 5, the conclusion is given.

2 System Description and Definition

The class of time delay uncertain nonlinear Markovian jump system under consideration is described by the following TS fuzzy models:

Plant Rule i: If $\nu_1(t)$ is M_{i1} and $\cdots$ and $\nu_\vartheta(t)$ is $M_{i\vartheta}$ then

$$\begin{aligned}
\dot{x}(t) &= [A_i(\eta(t)) + \Delta A_i(\eta(t))]x(t) + A_{d_i}(\eta(t))x(t-\tau(t)) \\
&\quad + B_{1_i}(\eta(t))w(t) + [B_{2_i}(\eta(t)) + \Delta B_{2_i}(\eta(t))]u(t), \quad x(0)=0, \\
z(t) &= [C_{1_i}(\eta(t)) + \Delta C_{1_i}(\eta(t))]x(t) + [D_{12_i}(\eta(t)) + \Delta D_{12_i}(\eta(t))]u(t) \\
x(t) &= \psi(t), \quad t \in [-\tau, 0], \quad \tau(t) \le \tau
\end{aligned} \tag{2.1}$$

where M_{iq} $(j = 1, 2, \ldots, \vartheta)$ is fuzzy sets q for rule i, $\nu_i(t)$ are the premise variables, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input, $w(t) \in R^p$ is the disturbance which belongs to $\mathcal{L}_2[0,\infty)$, $z(t) \in R^s$ is the controlled output, the matrices $A_i(\eta(t))$, $A_{d_i}(\eta(t))$, $B_{1_i}(\eta(t))$, $B_{2_i}(\eta(t))$, $C_{1_i}(\eta(t))$ and $D_{12_i}(\eta(t))$ are of appropriate dimensions, r is the number of IF-THEN rules, $\tau(t)$ is the bounded time-varying delay in the state with the following assumption

$$0 \le \tau(t) \le \tau \quad \text{and} \quad \dot{\tau}(t) \le \beta < 1$$

and $\psi(t)$ is a vector-valued initial continuous function defined on the interval $[-\tau, 0]$. $\{\eta(t)\}$, $t \ge 0$ is a continuous-time discrete-state homogenous Markov process taking values on a finite set $\mathcal{S} = \{1, 2, \dots, s\}$ with transition probability matrix $Pr = \{P_{\imath k}(t)\}$ given by

$$\begin{aligned} P_{\imath k}(t) &= Pr(\eta(t+\Delta) = k \mid \eta(t) = \imath) \\ &= \begin{cases} \lambda_{\imath k}\Delta + O(\Delta) & \text{if} \quad \imath \ne k, \\ 1 + \lambda_{\imath\imath}\Delta + O(\Delta) & \text{if} \quad \imath = k, \end{cases} \end{aligned} \tag{2.2}$$

and $\sum\limits_{k=1}^{s} P_{\imath k}(t) = 1$, where $\Delta > 0$; $\lim\limits_{\Delta \to 0} \dfrac{O(\Delta)}{\Delta} = 0$; $\lambda_{\imath k} \ge 0$, $\imath \ne k$ is the transition rate from mode $\imath$ to mode k; $\lambda_{\imath\imath} = -\sum\limits_{k=1,\, k\ne\imath}^{s} \lambda_{\imath k}$, $\imath, k \in \mathcal{S}$ gives the infinitesimal generator of the Markov process $\{\eta(t),\, t \ge 0\}$.

The matrices $\Delta A_i(\eta(t))$, $\Delta B_{2_i}(\eta(t))$, $\Delta C_{1_i}(\eta(t))$ and $\Delta D_{12_i}(\eta(t))$ represent the uncertainties in the system and satisfy the following assumption.

Assumption 2.1 Following equalities take place

$$\begin{aligned} \Delta A_i(\eta(t)) &= E_{1_i}(\eta(t))F(x(t),\eta(t),t)H_{1_i}(\eta(t)), \\ \Delta B_{2_i}(\eta(t)) &= E_{2_i}(\eta(t))F(x(t),\eta(t),t)H_{2_i}(\eta(t)), \\ \Delta C_{1_i}(\eta(t)) &= E_{3_i}(\eta(t))F(x(t),\eta(t),t)H_{3_i}(\eta(t)), \\ \Delta D_{12_i}(\eta(t)) &= E_{4_i}(\eta(t))F(x(t),\eta(t),t)H_{4_i}(\eta(t)), \end{aligned}$$

where $E_{j_i}(\eta(t))$ and $H_{j_i}(\eta(t))$, $j = 1, 2, \dots, 4$, are known matrix functions which characterize the structure of the uncertainties. Furthermore, the following inequality holds:

$$\|F(x(t),\eta(t),t)\| \le \rho(\eta(t)) \tag{2.3}$$

for any known positive constant $\rho(\eta(t))$.

Let

$$\varpi_i(\nu(t)) = \prod_{q=1}^{n} M_{iq}(\nu_q(t)), \quad \text{and} \quad \mu_i(\nu(t)) = \frac{\varpi_i(\nu(t))}{\sum\limits_{i=1}^{r} \varpi_i(\nu(t))},$$

where $M_{iq}(\nu_q(t))$ is the grade of membership of $\nu_q(t)$ in M_{iq}. It is assumed in this paper that

$$\varpi_i(\nu(t)) \ge 0, \quad i = 1, 2, \dots, n, \quad \text{and} \quad \sum_{i=1}^{r} \varpi_i(\nu(t)) > 0,$$

where r are the number of local plant rules, for all t. Therefore,

$$\mu_i(\nu(t)) \ge 0, \quad i = 1, 2, \dots, n, \quad \text{and} \quad \sum_{i=1}^{r} \mu_i(\nu(t)) = 1$$

for all t. For the convenience of notations, let $\varpi_i = \varpi_i(\nu(t))$, $\mu_i = \mu_i(\nu(t))$, $\eta = \eta(t)$ and any matrix $N(\mu, \eta(t) = \imath) = N(\mu, \imath)$.

The resulting fuzzy system model is inferred as the weighted average of the local models of the form

$$\begin{aligned} \dot{x}(t) &= [A(\mu,\imath) + \Delta A(\mu,\imath)]x(t) + A_d(\mu,\imath)\, x(t-\tau(t)) \\ &\quad + B_1(\mu,\imath)w(t) + [B_2(\mu,\imath) + \Delta B_2(\mu,\imath)]u(t), \quad x(0) = 0, \\ z(t) &= [C_1(\mu,\imath) + \Delta C_1(\mu,\imath)]\, x(t) + [D_{12}(\mu,\imath) + \Delta D_{12}(\mu,\imath)]\, u(t), \end{aligned} \tag{2.4}$$

where

$$A(\mu,\imath) = \sum_{i=1}^{r} \mu_i A_i(\imath), \quad A_d(\mu,\imath) = \sum_{i=1}^{r} \mu_i A_{d_i}(\imath), \quad B_1(\mu,\imath) = \sum_{i=1}^{r} \mu_i B_{1_i}(\imath),$$

$$B_2(\mu,\imath) = \sum_{i=1}^{r} \mu_i B_{2_i}(\imath), \quad C_1(\mu,\imath) = \sum_{i=1}^{r} \mu_i C_{1_i}(\imath), \quad D_{12}(\mu,\imath) = \sum_{i=1}^{r} \mu_i D_{12_i}(\imath),$$

$$\Delta A(\mu,\imath) = \sum_{i=1}^{r} \mu_i \Delta A_i(\imath) = E_1(\mu,\imath) F(x(t),\imath,t) H_1(\mu,\imath),$$

$$\Delta B_2(\mu,\imath) = \sum_{i=1}^{r} \mu_i \Delta B_{2_i}(\imath) = E_2(\mu,\imath) F(x(t),\imath,t) H_2(\mu,\imath),$$

$$\Delta C_1(\mu,\imath) = \sum_{i=1}^{r} \mu_i \Delta C_{1_i}(\imath) = E_3(\mu,\imath) F(x(t),\imath,t) H_3(\mu,\imath),$$

$$\Delta D_{12}(\mu,\imath) = \sum_{i=1}^{r} \mu_i \Delta D_{12_i}(\imath) = E_4(\mu,\imath) F(x(t),\imath,t) H_4(\mu,\imath)$$

with

$$E_1(\mu,\imath) = \sum_{i=1}^{r} \mu_i E_{1_i}(\imath), \quad E_2(\mu,\imath) = \sum_{i=1}^{r} \mu_i E_{2_i}(\imath), \quad E_3(\mu,\imath) = \sum_{i=1}^{r} \mu_i E_{3_i}(\imath),$$

$$E_4(\mu,\imath) = \sum_{i=1}^{r} \mu_i E_{4_i}(\imath), \quad H_1(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{1_i}(\imath), \quad H_2(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{2_i}(\imath),$$

$$H_3(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{3_i}(\imath), \quad H_4(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{4_i}(\imath).$$

Definition 2.1 Suppose γ is a given positive real number. A system of the form (2.4) is said to have $\mathcal{L}_2[0, T_f]$ gain less than or equal to γ if

$$E\left[\int_0^{T_f} \{ z^{\mathrm{T}}(t) z(t) - \gamma w^{\mathrm{T}}(t) w(t) \}\, dt \right] < 0, \tag{2.5}$$

where $E[\,\cdot\,]$ denotes as the expectation operator.

In this paper, we consider the following $\mathcal{H}_\infty$ fuzzy state-feedback which is inferred as the weighted average of the local models of the form:

$$u(t) = K(\mu, \imath)x(t), \tag{2.6}$$

where $K(\mu,\imath) = \sum_{j=1}^{r} \mu_j K_j(\imath)$. Before ending this section, we describe the problem under our study as follows.

Problem Formulation Given the system (2.4), design an $\mathcal{H}_\infty$ fuzzy state-feedback controller of the form (2.6) such that the $\mathcal{L}_2$ gain γ-performance (2.5) is guaranteed.

3 Main Result

First, let us consider the closed-loop state space form of the fuzzy system model (2.4) with the controller (2.6) which is given by

$$\begin{aligned} \dot{x}(t) &= [A(\mu,\imath) + B_2(\mu,\imath)K(\mu,\imath)]x(t) + A_d(\mu,\imath)x(t-\tau(t)) \\ &\quad + [\Delta A(\mu,\imath) + \Delta B_2(\mu,\imath)K(\mu,\imath)]x(t) + B_1(\mu,\imath)w(t), \quad x(0) = 0, \end{aligned} \tag{3.1}$$

or in a more compact form

$$\begin{aligned} &\dot{x}(t) = [A(\mu,\imath) + B_2(\mu,\imath)K(\mu,\imath)]x(t) + A_d(\mu,\imath)x(t-\tau(t)) + \widetilde{B}_1(\mu,\imath)\widetilde{w}(t), \\ &x(0) = 0, \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} \widetilde{B}_1(\mu,\imath) &= [\, E_1(\mu,\imath) \quad E_2(\mu,\imath) \quad B_1(\mu,\imath) \quad 0 \quad 0 \,], \\ \widetilde{w}(t) &= \begin{bmatrix} F(x(t),\imath,t)H_1(\mu,\imath)x(t) \\ F(x(t),\imath,t)H_2(\mu,\imath)K(\mu,\imath)x(t) \\ w(t) \\ F(x(t),\imath,t)H_3(\mu,\imath)x(t) \\ F(x(t),\imath,t)H_4(\mu,\imath)K(\mu,\imath)x(t) \end{bmatrix}. \end{aligned} \tag{3.3}$$

To provide LMI-based solutions to the problem of designing a robust $\mathcal{H}_\infty$ controller that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value for a class of time delay uncertainty nonlinear Markovian jump systems, the following theorem is given.

Theorem 3.1 *Given the system (2.4), the inequality (2.5) holds if there exist a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, positive definite symmetric matrices $P(\imath)$ and $W(\imath)$ for $\imath = 1, 2, \dots, s$, such that the following conditions hold:*

$$\Omega_{ii}(\imath) < 0, \quad i = 1, 2, \dots, r, \tag{3.5}$$

$$\Omega_{ij}(\imath) + \Omega_{ji}(\imath) < 0, \quad i < j \le r, \tag{3.6}$$

where

$$\Omega_{ij}(\imath) = \begin{pmatrix} \Psi_{ij}(\imath) & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} \\ \mathcal{B}_{ij}(\imath) & -\mathcal{M} + \widetilde{E}_i^{\mathrm{T}}(\imath)\widetilde{E}_j(\imath) & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} \\ W(\imath)A_{d_i}(\imath) & 0 & -(1-\beta)W(\imath) & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} \\ P(\imath) & 0 & 0 & -W(\imath) & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} \\ \Gamma_{ij}(\imath) & 0 & 0 & 0 & -I & (*)^{\mathrm{T}} & (*)^{\mathrm{T}} \\ \Upsilon_{ij}(\imath) & 0 & 0 & 0 & 0 & -I & (*)^{\mathrm{T}} \\ \mathcal{Z}^{\mathrm{T}}(\imath) & 0 & 0 & 0 & 0 & 0 & -\mathcal{P}(\imath) \end{pmatrix}, \tag{3.7}$$

$$\Psi_{ij}(\imath) = A_i(\imath)P(\imath) + P(\imath)A_i^{\mathrm{T}}(\imath) + B_{2_i}(\imath)Y_j(\imath) + Y_j^{\mathrm{T}}(\imath)B_{2_i}^{\mathrm{T}}(\imath)+\lambda_{\imath\imath}P(\imath), \tag{3.8}$$

$$\mathcal{B}_{ij}(\imath) = \widetilde{B}_{1_i}^{\mathrm{T}}(\imath) + \widetilde{E}_i^{\mathrm{T}}(\imath)C_{1_i}(\imath)P(\imath) + \widetilde{E}_i^{\mathrm{T}}(\imath)D_{12_i}(\imath)Y_j(\imath), \tag{3.9}$$

$$\Gamma_{ij}(\imath) = C_{1_i}(\imath)P(\imath) + D_{12_i}(\imath)Y_j(\imath), \tag{3.10}$$

$$\Upsilon_{ij}(\imath) = \widetilde{C}_i(\imath)P(\imath) + \widetilde{D}_i(\imath)Y_j(\imath), \tag{3.11}$$

$$\mathcal{M} = \mathrm{diag}\{I, I, \gamma I, I, I\}, \tag{3.12}$$

$$\mathcal{Z}(\imath) = \left(\sqrt{\lambda_{\imath 1}}P(\imath)\dots\sqrt{\lambda_{\imath(\imath-1)}}P(\imath)\sqrt{\lambda_{\imath(\imath+1)}}P(\imath)\dots\sqrt{\lambda_{\imath s}}P(\imath)\right), \tag{3.13}$$

$$\mathcal{P}(\imath) = \mathrm{diag}\left\{P(1),\dots,P(\imath-1),P(\imath+1),\dots,P(s)\right\}, \tag{3.14}$$

with

$$\widetilde{B}_{1_i}(\imath) = \left[\, E_{1_i}(\imath) \quad E_{2_i}(\imath) \quad B_{1_i}(\imath) \quad 0 \quad 0 \,\right], \tag{3.15}$$

$$\widetilde{C}_i(\imath) = \left[\, \rho(\imath)H_{1_i}^{\mathrm{T}}(\imath) \quad \rho(\imath)H_{3_i}^{\mathrm{T}}(\imath) \quad 0 \quad 0 \,\right]^{\mathrm{T}}, \tag{3.16}$$

$$\widetilde{D}_i(\imath) = \left[\, 0 \quad 0 \quad \rho(\imath)H_{2_i}^{\mathrm{T}}(\imath) \quad \rho(\imath)H_{4_i}^{\mathrm{T}}(\imath) \,\right]^{\mathrm{T}}, \tag{3.17}$$

$$\widetilde{E}_i(\imath) = [\, 0 \quad 0 \quad 0 \quad E_{3_i}(\imath) \quad E_{4_i}(\imath) \,]]. \tag{3.18}$$

Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{r} \mu_j K_j(\imath)x(t) \tag{3.19}$$

where

$$K_j(\imath) = Y_j(\imath)(P(\imath))^{-1}. \tag{3.20}$$

Proof Consider a Lyapunov-Krasovskii functional candidate as follows:

$$V(x(t),\imath) = x^{\mathrm{T}}(t)Q(\imath)x(t) + \int\limits_{t-\tau(t)}^{t} x^{\mathrm{T}}(v)G(\imath)x(v)\,dv, \quad \forall\, \imath \in \mathcal{S}, \tag{3.21}$$

where $Q(\imath) > 0$ and $G(\imath) > 0$. Now let us consider the weak infinitesimal operator $\widetilde{\Delta}$ of the joint process $\{(x(t),\imath),\ t \geq 0\}$, which is the stochastic analog of the deterministic derivative [28]. $\{(x(t),\imath),\ t \geq 0\}$ is a Markov process with infinitesimal operator given by [3]

$$\begin{aligned}\widetilde{\Delta}V(x(t),\imath) = {} & x^{\mathrm{T}}(t)[Q(\imath)(A(\mu,\imath) + B_2(\mu,\imath)K(\mu,\imath)) + (A(\mu,\imath) + B_2(\mu,\imath)K(\mu,\imath))^{\mathrm{T}}Q(\imath) \\ & + G(\imath)]x(t) + x^{\mathrm{T}}(t)Q(\imath)\widetilde{B}_1(\mu,\imath)\widetilde{w}(t) + \widetilde{w}^{\mathrm{T}}(t)\widetilde{B}_1^{\mathrm{T}}(\mu,\imath)Q(\imath)x(t) \\ & + x^{\mathrm{T}}(t)\sum_{k=1}^{s}\lambda_{\imath k}Q(k)x(t) - (1-\dot{\tau})x^{\mathrm{T}}(t-\tau(t))G(\imath)x(t-\tau(t)) \\ & + x^{\mathrm{T}}(t)Q(\imath)A_d(\mu,\imath)x(t-\tau(t)) + x^{\mathrm{T}}(t-\tau(t))A_d^{\mathrm{T}}(\mu,\imath)Q(\imath)x(t).\end{aligned} \tag{3.22}$$

Using the fact that for any vectors $x(t)$ and $x(t-\tau(t))$

$$\begin{aligned}
&x^{\mathrm{T}}(t)Q(\imath)A_d(\mu,\imath)x(t-\tau(t)) + x^{\mathrm{T}}(t-\tau(t))A_d^{\mathrm{T}}(\mu,\imath)Q(\imath)x(t)\\
&\quad \le \frac{1}{(1-\beta)}x^{\mathrm{T}}(t)Q(\imath)A_d(\mu,\imath)G^{-1}(\imath)A_d^{\mathrm{T}}(\mu,\imath)Q(\imath)x(t)\\
&\qquad + (1-\beta)x^{\mathrm{T}}(t-\tau(t))G(\imath)x(t-\tau(t)),
\end{aligned}$$

(3.22) becomes

$$\begin{aligned}
\widetilde{\Delta}V(x(t),\imath) \le x^{\mathrm{T}}(t)\Big[&Q(\imath)(A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)) + (A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath))^{\mathrm{T}}Q(\imath)\\
&+ \frac{1}{(1-\beta)}\,Q(\imath)A_d(\mu,\imath)G^{-1}(\imath)A_d^{\mathrm{T}}(\mu,\imath)Q(\imath) + G(\imath) + \sum_{k=1}^{s}\lambda_{\imath k}Q(k)\Big]x(t)\\
&+ x^{\mathrm{T}}(t)Q(\imath)\widetilde{B}_1(\mu,\imath)\widetilde{w}(t) + \widetilde{w}^{\mathrm{T}}(t)\widetilde{B}_1^{\mathrm{T}}(\mu,\imath)Q(\imath)x(t).
\end{aligned} \tag{3.23}$$

Adding and subtracting $-z^{\mathrm{T}}(t)z(t) + \widetilde{w}^{\mathrm{T}}(t)\mathcal{M}\widetilde{w}(t)$ to and from (3.23), we get

$$\begin{aligned}
\widetilde{\Delta}V(x(t),\imath) \le{}& -z^{\mathrm{T}}(t)z(t) + \widetilde{w}^{\mathrm{T}}(t)\mathcal{M}\widetilde{w}(t) + z^{\mathrm{T}}(t)z(t) + \begin{bmatrix} x(t)\\ \widetilde{w}(t)\end{bmatrix}^{\mathrm{T}}\\
&\times \begin{bmatrix}
\begin{array}{c}[A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)]^{\mathrm{T}}Q(\imath)\\ +Q(\imath)[A(\imath)+B_2(\mu,\imath)K(\mu,\imath)]\\ +\sum_{k=1}^{s}\lambda_{\imath k}Q(k)+G(\imath)\\ +\frac{1}{(1-\beta)}\,Q(\imath)A_d(\mu,\imath)G^{-1}(\imath)A_d^{\mathrm{T}}(\mu,\imath)Q(\imath)\end{array} & (*)^{\mathrm{T}}\\
\widetilde{B}_1^{\mathrm{T}}(\mu,\imath)Q(\imath) & -\mathcal{M}
\end{bmatrix}\begin{bmatrix} x(t)\\ \widetilde{w}(t)\end{bmatrix},
\end{aligned} \tag{3.24}$$

where $\mathcal{M} = \mathrm{diag}\{I, I, \gamma I, I, I\}$.

Now let us consider the following terms

$$\begin{aligned}
\widetilde{w}^{\mathrm{T}}(t)\mathcal{M}\widetilde{w}(t) &= \begin{bmatrix} F(x(t),\imath,t)H_1(\mu,\imath)x(t)\\ F(x(t),\imath,t)H_2(\mu,\imath)K(\mu,\imath)x(t)\\ w(t)\\ F(x(t),\imath,t)H_3(\mu,\imath)x(t)\\ F(x(t),\imath,t)H_4(\mu,\imath)K(\mu,\imath)x(t)\end{bmatrix}^{\mathrm{T}} \mathcal{M}\begin{bmatrix} F(x(t),\imath,t)H_1(\mu,\imath)x(t)\\ F(x(t),\imath,t)H_2(\mu,\imath)K(\mu,\imath)x(t)\\ w(t)\\ F(x(t),\imath,t)H_3(\mu,\imath)x(t)\\ F(x(t),\imath,t)H_4(\mu,\imath)K(\mu,\imath)x(t)\end{bmatrix}\\
&\le \rho^2(\imath)x^{\mathrm{T}}(t)\{H_1^{\mathrm{T}}(\mu,\imath)H_1(\mu,\imath) + K^{\mathrm{T}}(\mu,\imath)H_2^{\mathrm{T}}(\mu,\imath)H_2(\mu,\imath)K(\mu,\imath)\\
&\quad + H_3^{\mathrm{T}}(\mu,\imath)H_3(\mu,\imath) + K^{\mathrm{T}}(\mu,\imath)H_4^{\mathrm{T}}(\mu,\imath)H_4(\mu,\imath)K(\mu,\imath)\}x(t) + \gamma w^{\mathrm{T}}(t)w(t)
\end{aligned} \tag{3.25}$$

and

$$\begin{aligned}
z^{\mathrm{T}}(t)z(t) ={}& x^{\mathrm{T}}(t)[C_1(\mu,\imath) + E_3(\mu,\imath)F(x(t),\imath,t)H_3(\mu,\imath) + D_{12}(\mu,\imath)K(\mu,\imath)\\
&+ E_4(\mu,\imath)F(x(t),\imath,t)H_4(\mu,\imath)K(\mu,\imath)^{\mathrm{T}}[C_1(\mu,\imath) + E_3(\mu,\imath)F(x(t),\imath,t)H_3(\mu,\imath)\\
&+ D_{12}(\mu,\imath)K(\mu,\imath) + E_4(\mu,\imath)F(x(t),\imath,t)H_4(\mu,\imath)K(\mu,\imath)]x(t)\\
={}& \begin{bmatrix} x(t)\\ \widetilde{w}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}
\begin{array}{c}[C_1(\mu,\imath)+D_{12}(\mu,\imath)K(\mu,\imath)]^{\mathrm{T}}\times\\ [C_1(\mu,\imath)+D_{12}(\mu,\imath)K(\mu,\imath)]\end{array} & (*)^{\mathrm{T}}\\
\widetilde{E}^{\mathrm{T}}(\mu,\imath)[C_1(\mu,\imath)+D_{12}(\mu,\imath)K(\mu,\imath)] & \widetilde{E}^{\mathrm{T}}(\mu,\imath)\widetilde{E}(\mu,\imath)
\end{bmatrix}\begin{bmatrix} x(t)\\ \widetilde{w}(t)\end{bmatrix},
\end{aligned} \tag{3.26}$$

where

$$\widetilde{E}(\mu,\imath) = [\,0 \quad 0 \quad 0 \quad E_3(\mu,\imath) \quad E_4(\mu,\imath)\,].$$

Substituting (3.25) and (3.26) into (3.24), we have

$$\widetilde{\Delta}V(x(t),\imath) \le -z^{\mathrm{T}}(t)z(t) + \gamma w^{\mathrm{T}}(t)w(t) + \begin{bmatrix} x(t) \\ \widetilde{w}(t) \end{bmatrix}^{\mathrm{T}} \Phi(\mu,\imath) \begin{bmatrix} x(t) \\ \widetilde{w}(t) \end{bmatrix}, \tag{3.27}$$

where

$$\Phi(\mu,\imath) = \begin{bmatrix} \begin{matrix} [A(\mu,\imath) + B_2(\mu,\imath)K(\mu,\imath)]^{\mathrm{T}}Q(\imath) \\ + Q(\imath)[A(\mu,\imath) + B_2(\mu,\imath)K(\mu,\imath)] \\ + [C_1(\mu,\imath) + D_{12}(\mu,\imath)K(\mu,\imath)]^{\mathrm{T}} \\ \times [C_1(\mu,\imath) + D_{12}(\mu,\imath)K(\mu,\imath)] \\ + [\widetilde{C}(\mu,\imath) + \widetilde{D}(\mu,\imath)K(\mu,\imath)]^{\mathrm{T}} \\ \times [\widetilde{C}(\mu,\imath) + \widetilde{D}(\mu,\imath)K(\mu,\imath)] \\ + \sum_{k=1}^{s} \lambda_{\imath k} Q(k) + G(\imath) \\ + \frac{1}{(1-\beta)} Q(\imath)A_d(\mu,\imath)G^{-1}(\imath)A_d^{\mathrm{T}}(\mu,\imath)Q(\imath) \end{matrix} & (*)^{\mathrm{T}} \\ \begin{matrix} \widetilde{B}_1^{\mathrm{T}}(\mu,\imath)Q(\imath) + \\ \widetilde{E}^{\mathrm{T}}(\mu,\imath)[C_1(\mu,\imath) + D_{12}(\mu,\imath)K(\mu,\imath)] \end{matrix} & -\mathcal{M} + \widetilde{E}^{\mathrm{T}}(\mu,\imath)\widetilde{E}(\mu,\imath) \end{bmatrix} \tag{3.28}$$

with

$$\begin{aligned} \widetilde{C}(\mu,\imath) &= [\,\rho(\imath)H_1^{\mathrm{T}}(\mu,\imath) \quad \rho(\imath)H_3^{\mathrm{T}}(\mu,\imath) \quad 0 \quad 0\,]^{\mathrm{T}}, \\ \widetilde{D}(\mu,\imath) &= [\,0 \quad 0 \quad \rho(\imath)H_2^{\mathrm{T}}(\mu,\imath) \quad \rho(\imath)H_4^{\mathrm{T}}(\mu,\imath)\,]^{\mathrm{T}}. \end{aligned}$$

Using the fact

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n M_{ij}^{\mathrm{T}}(\imath)N_{mn}(\imath) \le \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j[M_{ij}^{\mathrm{T}}(\imath)M_{ij}(\imath) + N_{ij}(\imath)N_{ij}^{\mathrm{T}}(\imath)],$$

we can rewrite (3.27) as follows:

$$\begin{aligned} \widetilde{\Delta}V(x(t),\imath) &\le -z^{\mathrm{T}}(t)z(t) + \gamma w^{\mathrm{T}}(t)w(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j \begin{bmatrix} x(t) \\ \widetilde{w}(t) \end{bmatrix}^{\mathrm{T}} \Phi_{ij}(\imath) \begin{bmatrix} x(t) \\ \widetilde{w}(t) \end{bmatrix} \\ &= -z^{\mathrm{T}}(t)z(t) + \gamma w^{\mathrm{T}}(t)w(t) + \sum_{i=1}^{r}\mu_i^2 \begin{bmatrix} x(t) \\ \widetilde{w}(t) \end{bmatrix}^{\mathrm{T}} \Phi_{ii}(\imath) \begin{bmatrix} x(t) \\ \widetilde{w}(t) \end{bmatrix} \\ &\quad + \sum_{i=1}^{r}\sum_{i<j}^{r}\mu_i\mu_j \begin{bmatrix} x(t) \\ \widetilde{w}(t) \end{bmatrix}^{\mathrm{T}} (\Phi_{ij}(\imath) + \Phi_{ji}(\imath)) \begin{bmatrix} x(t) \\ \widetilde{w}(t) \end{bmatrix}, \end{aligned} \tag{3.29}$$

where

$$\Phi_{ij}(\imath) = \begin{bmatrix} \begin{array}{c} [A_i(\imath) + B_{2_i}(\imath)K_j(\imath)]^{\mathrm{T}}Q(\imath) \\ + Q(\imath)[A_i(\imath) + B_{2_i}(\imath)K_j(\imath)] \\ + [C_{1_i}(\imath) + D_{12_i}(\imath)K_j(\imath)]^{\mathrm{T}} \\ \times [C_{1_i}(\imath) + D_{12_i}(\imath)K_j(\imath)] \\ + [\widetilde{C}_i(\imath) + \widetilde{D}_i(\imath)K_j(\imath)]^{\mathrm{T}} \\ \times [\widetilde{C}_i(\imath) + \widetilde{D}_i(\imath)K_j(\imath) \\ + \sum\limits_{k=1}^{s} \lambda_{\imath k}Q(k) + G(\imath) \\ + \frac{1}{(1-\beta)}Q(\imath)A_{d_i}(\imath)G^{-1}(\imath)A_{d_i}^{\mathrm{T}}(\imath)Q(\imath) \end{array} & (*)^{\mathrm{T}} \\ \widetilde{B}_{1_i}^{\mathrm{T}}(\imath)Q(\imath) + \widetilde{E}_i^{\mathrm{T}}(\imath)[C_{1_i}(\imath) + D_{12_i}(\imath)K_j(\imath)] & -\mathcal{M} + \widetilde{E}_i^{\mathrm{T}}(\imath)\widetilde{E}_j(\imath) \end{bmatrix}. \tag{3.30}$$

Using (3.20) and pre and post multiplying (3.30) by

$$\Xi(\imath) = \begin{bmatrix} P(\imath) & 0 \\ 0 & I \end{bmatrix},$$

we obtain

$$\Xi(\imath)\Phi_{ij}(\imath)\Xi(\imath) = \begin{bmatrix} \begin{array}{c} P(\imath)A_i^{\mathrm{T}}(\imath) + Y_j^{\mathrm{T}}(\imath)B_{2_i}^{\mathrm{T}}(\imath) \\ +A_i(\imath)P(\imath) + B_{2_i}(\imath)Y_j(\imath) \\ +[C_{1_i}(\imath)P(\imath) + D_{12_i}(\imath)Y_j(\imath)]^{\mathrm{T}} \\ \times[C_{1_i}(\imath)P(\imath) + D_{12_i}(\imath)Y_j(\imath) \\ +[\widetilde{C}_i(\imath)P(\imath) + \widetilde{D}_i(\imath)Y_j(\imath)]^{\mathrm{T}} \\ \times[\widetilde{C}_i(\imath)P(\imath) + \widetilde{D}_i(\imath)Y_j(\imath)] \\ + \sum\limits_{k=1}^{s} \lambda_{\imath k}P(\imath)P^{-1}(k)P(\imath) \\ +P(\imath)G(\imath)P(\imath) + \frac{1}{(1-\beta)}A_{d_i}(\imath)G^{-1}(\imath)A_{d_i}^{\mathrm{T}}(\imath) \end{array} & (*)^{\mathrm{T}} \\ \widetilde{B}_{1_i}^{\mathrm{T}}(\imath) + \widetilde{E}_i^{\mathrm{T}}(\imath)C_{1_i}(\imath)P(\imath) + \widetilde{E}_i^{\mathrm{T}}(\imath)D_{12_i}(\imath)Y_j(\imath) & -\mathcal{M} + \widetilde{E}_i^{\mathrm{T}}(\imath)\widetilde{E}_j(\imath) \end{bmatrix}. \tag{3.31}$$

Note that (3.31) is the Schur complement of $\Omega_{ij}(\imath)$ defined in (3.7). Using (3.5), (3.6) and (3.31), we learn that

$$\Phi_{ii}(\imath) < 0, \tag{3.32}$$

$$\Phi_{ij}(\imath) + \Phi_{ji}(\imath) < 0. \tag{3.33}$$

Following from (3.29), (3.32) and (3.33), we know that

$$\widetilde{\Delta}V(x(t), \imath) < -z^{\mathrm{T}}(t)z(t) + \gamma w^{\mathrm{T}}(t)w(t). \tag{3.34}$$

Applying the operator $E\left[\int\limits_0^{T_f}(\cdot)\,dt\right]$ on both sides of (3.34), we obtain

$$E\left[\int\limits_0^{T_f}\widetilde{\Delta}V(x(t), \imath)\,dt\right] < E\left[\int\limits_0^{T_f}(-z^{\mathrm{T}}(t)z(t) + \gamma w^{\mathrm{T}}(t)w(t))\,dt\right]. \tag{3.35}$$

From the Dynkin's formula [29], it follows that

$$E\left[\int_0^{T_f} \widetilde{\Delta} V(x(t),\imath)\,dt\right] = E[V(x(T_f),\imath(T_f))] - E[V(x(0),\imath(0))]. \tag{3.36}$$

Substitute (3.36) into (3.35) yields

$$0 < E\left[\int_0^{T_f} (-z^{\mathrm{T}}(t)z(t) + \gamma w^{\mathrm{T}}(t)w(t))\,dt\right] - E[V(x(T_f),\imath(T_f))] + E[V(x(0),\imath(0))].$$

Using (3.34) and the fact that $V(x(0)=0,\,\imath(0))=0$ and $V(x(T_f),\imath(T_f))>0,$ we have

$$E\left[\int_0^{T_f} \left\{z^{\mathrm{T}}(t)z(t) - \gamma w^{\mathrm{T}}(t)w(t)\right\}dt\right] < 0. \tag{3.37}$$

Hence, the inequality (2.5) holds. This completes the proof of Theorem 3.1.

In order to demonstrate the effectiveness and advantages of the proposed design methodology, an illustrative example is given in next section.

4 An Illustrative Example

Consider an uncertain nonlinear system which is governed by the following state equation [21]

$$\begin{aligned}
\dot{x}_1(t) &= -0.1c(t)x_1^3(t) - \alpha(\eta(t))x_1(t-\tau(t)) - 0.02x_2(t) - 0.67x_2^3(t)\\
&\quad - 0.1x_2^3(t-\tau(t)) - 0.005x_2(t-\tau(t)) + u(t) + 0.1w_1(t),\\
\dot{x}_2(t) &= x_1(t) + 0.1w_2(t),\\
z(t) &= \begin{bmatrix} x_1(t)\\ x_2(t)\end{bmatrix},
\end{aligned} \tag{4.1}$$

where $x_1(t)$ and $x_2(t)$ are the state vectors, $u(t)$ is the control input, $w_1(t)$ and $w_2(t)$ are the disturbance input, $z(t)$ is the regulated output, $\eta(t)$ is the discrete state of the Markov process, $\tau(t) = 4 + 0.5\cos(0.9t)$ and $c(t)$ is the uncertain term, that is, $c(t) \in [0 \quad 2.25]$. It is assumed that

$$x_1(t) \in [-1.5 \quad 1.5] \quad \text{and} \quad x_2(t) \in [-1.5 \quad 1.5].$$

Using the same procedure as in [14], the nonlinear term can be represented as

$$\begin{aligned}
-0.67x_2^3(t) &= M_1 \cdot 0 \cdot x_2(t) - (1-M_1)\cdot 1.5075x_2(t),\\
-0.1x_2^3(t-\tau(t)) &= M_1 \cdot 0 \cdot x_2(t-\tau(t)) - (1-M_1)\cdot 0.225x_2(t-\tau(t)).
\end{aligned}$$

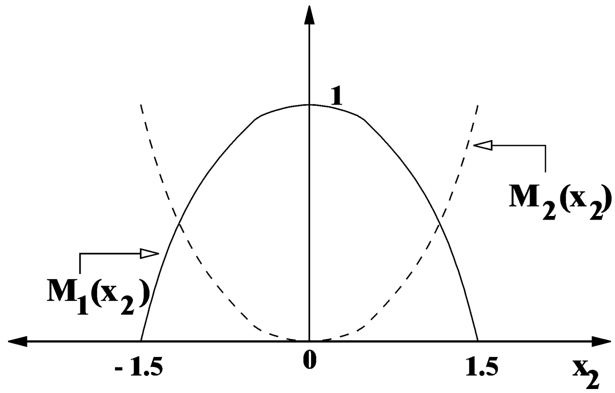

Figure 4.1. Membership functions for two fuzzy set.

Solving the above equations, M_1 is obtained as follows:

$$M_1(x_2(t)) = 1 - \frac{x_2^2(t)}{2.25},$$
$$M_2(x_2(t)) = 1 - M_1(x_2(t)) = \frac{x_2^2(t)}{2.25}.$$

Note that $M_1(x_2(t))$ and $M_1(x_2(t))$ can be interpret as the membership functions of fuzzy set.

Using these two fuzzy set, the uncertain nonlinear Markovian jump system with time-varying delay can be represented by the following TS fuzzy model:

Plant Rule 1: If $x_2(t)$ is $M_1(x_2(t))$ then

$$\dot{x}(t) = [A_1(\imath) + \Delta A_1(\imath)]x(t) + A_{d_1}(\imath)x(t-\tau(t)) + B_1(\imath)w(t) + B_2(\imath)u(t), \quad x(0) = 0,$$
$$z(t) = C_1(\imath)x(t),$$

Plant Rule 2: If $x_2(t)$ is $M_2(x_2(t))$ then

$$\dot{x}(t) = [A_2(\imath) + \Delta A_2(\imath)]x(t) + A_{d_2}(\imath)x(t-\tau(t)) + B_1(\imath)w(t) + B_2(\imath)u(t), \quad x(0) = 0,$$
$$z(t) = C_1(\imath)x(t),$$

where

$$A_1(\imath) = \begin{bmatrix} -0.1125 & -0.02 \\ 1 & 0 \end{bmatrix}, \qquad A_2(\imath) = \begin{bmatrix} -0.1125 & -1.5275 \\ 1 & 0 \end{bmatrix},$$
$$A_{d_1}(\imath) = \begin{bmatrix} -\alpha(\imath) & -0.005 \\ 0 & 0 \end{bmatrix}, \qquad A_{d_2}(\imath) = \begin{bmatrix} -\alpha(\imath) & -0.23 \\ 0 & 0 \end{bmatrix},$$
$$B_1(\imath) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad B_2(\imath) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad C_1(\imath) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\Delta A_1(\imath) = E_{1_1}(\imath)F(x(t),\imath,t)H_{1_1}(\imath), \qquad \Delta A_2(\imath) = E_{1_2}(\imath)F(x(t),\imath,t)H_{1_2}(\imath),$$

$$x(t) = [x_1^{\mathrm{T}}(t) \quad x_2^{\mathrm{T}}(t)]^{\mathrm{T}} \quad \text{and} \quad w(t) = [w_1^{\mathrm{T}}(t) \quad w_2^{\mathrm{T}}(t)]^{\mathrm{T}}.$$

Assuming $\|F(x(t), \imath, t)\| \le \rho(\imath) = 1$ and letting

$$E_{1_1}(\imath) = E_{1_2}(\imath) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

we have

$$H_{1_1}(\imath) = H_{1_2}(\imath) = \begin{bmatrix} -1.1250 & 0 \\ 0 & 0 \end{bmatrix}.$$

Assume that the system is a three modes Markov process as shown in Table 4.1.

Table 4.1 Modes of the Markov process.

Mode $\imath$	$\alpha(\imath)$
1	0.0120
2	0.0125
3	0.0130

The transition probability matrix that relates the three modes is given as follows:

$$P_{\imath k} = \begin{bmatrix} 0.67 & 0.17 & 0.16 \\ 0.30 & 0.47 & 0.23 \\ 0.26 & 0.10 & 0.64 \end{bmatrix}.$$

Using the LMI optimization algorithm and Theorem 3.1 with $\beta = 0.6$, we obtain $\gamma = 0.1680$

$$P(1) = \begin{bmatrix} 2.4912 & -0.2673 \\ -0.2673 & 0.0718 \end{bmatrix}, \qquad W(1) = \begin{bmatrix} 1.1072 & -0.1535 \\ -0.1535 & 16.1836 \end{bmatrix},$$

$$Y_1(1) = [\,-16.9067 \quad -0.1051\,], \qquad Y_2(1) = [\,-17.2552 \quad -0.0235\,],$$

$$K_1(1) = [\,-11.5635 \quad -44.5276\,], \qquad K_2(1) = [\,-11.5934 \quad -43.5022\,],$$

$$P(2) = \begin{bmatrix} 2.3815 & -0.2881 \\ -0.2881 & 0.0841 \end{bmatrix}, \qquad W(2) = \begin{bmatrix} 1.1489 & -0.1931 \\ -0.1931 & 16.4120 \end{bmatrix},$$

$$Y_1(2) = [\,-15.9725 \quad 0.0589\,], \qquad Y_2(2) = [\,-16.3401 \quad 0.1485\,],$$

$$K_1(2) = [\,-11.3092 \quad -38.0433\,], \qquad K_2(2) = [\,-11.3526 \quad -37.1260\,],$$

$$P(3) = \begin{bmatrix} 2.4793 & -0.2638 \\ -0.2638 & 0.0857 \end{bmatrix}, \qquad W(3) = \begin{bmatrix} 0.9718 & -0.1883 \\ -0.1883 & 15.8428 \end{bmatrix},$$

$$Y_1(3) = [\,-17.0602 \quad -0.0867\,], \qquad Y_2(3) = [\,-17.4006 \quad 0.0530\,],$$

$$K_1(3) = [\,-10.3932 \quad -33.0111\,], \qquad K_2(3) = [\,-10.3394 \quad -31.2150\,].$$

The resulting fuzzy controller is

$$u(t) = \sum_{j=1}^{2} \mu_j K_j(\imath) x(t) \tag{4.2}$$

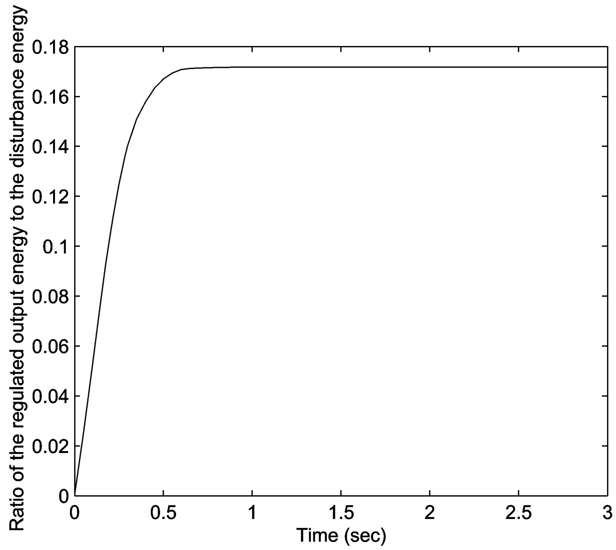

Figure 4.2. The result of the changing between modes during the simulation with the initial mode at Mode 2.

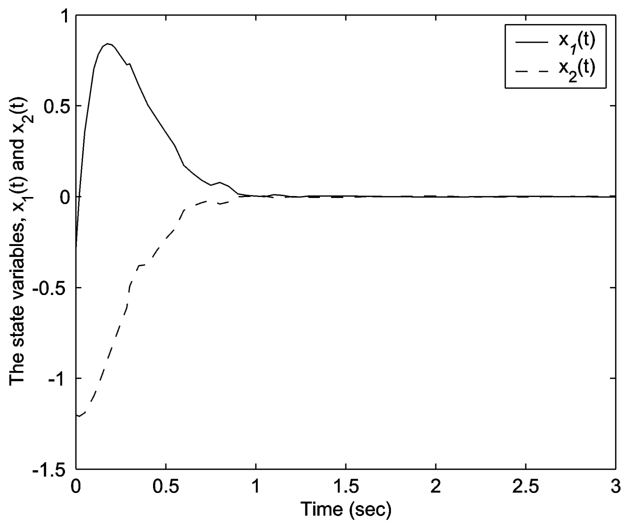

Figure 4.3. The histories of the state variables, $x_1(t)$ and $x_2(t)$.

where

$$\mu_1 = M_1(x_2(t)) \quad \text{and} \quad \mu_2 = M_2(x_2(t)).$$

Remark 4.1 Figure 4.2 shows the changing between modes with the initial mode at Mode 2. The histories of the state variables, $x_1(t)$ and $x_2(t)$ are given in Figure 4.3. The disturbance input signal, $w(t)$, which was used during simulation is given in Figure 4.4. The ratio of the regulated output energy to the disturbance input noise energy obtained by using the $\mathcal{H}_\infty$ fuzzy controller (4.2) is depicted in Figure 4.5. After 3 seconds, the ratio

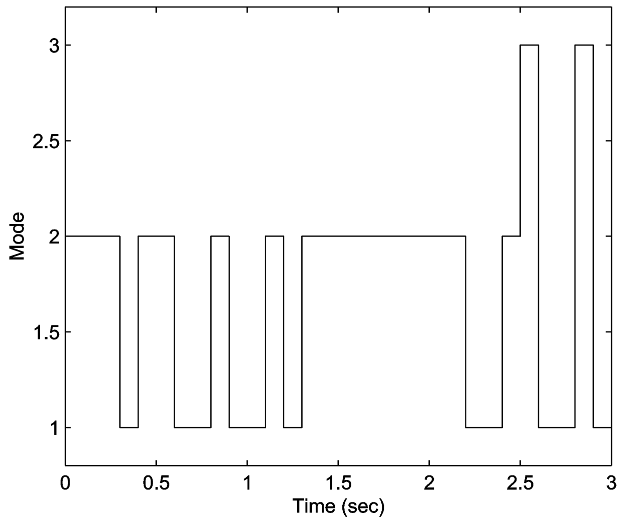

Figure 4.4. The disturbance input noise, $w(t)$.

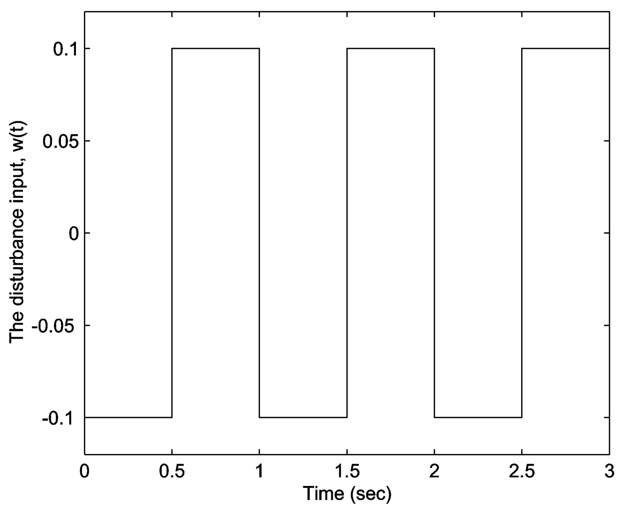

Figure 4.5. The ratio of the regulated output energy to the disturbance noise energy, $\left(\int_0^{T_f} z^{\mathrm{T}}(t)z(t)\,dt \Big/ \int_0^{T_f} w^{\mathrm{T}}(t)w(t)\,dt\right)$.

of the regulated output energy to the disturbance input noise energy tends to a constant value which is about 0.1680. From Figure 4.5, we can conclude that the inequality (2.5) is guaranteed by the fuzzy controller (4.2).

5 Conclusion

In this paper, we have developed a technique for designing a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller for a class of time delay nonlinear Markovian jump systems that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value. In addition, solutions to the problem are given in terms of linear matrix inequalities which make them more useful. Finally, an illustrative example is provided to demonstrate the effectiveness and advantages of the proposed design methodology.

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