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Designing by Control Law without Model for Dynamic IS-LM Model

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Abstract: IS-LM model is used for comparative static analysis and many dynamic factors are not considered, so dynamic analysis is introduced to IS-LM model to analyze economy more deeply. Control without model proposed by Mr. Han Zhi-gang has many advantages, such as strong adaptability, strong tracking ability, strong anti-disturbance ability, time lag controlling and so on, so it is fit for macro-economy dynamic analysis. The property of maximal energy saving of control law without model makes it possible to save more fund when government uses finance policy and currency policy.

Keywords: *Dynamic IS-LM; control without model.*

Mathematics Subject Classification (2000): 91B28, 91B62, 91B64.

1 Introduction

As it is well known the IS-LM model is the core of modern macro-economy [1]. The IS-LM model can be used to analyze every kind of problems of the public finance policy and currency policy and the match of these policies so that the national macro-economy can attain the aim of high economy growth rate and low inflation rate. But this model has some weakness, it is a kind of static balanced analysis, and does not consider many dynamic factors (for example, time lag) within economy, so it is difficult to do more in-depth analysis of economy. The macro-economic system is a complicated one, and is nonlinear with time lag. It is difficult to establish an available mathematical model, and along with the economic reformation going deep and system innovating, the model's structure changes constantly too.

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Therefore, the control law for regulating macro-economy must not only adapt to the model parameters change, and also adapt to the variety of the model structure. Control without model is an effective tool for solving this kind of problem. The mathematical model of dynamic system generally need to be established before designing the control law. The classical method requests this kind of mathematical model be established in advance, at least its construction must be certain in advance, and the model parameters are as accurate as possible. When designing control law without model, it breaks the restrict that control law must be based on the accurate mathematical model. The process of establishing model goes along with feedback control [2]. The initial mathematical model can be not accurate, but it must guarantee the control law designed having proper astringency.

The control law without model works along with establishing model. After getting new data, the model is established again and control is established again. Going on like this, and making the model gradually accurate the performance of system under the control law improves. Real-time establishing model and feedback controlling become integral. The means make the control law have structure self-adaptability with real-time identification i.e. real-time feedback control used by control without model combines identification with control law designing. At the same time we can prove that the control law without model has the property of maximal "energy saving" so that government can use minimum fund to do the same thing when government makes use of finance policy and currency policy.

2 Some Control Without Model Theory

In reference [3], the following lemma has been proved.

Lemma 1 *For any dynamic system with one step delay, if input-output data* $\{u(k-2), y(k-1)\}$, $\{u(k-1), y(k)\}$ *are given and* $u(k-2) \neq u(k-1)$, *then there exists a vector* $\varphi(k)$ *such that*

$$y(k) - y(k-1) = \varphi(k-1)^{\tau}[u(k-1) - u(k-2)] \tag{1}$$

where $(^{\tau})$ *is a symbol of transposition,* $y(k)$ *is one-dimension output of system and* $u(k)$ *is input vector,* $\varphi(r)$ *is called pseudo-gradient.*

By the following way, $\varphi(k)$ can be estimated.

Let

$$z(k) = y(k) - y(k-1),$$
$$\phi(k) = u(k-1) - u(k-2).$$

Using the above notation, we now can rewrite (1) as

$$z(k) = \phi(k)^{\tau}\varphi(k-1). \tag{2}$$

Real-time observed values $y(k)$ and $u(k-1)$, $z(k)$ and $\phi(k)$ are obtained. Therefore $\varphi(k-1)$ is estimated by the value $\hat{\varphi}(k-1)$ as follows

$$\begin{aligned}
\hat{\varphi}(k-1) &= \hat{\varphi}_1(k-1) + \frac{\delta}{\eta_k + \|\phi(k)\|^2}\,\phi(k)\{z(k) - \phi(k)^\tau \hat{\varphi}_1(k-1)\},\\
\hat{\varphi}_1(k-1) &= \hat{\varphi}_1(k-2) + M(k-1)\{z(k) - \phi(k)^\tau \hat{\varphi}_1(k-2)\},\\
M(k-1) &= \frac{p(k-2)\phi(k)}{\lambda + \phi(k)^\tau p(k-2)\phi(k)}\,,\\
p(k-1) &= \frac{1}{\lambda}[I - M(k-1)\phi(k)^\tau]\,p(k-2)
\end{aligned} \tag{3}$$

where η_k is a suitable small positive value and δ is a proper constant.

Then we find forecasting value of $\hat{\varphi}(k-1)$ signed as $\hat{\varphi}^*(k)$. A simple method is

$$\hat{\varphi}^*(k) = \hat{\varphi}(k-1).$$

When we design control law, also sign $\hat{\varphi}^*(k)$ as $\hat{\varphi}(k)$. So using the basic form of control law without model

$$u(k) = u(k-1) + \frac{\lambda_k}{a + \|\hat{\varphi}(k)\|^2}\,\hat{\varphi}(k)\{y_0(k+1) - y(k)\}, \tag{4}$$

where λ_k is called control parameter, $y_0(k+1)$ is expectation output at $k+1$ time, and a is suitable small positive constant which makes denominator not equal to zero, we can obtain control vector $u(k)$. It acts on the system, so we can obtain new output $y(k+1)$ and a new group of data $\{y(k+1), u(k)\}$.

The next theorem shows that control variation has the property of minimum.

Theorem 1 *If $y_0(k+1)$, $y(k)$, $u(k-1)$, $\varphi(k)$ are known and $\|\varphi(k)\|^2 \neq 0$, then vector of control is defined by*

$$u(k) = u(k-1) + \frac{1}{\|\varphi(k)\|^2}\,\varphi(k)\{y_0(k+1) - y(k)\}$$

and satisfies the conditions

$$\begin{aligned}
y_0(k+1) - y(k) &= \varphi(k)^\tau[u(k) - u(k-1)],\\
\|u(k) - u(k-1)\|^2 &= \min_u \|u - u(k-1)\|^2.
\end{aligned}$$

Proof The Lagrangian multiplier can be used here.

Let

$$f(u,\lambda) = \|u - u(k-1)\|^2 + \lambda\{y_0(k+1) - y(k) - \varphi(k)^\tau[u - u(k-1)]\}$$

be Lagrangian function. For this case, it can be shown that

$$\begin{aligned}
\frac{\partial f}{\partial u} &= 2(u - u(k-1)) - \lambda\varphi(k),\\
\frac{\partial f}{\partial \lambda} &= y_0(k+1) - y(k) - \varphi(k)^\tau[u - u(k-1)].
\end{aligned}$$

Let

$$\frac{\partial f}{\partial u} = 0 \quad \text{and} \quad \frac{\partial f}{\partial \lambda} = 0.$$

We can compute

$$\begin{aligned} u - u(k-1) &= \frac{\lambda}{2}\,\varphi(k), \\ y_0(k+1) - y(k) = \varphi(k)^\tau [u - u(k-1)] &= \frac{\lambda}{2}\,\|\varphi(k)\|^2 . \end{aligned} \tag{5}$$

Hence, when $\|\varphi(k)\|^2 \neq 0$ we obtain

$$\lambda = \frac{2}{\|\varphi(k)\|^2}\,\{y_0(k+1) - y(k)\}.$$

Thus, from (5) it can be obtained that

$$u(k) = u(k-1) + \frac{1}{\|\varphi(k)\|^2}\varphi(k)\{y_0(k+1) - y(k)\}$$

which satisfies the conclusion of Theorem 1, since there is only minimum point for the function $\|u - u(k-1)\|^2$.

Above is the case of multi-input and single-output. Reference [4] extended it to MIMO system. Suppose that the dimension of system output variable $y(k)$ is n, the dimension of input (control) variable $u(k)$ is m, and that $n \leq m$. Suppose the time lag of system is 1, so the model can be written as

$$y(k+1) - y(k) = \varphi(k)[\hat{u}(k) - \hat{u}(k-1)],$$

where

$$\varphi(k) = \begin{bmatrix} \varphi_1(k)^\tau \\ \varphi_2(k)^\tau \\ \cdots\cdots \\ \varphi_n(k)^\tau \end{bmatrix} = \begin{bmatrix} \varphi_{11}(k) & \varphi_{12}(k) & \dots & \varphi_{1m}(k) \\ \varphi_{21}(k) & \varphi_{22}(k) & \dots & \varphi_{2m}(k) \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{n1}(k) & \varphi_{n2}(k) & \dots & \varphi_{nm}(k) \end{bmatrix},$$

i.e. $\varphi(k)$ is called pseudo-gradient matrix. Set

$$r(k) = \operatorname{rank}\{\varphi(k)\}.$$

Apparently $r(k) \leq n$. Suppose $D_t(k)$ is $r(k)$ full-rank submatrix of $\varphi(k)$, $t = 1, 2, \dots, N$, N is the number of $r(k)$ full-rank submatrix of $\varphi(k)$. Let $\|D_t(k)\|$ denote a kind of norm of $D_t(k)$. There must be one $r(k)$ full-rank submatrix

$$D(k) = \begin{bmatrix} \varphi_{i_1 j_1}(k) & \varphi_{i_1 j_2}(k) & \dots & \varphi_{i_1 j_r}(k) \\ \varphi_{i_2 j_1}(k) & \varphi_{i_2 j_2}(k) & \dots & \varphi_{i_2 j_r}(k) \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{i_r j_1}(k) & \varphi_{i_r j_2}(k) & \dots & \varphi_{i_r j_r}(k) \end{bmatrix}$$

that makes

$$\|D(k)\| = \max_{1 \leq t \leq N} \|D_t(k)\| .$$

We call $D(k)$ dominant $r(k)$ full-rank submatrix of $\varphi(k)$. Its corresponding $u_{j_1}(k)$, $u_{j_2}(k), \dots, u_{j_r}(k)$ are called dominant control variables. Its corresponding output variables are $y_{i_1}(k), y_{i_2}(k), \dots, y_{i_r}(k)$, set

$$\begin{aligned} y^*(k+1) &= (y_{i_1}(k+1), y_{i_2}(k+1), \dots, y_{i_r}(k+1))^\tau, \\ u^*(k) &= (u_{j_1}(k), u_{j_2}(k), \dots, u_{j_r}(k))^\tau. \end{aligned}$$

Eliminating $y^*(k+1)$ from $y(k+1)$, the rest can be written as vector $y^-(k+1)$. Similarly eliminating $u^*(k)$ from $u(k)$, the rest can be written as vector $u^-(k)$. Ordering $y(k+1)$ and $u(k)$ properly, there exists

$$y(k+1) = (y^*(k+1)^\tau, y^-(k+1)^\tau)^\tau, \quad u(k) = (u^*(k)^\tau, u^-(k)^\tau)^\tau.$$

So we can acquire MIMO control law without model

$$\begin{aligned} &\hat{u}^*(k) = \hat{u}^*(k-1) + \frac{\lambda_k}{a + |\widehat{D}(k)|}\hat{D}^*(k)\{y^*(k+1) - y^*(k)\}, \\ &\hat{u}^-(k) = \hat{u}^-(k-1), \end{aligned} \tag{6}$$

where $\widehat{D}^*(k)$ denotes adjoint of $\widehat{D}(k)$, $|\hat{D}(k)|$ denotes determinant of $\hat{D}(k)$, and $y^*(k+1)$ denotes expectation value of the component determined by $y(k+1)$ independently. We have the matrix

$$\lambda_k = \begin{bmatrix} \lambda_1(k) & & & 0 \\ & \lambda_2(k) & & \\ & & \ddots & \\ 0 & & & \lambda_r(k) \end{bmatrix},$$

where $\lambda_1(k), \lambda_2(k), \dots, \lambda_r(k)$ are proper parameters, λ_k is called control parameter matrix.

3 Control without Model Application in Macro-Economy

In the model IS-LM, finance policy variable (M) and currency policy variable (G) can be taken as control(input) variables and Gross Domestic Product (GDP) and nominal interest rate (i) can be taken as output variables. Nominal interest rate is equal to actual interest rate plus inflation rate. What shows economy running well is high economic growth rate and low inflation rate, so the control aim of macro-economy system can be

$$aim\colon \quad \begin{cases} i(t) = i^*, \\ Y(t) = Y^*(1+\alpha)^t, \end{cases}$$

where i^* and Y^* are given constants, α is given economy growth rate. According to actual situation of China, annual interest rate is 2.25% and expected inflation rate is under 3%, so set $i^* = 5\%$ and economic growth rate is $\alpha = 8\%$. So the model may be written as

$$\begin{bmatrix} Y(k+1) - Y(k) \\ i(k+1) - i(k) \end{bmatrix} = \varphi(k) \begin{bmatrix} G(k) - G(k-1) \\ M(k) - M(k-1) \end{bmatrix}.$$

According to the data of National Bureau of Statistics of China, see Table 3.1, by formula (3), $\hat{\varphi}(k)$ can be obtained. Suppose target value is

$$\begin{bmatrix} Y_0(k+1) \\ i_0(k+1) \end{bmatrix} = \begin{bmatrix} 117.25\,(1+0.08) \\ 5\% \end{bmatrix}.$$

By formula (6), $\widehat{G}(k)$ and $\widehat{M}(k)$ that meet the target can be obtained. In formula (6), $\lambda_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ denotes neutral finance policy and balanced currency policy.

Table 3.1. 1990 – 2003 statistics datum.

year	Gross Domestic Product GDP billion yuan	Finance Payout G billion yuan	Money Supply M2 billion yuan	Consumer Price Index CPI	Annual Interest %
1990	1854.79	308.359	1529.34	103.1	8.64
1991	2161.78	338.662	1934.99	103.4	7.92
1992	2663.81	374.22	2540.22	106.4	7.56
1993	3463.44	464.23	3487.98	114.7	9.26
1994	4675.94	579.262	4692.35	124.1	10.98
1995	5847.81	682.372	6075.05	117.1	10.98
1996	6788.46	793.755	7609.49	108.3	9.21
1997	7446.26	923.356	9099.53	102.8	7.17
1998	7834.52	1079.818	10449.85	99.2	5.03
1999	8206.75	1318.767	11989.79	98.6	2.89
2000	8946.81	1588.65	13461.04	100.4	2.25
2001	9731.48	1890.258	15830.19	100.7	2.25
2002	10517.23	2205.315	18500.70	99.2	2.03
2003	11725.19	2464.995	22122.28	101.2	1.98

Note: Annual Interest is arithmetic mean.

Going on with

$$\begin{bmatrix} \widehat{Y}(k+1) \\ \hat{i}(k+1) \end{bmatrix} = \begin{bmatrix} Y(k) \\ i(k) \end{bmatrix} + \hat{\varphi}(k) \begin{bmatrix} \widehat{G}(k) - G(k-1) \\ \widehat{M}(k) - M(k-1) \end{bmatrix},$$

estimated values of next year can be obtained. Repeating formulas (3),(6) graph 1 can be obtained. From the graph we can draw the following conclusions:

1. From (c) and (d), we can see that system tracking ability is very good, estimated values superimpose with target values.
2. Estimated finance payout amplitude is 9.4% on the average, money supply amplitude is 14.8% on the average. They are less than the average value 18.0% and 16.2% of past 5 years.

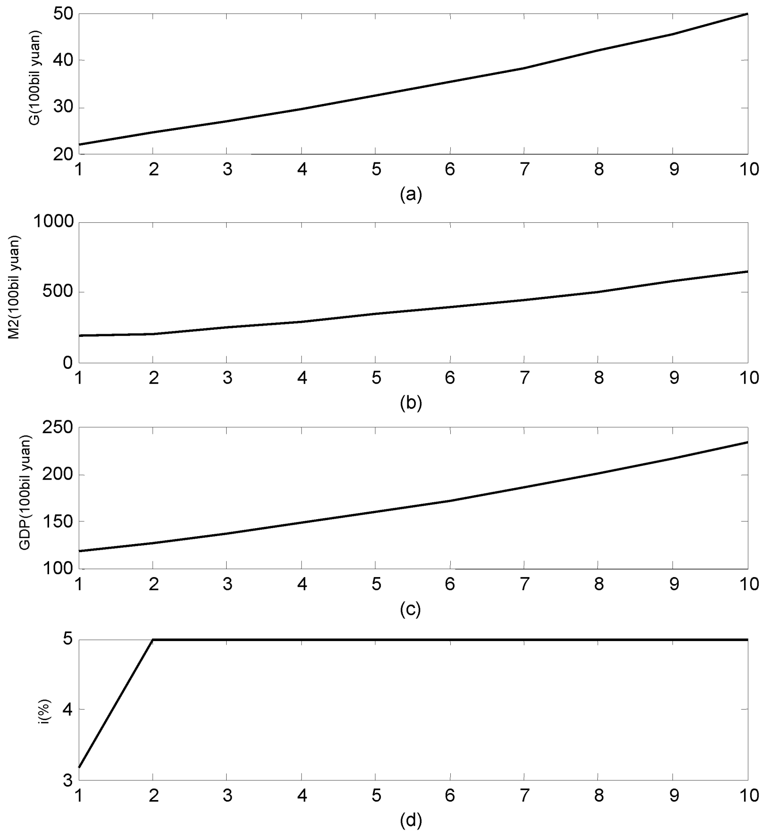

Figure 3.1. System simulation curve.

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