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An LMI Approach to H_∞ Filtering for Linear Parameter-Varying Systems with Delayed States and Outputs

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Abstract: This paper considers the problem of delay-dependent robust H_∞ filtering for linear parameter-varying (LPV) systems with time-invariant delay in the states and outputs. It is assumed that the state-space matrices affinely depend on parameters that are measurable in real-time. By taking the relationship between the terms in the Leibniz-Newton formula and a suitable change of variables into account, some new parameter-dependent delay-dependent stability conditions are established in terms of linear matrix inequalities so that the filtering process remains asymptotically stable and satisfies a prescribed H_∞ performance level. Using polynomially parameter-dependent quadratic functions and some multiplier matrices, we establish the parameter-independent delay-dependent conditions with high precision under which the desired robust H_∞ filters exist and derive the explicit expression of these filters. A numerical example is provided to demonstrate the validity of the proposed design approach.

Keywords: *LPV systems; H_∞ filtering; delay; LMI; polynomially parameter-dependent quadratic functions.*

Mathematics Subject Classification (2000): 34D20, 93A30, 93E11.

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1 Introduction

One of the problems with optimal Kalman filters, which has now been well recognized, is that they can be sensitive to the system data and the spectral densities of noise processes, or in other words, they may lack robustness [1]. Therefore, in the past decade, a number of papers have attempted to develop robust filters that are capable of guaranteeing satisfactory estimation in the presence of modeling errors and unknown signal statistics [35]. Concerning the energy bounded deterministic noise inputs, the H_∞ filtering theory has been developed which minimizes (or, in the suboptimal case, bound) the worst-case energy gain from the energy-bounded disturbances (without the need for knowledge of noise statistics) to the estimation errors [16]. Furthermore, the robust H_∞ filtering problem has recently received considerable attention. The aim of this problem is to pursue the enforcement of the upper bound constraint on the H_∞ norm where the system is affected by parameter uncertainties (see for instance [22], and the references therein).

The stability analysis and control design of linear parameter-varying (LPV) systems where the state-space matrices depend affinely on parameter vector, whose values are not known a priori, but can be measured online for control process, have received considerable attention recently (see for instance [3, 5, 17, 21, 31] and the references therein). Establishing stability via the use of classical quadratic Lyapunov function is conservative for the LPV systems. To investigate the stability of LPV systems one needs to resort the use of parameter-dependent Lyapunov functions to achieve necessary and sufficient conditions of system stability, see [4, 6, 7, 10, 13, 19, 23]. However, Bliman in [7] proposed robust stability analysis for LPV systems with polytopic uncertain parameters. He also developed some conditions for robust stability in terms of solvability of some linear matrix inequalities (LMIs) without conservatism. Concerning unknown parameter vector, an adaptive method has been presented in [21] for robust stabilization with H_∞ performance of LPV systems. Moreover, the existence of a polynomially parameter-dependent quadratic (PPDQ) Lyapunov function for parameter-dependent systems, which are robustly stable, is stated in [8]. Recently, sufficient conditions for robust stability of the linear state-space models affected by polytopic uncertainty have been provided in [9] using homogeneous PPDQ Lyapunov functions, which are formulated in terms of LMI feasibility tests.

On the other hand, in addition to the system uncertainties, it is well known that the time-delay is also often the main cause of instability and poor performance of dynamical systems [11, 12, 25, 37]. Stability criteria for time-delay systems can be classified into two categories: delay-dependent and delay-independent criteria. The stability and the performance issues of the LPV state-delayed systems are then both theoretically and practically important and are a field of intense research. Recently, some appreciable works have been performed to analyze and synthesize LPV time-delay systems (e.g. see [18, 20, 32, 34, 36, 38]). It is known that the conservatism of the delay-dependent stability conditions stems from two causes: one is the model transformation used and the other is the inequality bounding technique employed for some cross terms encountered in the analysis. Considering these, in [38], a model which is equivalent to the original delay system was proposed and the bounding technique in [26, 28] was used. However, conservatism still remains in these results, which motivates the present study.

The filter design problems of uncertain time-delay systems have received much less attention although they are important in control design and signal processing applications (e.g. see [15]). Recently, Pila et al. [27] have considered the problem of filtering for linear time-varying system with time-delay measurements. Moreover, the robust filtering problem for uncertain linear systems with delayed sates and outputs for both time-invariant and time-varying cases were investigated in [35]. It is also worth citing that few studies have been done for the design of robust H_∞ filters for LPV systems [24, 33]. However, the robust H_∞ filtering problem for LPV systems with delayed states and outputs has not been fully investigated and remains to be important and challenging.

In this work, we are concerned with the delay-dependent robust H_∞ filtering problem for a class of LPV systems with time-invariant delay in the states and outputs. It is assumed that the state-space data affinely depend on parameter vector that are measurable in real-time. Some new delay dependent stability conditions are established based on a new method with some interesting features. First, it is obtained without resorting to any model transformations and bounding techniques for some cross terms, thus reducing the conservatism in the derivation of the stability condition. Second, some free weighting matrices are employed to express the influence of the terms in the Leibniz-Newton formula which are determined by solving LMIs. Third, using a suitable change of variables the delay-dependent stability conditions are formulated in terms of LMIs such that the filtering process remains asymptotically stable and satisfies a prescribed H_∞ performance level. Forth, using polynomially parameter-dependent quadratic (PPDQ) functions and some multiplier matrices, the parameter-dependent delay-dependent conditions are relaxed to the parameter-independent delay-dependent conditions with high precision under which the desired robust H_∞ filters exist and the explicit expression of these filters is derived. Accordingly, the designed filters have the ability to track the plant states in the presence of external disturbances. Eventually, an illustrative example is given to show the qualification of our design methodology.

Notations. The symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix. Also, the symbol $\otimes$ denotes Kronecker product, the power of Kronecker products being used with the natural meaning $M^{0\otimes} = 1$, $M^{p\otimes} := M^{(p-1)\otimes} \otimes M$. Let $\{\hat{J}_k, \tilde{J}_k\} \in \Re^{k\times(k+1)}$, and $v^{[k]}$ be defined by $\hat{J}_k := [I_k, \quad 0_{k\times 1}]$, $\tilde{J}_k := [0_{k\times 1}, \quad I_k]$ and $v^{[k]} = \mathrm{col}\,\{1,\, v, \ldots,\, v^{k-1}\}$, respectively, which have essential roles for polynomial manipulations [7].

2 Problem Description

Consider a class of LPV systems with delayed states and outputs as

$$
\begin{aligned}
\dot{x}(t) &= A(\rho)x(t) + A_d(\rho)x(t-h) + E_1(\rho)w(t),\\
x(t) &= \phi(t), \quad t \in [-h, 0],\\
z(t) &= L(\rho)x(t) + L_d(\rho)x(t-h) + E_3(\rho)w(t),\\
y(t) &= C(\rho)x(t) + C_d(\rho)x(t-h) + E_2(\rho)w(t),
\end{aligned}
\tag{1}
$$

where $x(t) \in \Re^n$, $w(t) \in L_2[0,\infty)$, $z(t) \in \Re^z$ and $y(t) \in \Re^p$ are state vector, disturbance input, estimated output and measured output, respectively. $\phi(t)$ is continuous vector valued initial function. Moreover, the parameter $h > 0$ is the constant time-delay and the vector $\rho = \mathrm{col}\,\{\rho_1, \rho_2, \ldots, \rho_m\} \in \zeta \subset \Re^m$ is uncertain but the parameters ρ_i are measurable in real-time with ζ being a compact set. In (1), the parameter-dependent matrices are unknown real continuous matrix functions, which affinely depend on the vector ρ, that are

$$\begin{bmatrix} A(\rho) & A_d(\rho) & E_1(\rho) \\ L(\rho) & L_d(\rho) & E_3(\rho) \\ C(\rho) & C_d(\rho) & E_2(\rho) \end{bmatrix} = \begin{bmatrix} A_0 & A_{0d} & E_{01} \\ L_0 & L_{0d} & E_{03} \\ C_0 & C_{0d} & E_{02} \end{bmatrix} + \sum_{j=1}^{m} \rho_j \begin{bmatrix} A_j & A_{jd} & E_{j1} \\ L_j & L_{jd} & E_{j3} \\ C_j & C_{jd} & E_{j2} \end{bmatrix}. \tag{2}$$

In this paper, we focus on the design of an n-th order H_∞ filter with delayed states and outputs with the following equations

$$\begin{aligned} \dot{\hat{x}}(t) &= F(\rho)\hat{x}(t) + F_d(\rho)\hat{x}(t-h) + G(\rho)y(t), \\ \hat{x}(t) &= 0, \quad t \in [-h, 0], \\ \hat{z}(t) &= L(\rho)\hat{x}(t) + L_d(\rho)\hat{x}(t-h) + E_3(\rho)w(t), \end{aligned} \tag{3}$$

where the state-space parameter-dependent matrices $F(\rho)$, $F_d(\rho)$ and $G(\rho)$ of the appropriate dimensions are the filter design objectives to be determined. In (3), it is assumed that $\hat{x}(t) \in \Re^n$ is the estimation of the plant's state. By defining $e(t) = x(t) - \hat{x}(t)$ as the estimation error, then we obtain the following state-space model:

$$\begin{aligned} \dot{x}_f(t) &= A_{f\rho}x_f(t) + A_{df\rho}x_f(t-h) + E_{f\rho}w(t), \\ z(t) - \hat{z}(t) &= L_{f\rho}x_f(t) + L_{df\rho}x_f(t-h), \end{aligned} \tag{4}$$

where $x_f(t) = \mathrm{col}\,\{x(t), e(t)\}$ and $L_{f\rho} := L_f(\rho) = [0,\ L(\rho)]$, $L_{df\rho} := L_{df}(\rho) = [0,\ L_d(\rho)]$, and

$$\begin{aligned} A_{f\rho} &:= A_f(\rho) = \begin{bmatrix} A(\rho) & 0 \\ A(\rho) - F(\rho) - G(\rho)C(\rho) & F(\rho) \end{bmatrix}, \\ A_{df\rho} &:= A_{df}(\rho) = \begin{bmatrix} A_d(\rho) & 0 \\ A_d(\rho) - F_d(\rho) - G(\rho)C_d(\rho) & F_d(\rho) \end{bmatrix}, \\ E_{f\rho} &:= E_f(\rho) = \begin{bmatrix} E_1(\rho) \\ E_1(\rho) - G(\rho)E_2(\rho) \end{bmatrix}. \end{aligned}$$

Remark 2.1 In the case of a free-delay filter, the delay-dependent filter (3) is written in the form

$$\begin{aligned} \dot{\hat{x}}(t) &= F(\rho)\hat{x}(t) + G(\rho)y(t), \\ \hat{x}(t) &= 0, \quad t \in [-h, 0], \\ \hat{z}(t) &= L(\rho)\hat{x}(t) + E_3(\rho)w(t). \end{aligned}$$

Definition 2.1 The delay-dependent robust H_∞ filter of the type (3) is said to *guarantee robust disturbance attenuation* if under zero initial condition

$$\limsup_{\rho \in \zeta} \limsup_{\|w\|_2 \neq 0} \frac{\|z(t) - \hat{z}(t)\|}{\|w(t)\|_2} \leq \gamma$$

for all bounded energy disturbances and a prescribed positive value γ.

Therefore, the main objective of the paper is to seek the state-space parameter-dependent matrices of the delay-dependent robust H_∞ filter (3) guarantees a prescribed H_∞ performance for the augmented system (4). To investigate the Lyapunov-based stability of the augmented system, one important role will be played by the search for PPDQ Lyapunov functions chosen within the following class.

Definition 2.2 We call a *polynomial parameter-dependent quadratic (PPDQ) function* any quadratic function $x^T S(\rho)x(t)$ such that

$$S(\rho) := (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)^T S_k (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)$$

for every $x(t) \in \Re^n$ and a certain $S_k \in \Re^{k^m n \times k^m n}$. The integer $k-1$ is called *the degree of the PPDQ function* $S(\rho)$ [7].

3 Delay-Dependent Robust H_∞ Filtering

In the following, it will be assumed that the delay-dependent robust H_∞ filter (3) is known and the delay-dependent stability conditions will be investigated under which the augmented system (4) is stable and satisfies the prescribed H_∞ performance for all admissible vectors $\rho \in \zeta$.

The approach employed here is to investigate the delay-dependent stability analysis of the augmented system (4) in the presence of the disturbance (or exogenous input). In the literature, extensions of the quadratic Lyapunov functions to the quadratic Lyapunov-Krasovskii functionals have been proposed for time-delayed systems [11, 12]. Now, we choose a Lyapunov-Krasovskii functional candidate for the LPV system with delayed states and outputs as

$$V(x_f(t)) = x_f(t)^T P_\rho x_f(t) + \int_{t-h}^{t} x_f(\sigma)^T Q_\rho x_f(\sigma)\, d\sigma + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}_f(\sigma)^T Z_\rho \dot{x}_f(\sigma)\, d\sigma\, d\theta \tag{5}$$

with the positive definite matrices

$$P_\rho := P(\rho) = \begin{bmatrix} P_{1\rho} & 0 \\ * & P_{2\rho} \end{bmatrix} \in \Re^{2n\times 2n}, \tag{6}$$

$$Q_\rho := Q(\rho) = \begin{bmatrix} Q_{11\rho} & Q_{12\rho} \\ * & Q_{22\rho} \end{bmatrix} \in \Re^{2n\times 2n}, \tag{7}$$

$$Z_\rho := Z(\rho) = \begin{bmatrix} Z_{11\rho} & 0 \\ * & P_{2\rho} \end{bmatrix} \in \Re^{2n\times 2n}, \tag{8}$$

where the PPDQ functions $P_{1\rho}$, $P_{2\rho}$, $Q_{11\rho}$, $Q_{22\rho}$, $Q_{12\rho}$ and $Z_{11\rho}$ satisfying the following representation forms:

$$\begin{aligned}
P_{1\rho} &:= P_1(\rho) = (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)^T P_{1,k} (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n),\\
P_{2\rho} &:= P_2(\rho) = (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)^T P_{2,k} (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n),\\
Q_{11\rho} &:= Q_{11}(\rho) = (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)^T Q_{11,k} (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n),\\
Q_{22\rho} &:= Q_{22}(\rho) = (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)^T Q_{22,k} (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n),\\
Q_{12\rho} &:= Q_{12}(\rho) = (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)^T Q_{12,k} (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n),\\
Z_{11\rho} &:= Z_{11}(\rho) = (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)^T Z_{11,k} (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)
\end{aligned} \tag{9}$$

with parameter-independent positive definite matrices
$\{P_{1,k}, P_{2,k}, Q_{11,k}, Q_{22,k}, Q_{12,k}, Z_{11,k}\} \in \Re^{k^m n\times k^m n}$ of the order $k-1$. Now, let us define a Hamiltonian function $H(x_f, w, \rho)$ as:

$$H(x_f, w, \rho) = \frac{d}{dt} V(x_f) + (z-\hat{z})^T (z-\hat{z}) - \gamma^2 w^T w. \tag{10}$$

It is known that the inequity

$$H(x_f, w, \rho) < 0 \tag{11}$$

implies the following inequality

$$\int_0^T (z - \hat{z})^T (z - \hat{z})\, dt < \gamma^2 \int_0^T w^T w\, dt + V(x_f(0)) - V(x_f(T)) < \gamma^2 \int_0^T w^T w\, dt,$$
$$\forall T > 0,\ \forall w$$

that is identical to the performance specification in Definition 2.1.

Using the Leibniz-Newton formula, we write

$$x_f(t-h) = x_f(t) - \int_{t-h}^{t} \dot{x}_f(\sigma)\, d\sigma,$$

then, for any appropriately dimensioned matrices Y_ρ, T_ρ and S_ρ, we have

$$2(x_f(t)^T Y_\rho + x_{fh}(t)^T T_\rho + w(t)^T S_\rho)\left(x_f(t) - x_f(t-h) - \int_{t-h}^{t} \dot{x}_f(\sigma)\, d\sigma\right) = 0, \tag{12}$$

which is added to the Hamiltonian function $H(x_f, w, \rho)$. On the other hand, for any semi-positive definite matrix

$$X_\rho = \begin{bmatrix} X_{11\rho} & X_{12\rho} & X_{13\rho} \\ * & X_{22\rho} & X_{23\rho} \\ * & * & X_{33\rho} \end{bmatrix} \geq 0, \tag{13}$$

the following holds

$$h\xi(t)^T X_\rho \xi(t) - \int_{t-h}^{t} \xi(t)^T X_\rho \xi(t)\, d\sigma = 0, \tag{14}$$

where $\xi(t) = \mathrm{col}\,\{x_f(t),\, x_f(t-h),\, w(t)\}$.

Calculating the time derivative of $V(x_f(t))$ along the trajectory of the augmented system (4) and replacing in Eq. (10), results in

$$H(x_f, w, \rho) = \xi(t)^T \Xi_\rho \xi(t) - \int_{t-h}^{t} \xi(t,\sigma)^T \Omega_\rho \xi(t,\sigma)\, d\sigma, \tag{15}$$

where $\xi(t,\sigma) = \mathrm{col}\,\{x_f(t),\, x_f(t-h),\, w(t),\, \dot{x}_f(\sigma)\}$,

$$\Omega_\rho = \begin{bmatrix} X_{11\rho} & X_{12\rho} & X_{13\rho} & Y_\rho \\ * & X_{22\rho} & X_{23\rho} & T_\rho \\ * & * & X_{33\rho} & S_\rho \\ * & * & * & Z_\rho \end{bmatrix},$$

and

$$\Xi_\rho = \begin{bmatrix} \Delta_{11} & \Delta_{12} & P_\rho E_{f\rho} + S_\rho^T + hX_{13\rho} + hA_{f\rho}^T Z_\rho E_{f\rho} \\ * & \Delta_{22} & -S_\rho^T + hX_{23\rho} + hA_{df\rho}^T Z_\rho E_{f\rho} \\ * & * & -\gamma^2 I_s + hX_{33\rho} + hE_{f\rho}^T Z_\rho E_{f\rho} \end{bmatrix},$$

with

$$\Delta_{11} = P_\rho A_{f\rho} + A_{f\rho}^T P_\rho + Y_\rho + Y_\rho^T + Q_\rho + hX_{11\rho} + hA_{f\rho}^T Z_\rho A_{f\rho} + L_{f\rho}^T L_{f\rho},$$
$$\Delta_{12} = P_\rho A_{df\rho} - Y_\rho + hX_{12\rho} + hA_{f\rho}^T Z_\rho A_{df\rho} + L_{f\rho}^T L_{df\rho},$$
$$\Delta_{22} = -T_\rho - T_\rho^T - Q_\rho + hX_{22\rho} + hA_{df\rho}^T Z_\rho A_{df\rho} + L_{df\rho}^T L_{df\rho}.$$

According to partitioning the existing matrices if $\Xi_\rho < 0$ and $\Omega_\rho \geq 0$, then $H(x_f, w, \rho) < 0$ for any $\xi(t) \neq 0$. Applying the Schur complement Lemma shows that inequality $\Xi_\rho < 0$ implies

$$\hat{\Pi}_\rho = \begin{bmatrix} \hat{\Delta}_{11} & \hat{\Delta}_{12} & P_\rho E_{f\rho} + S_\rho^T + hX_{13\rho} & hA_{f\rho}^T Z_\rho \\ * & \hat{\Delta}_{22} & -S_\rho^T + hX_{23\rho} & hA_{df\rho}^T Z_\rho \\ * & * & -\gamma^2 I_s + hX_{33\rho} & hE_{f\rho}^T Z_\rho \\ * & * & * & -hZ_\rho \end{bmatrix} < 0, \tag{16}$$

with $\hat{\Delta}_{11} = P_\rho A_{f\rho} + A_{f\rho}^T P_\rho + Y_\rho + Y_\rho^T + Q_\rho + hX_{11\rho} + L_{f\rho}^T L_{f\rho}$, $\hat{\Delta}_{12} = P_\rho A_{df\rho} - Y_\rho + hX_{12\rho} + L_{f\rho}^T L_{df\rho}$ and $\hat{\Delta}_{22} = -T_\rho - T_\rho^T - Q_\rho + hX_{22\rho} + L_{df\rho}^T L_{df\rho}$.

Notice that the matrix inequality (16) includes multiplication of filter matrices and Lyapunov matrices. In the literature, more attention has been paid to the problems having this nature, which called bilinear matrix inequality (BMI) problems [29]. In the sequel, it is shown that, by a suitable change of variables, the robust H_∞ filtering problem can be converted into convex programming problems written in terms of LMIs.

Remark 3.1 Considering the parameter-dependent BMI (16) in addition to partitioning the existing matrices P_ρ, Q_ρ and Z_ρ and assuming

$$\begin{bmatrix} W_{1\rho} & W_{2\rho} & W_{3\rho} \end{bmatrix} = P_{2\rho} \begin{bmatrix} F_\rho & G_\rho & F_{d\rho} \end{bmatrix}, \tag{17}$$

where $\{W_{1\rho}, W_{3\rho}\} \in \Re^{n\times n}$ and $W_{2\rho} \in \Re^{n\times p}$ leads to

$$\Pi_\rho = \begin{bmatrix} \bar{\Delta}_{11} & \bar{\Delta}_{12} & \bar{\Delta}_{13} & \bar{\Delta}_{14} & \bar{\Delta}_{15} & hA_\rho^T Z_{11\rho} & \bar{\Delta}_{17} \\ * & \bar{\Delta}_{22} & \bar{\Delta}_{23} & \bar{\Delta}_{24} & \bar{\Delta}_{25} & 0 & hW_{1\rho}^T \\ * & * & \bar{\Delta}_{33} & \bar{\Delta}_{34} & -S_{11\rho}^T + hX_{23,11\rho} & hA_{d\rho}^T Z_{11\rho} & \bar{\Delta}_{37} \\ * & * & * & \bar{\Delta}_{44} & -S_{12\rho}^T + hX_{23,21\rho} & 0 & hW_{3\rho}^T \\ * & * & * & * & -\gamma^2 I_s + hX_{33\rho} & hE_{1\rho}^T Z_{11\rho} & \bar{\Delta}_{57} \\ * & * & * & * & * & -hZ_{11\rho} & 0 \\ * & * & * & * & * & * & -hP_{2\rho} \end{bmatrix} < 0. \tag{18}$$

In (18)

$$\begin{aligned}
&\bar{\Delta}_{11} = A_\rho^T P_{1\rho} + P_{1\rho} A_\rho + Q_{11\rho} + Y_{11\rho} + Y_{11\rho}^T + hX_{11,11\rho}, \\
&\bar{\Delta}_{12} = A_\rho^T P_{2\rho} - W_{1\rho}^T - C_\rho^T W_{2\rho}^T + Y_{12\rho} + Y_{21\rho}^T + Q_{12\rho} + hX_{11,12\rho}, \\
&\bar{\Delta}_{13} = P_{1\rho} A_{d\rho} - Y_{11\rho} + hX_{12,11\rho}, \quad \bar{\Delta}_{14} = -Y_{12\rho} + hX_{12,12\rho}, \\
&\bar{\Delta}_{15} = P_{1\rho} E_{1\rho} + S_{11\rho}^T + hX_{13,11\rho}, \quad \bar{\Delta}_{17} = h(A_\rho^T P_{2\rho} - W_{1\rho}^T - C_\rho^T W_{2\rho}^T), \\
&\bar{\Delta}_{22} = W_{1\rho} + W_{1\rho}^T + L_\rho^T L_\rho + Q_{22\rho} + Y_{22\rho} + Y_{22\rho}^T + hX_{11,22\rho}, \\
&\bar{\Delta}_{23} = P_{2\rho} A_{d\rho} - W_{3\rho} - W_{2\rho} C_{d\rho}, \quad \bar{\Delta}_{24} = W_{3\rho} + L_\rho^T L_{d\rho} - Y_{22\rho} + hX_{12,22\rho}, \\
&\bar{\Delta}_{25} = P_{2\rho} E_{1\rho} - W_{2\rho} E_{2\rho} + S_{12\rho}^T + hX_{13,21\rho}, \\
&\bar{\Delta}_{33} = -Q_{11\rho} - T_{11\rho} - T_{11\rho}^T + hX_{22,11\rho}, \\
&\bar{\Delta}_{34} = -Q_{12\rho} - T_{12\rho} - T_{21\rho}^T + hX_{22,12\rho}, \quad \bar{\Delta}_{37} = h(A_{d\rho}^T P_{2\rho} - W_{3\rho}^T - C_{d\rho}^T W_{2\rho}), \\
&\bar{\Delta}_{44} = -Q_{22\rho} - T_{22\rho} - T_{22\rho}^T + L_{d\rho}^T L_{d\rho} + hX_{22,22\rho}, \\
&\bar{\Delta}_{57} = h(E_{1\rho}^T P_{2\rho} - E_{2\rho}^T W_{2\rho}^T),
\end{aligned}$$

where

$$Y_\rho = \begin{bmatrix} Y_{11\rho} & Y_{12\rho} \\ Y_{21\rho} & Y_{22\rho} \end{bmatrix}, \quad T_\rho = \begin{bmatrix} T_{11\rho} & T_{12\rho} \\ T_{21\rho} & T_{22\rho} \end{bmatrix}, \quad S_\rho = \begin{bmatrix} S_{11\rho} & S_{12\rho} \end{bmatrix},$$

$$X_{ij\rho} = \begin{bmatrix} X_{ij,11\rho} & X_{ij,12\rho} \\ * & X_{ij,22\rho} \end{bmatrix} \quad \text{and} \quad X_{i3\rho} = \begin{bmatrix} X_{i3,11\rho} \\ X_{i3,21\rho} \end{bmatrix} \quad \text{for} \quad i,j = 1,2.$$

Now, our results are summarized in the following Theorem.

Theorem 3.1 *The augmented system (4) obtained from the interconnection of the plant (1) and the filter (3) is stable and achieves the H_∞ performance for a given performance bound γ in the sense of Definition 2.1 if there exist the parameter-dependent positive definite matrices $P_\rho = P_\rho^T > 0$, $Q_\rho = Q_\rho^T > 0$ and $Z_\rho = Z_\rho^T > 0$, a symmetric semi-positive definite matrix $X_\rho \geq 0$, the parameter-dependent matrixes $W_{1\rho}$, $W_{2\rho}$, $W_{3\rho}$ and any appropriately dimensioned matrices Y_ρ, T_ρ and S_ρ such that the parameter-dependent LMIs $\Pi_\rho < 0$ and $\Omega_\rho \geq 0$ are satisfied, respectively.*

Remark 3.2 In the LPV system (1) if an uncertain time-invariant delay lies in $[0, \bar{h}]$, i.e., $h \in [0, \bar{h}]$, then according to the procedure of above it can be shown that the following inequality, which is delay-dependent H_∞ filter design criterion is concluded instead of (16):

$$\hat{\Pi}_\rho = \begin{bmatrix} \tilde{\Delta}_{11} & \tilde{\Delta}_{12} & P_\rho E_{f\rho} + S_\rho^T + \bar{h} X_{13\rho} & \bar{h} A_{f\rho}^T Z_\rho \\ * & \tilde{\Delta}_{22} & -S_\rho^T + \bar{h} X_{23\rho} & \bar{h} A_{df\rho}^T Z_\rho \\ * & * & -\gamma^2 I_s + \bar{h} X_{33\rho} & \bar{h} E_{f\rho}^T Z_\rho \\ * & * & * & -\bar{h} Z_\rho \end{bmatrix} < 0,$$

where $\tilde{\Delta}_{11} = P_\rho A_{f\rho} + A_{f\rho}^T P_\rho + Y_\rho + Y_\rho^T + Q_\rho + \bar{h} X_{11\rho} + L_{f\rho}^T L_{f\rho}$, $\tilde{\Delta}_{12} = P_\rho A_{df\rho} - Y_\rho + \bar{h} X_{12\rho} + L_{f\rho}^T L_{df\rho}$ and $\tilde{\Delta}_{22} = -T_\rho - T_\rho^T - Q_\rho + \bar{h} X_{22\rho} + L_{df\rho}^T L_{df\rho}$.

Remark 3.3 If the matrices X_ρ, Y_ρ, T_ρ and S_ρ in the matrix Ω_ρ are set to zero, and $Z_{11\rho} = \epsilon I_n$ (ϵ is a sufficiently small positive scalar), then Theorem 1 is identical to the delay-independent H_∞ filter design criterion such that the inequality (16) is stated in the following delay-independent form:

$$\begin{bmatrix} P_\rho A_{f\rho} + A_{f\rho}^T P_\rho + Q_\rho + L_{f\rho}^T L_{f\rho} & P_\rho A_{df\rho} + L_{f\rho}^T L_{df\rho} & P_\rho E_{f\rho} \\ * & -Q_\rho + L_{df\rho}^T L_{df\rho} & 0 \\ * & * & -\gamma^2 I_s \end{bmatrix} < 0.$$

In the next Section, a new framework for relaxing parameter-dependent matrix inequalities into conventional LMI problems is stated using the PPDQ functions.

4 Parameter-Dependent LMI Relaxations

This section is devoted to solve the parameter-dependent LMIs to finding the parameter-dependent state-space matrices F_ρ, $F_{d\rho}$ and G_ρ. These parameter-dependent LMIs are corresponded to infinite-dimensional convex problems. In the literature, there are some attempts to obtain a finite-dimensional optimization problem such the parameter-dependent Lyapunov functions are approximated using a finite set of basis functions [2, 30, 32, 36].

The main approach employed here is using the PPDQ functions as the basis functions to relax parameter-dependent LMIs into parameter-independent LMI forms by utilizing some multiplier matrices.

Lemma 4.1 *Let the degree of the PPDQ function $P_{1\rho}$ be $k-1$. A PPDQ function of degree k for parameter-dependent matrix $P_{1\rho}T_\rho$ is given by*

$$P_{1\rho}T_\rho := (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T S_k (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_q),$$

where $T_\rho = T_0 + \sum_{i=1}^{m} \rho_i T_i$ and $T_i \in \Re^{n\times q}$, then the parameter-independent matrix $S_k \in \Re^{(k+1)^m n \times (k+1)^m q}$ which depends on the parameter-independent matrix $P_{1,k}$ linearly is defined as

$$S_k = (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{1,k} (\hat{J}_k^{m\otimes} \otimes T_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes T_i).$$

According to Lemma 4.1 for the parameter-dependent matrices $E_{1\rho}$, A_ρ and $A_{d\rho}$, we obtain

$$\begin{aligned}
P_{1\rho}E_{1\rho} &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T \Xi_{1,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_s),\\
P_{2\rho}E_{1\rho} &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T \Xi_{2,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_s),\\
Z_{11\rho}E_{1\rho} &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T \Xi_{3,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_s),\\
P_{2\rho}A_{\rho} &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T S_{1,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n),\\
Z_{11\rho}A_{\rho} &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T S_{2,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n),\\
P_{1\rho}A_{d\rho} &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T S_{1d,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n),\\
P_{2\rho}A_{d\rho} &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T S_{2d,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n),\\
Z_{11\rho}A_{d\rho} &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T S_{3d,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n),
\end{aligned} \tag{19}$$

where the parameter-independent matrices $\Xi_{1,k}$, $\Xi_{2,k}$, $\Xi_{3,k}$, $S_{1,k}$, $S_{2,k}$, $S_{1d,k}$, $S_{2d,k}$ and $S_{3d,k}$ are represented in the following forms:

$$\begin{aligned}
\Xi_{1,k} &= (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{1,k} \left(\hat{J}_k^{m\otimes} \otimes E_{01} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes E_{j1} \right),\\
\Xi_{2,k} &= (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{2,k} \left(\hat{J}_k^{m\otimes} \otimes E_{01} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes E_{j1} \right),\\
\Xi_{3,k} &= (\hat{J}_k^{m\otimes} \otimes I_n)^T Z_{11,k} \left(\hat{J}_k^{m\otimes} \otimes E_{01} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes E_{j1} \right),\\
S_{1,k} &= (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{2,k} \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_j \right),\\
S_{2,k} &= (\hat{J}_k^{m\otimes} \otimes I_n)^T Z_{11,k} \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_j \right),\\
S_{1d,k} &= (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{1,k} \left(\hat{J}_k^{m\otimes} \otimes A_{0d} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_{jd} \right),\\
S_{2d,k} &= (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{2,k} \left(\hat{J}_k^{m\otimes} \otimes A_{0d} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_{jd} \right),
\end{aligned} \tag{20}$$

$$S_{3d,k} = (\hat{J}_k^{m\otimes} \otimes I_n)^T Z_{11,k}\left(\hat{J}_k^{m\otimes} \otimes A_{0d} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_{jd}\right).$$

Remark 4.1 For the parameter-dependent matrix $R_{1\rho} := A_\rho^T P_{1\rho} + P_{1\rho} A_\rho$ the PPDQ function of degree k is given by

$$R_{1\rho} = (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T R_{1,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n),$$

and from Lemma 4.1, the parameter-independent positive definite matrices $R_{1,k} \in \Re^{(k+1)^m n \times (k+1)^m n}$ which depends on the parameter-independent matrix $P_{1,k}$ linearly is obtained as follows:

$$\begin{aligned} R_{1,k} = (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{1,k}\left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_i\right) \\ + \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_i\right)^T P_{1,k} (\hat{J}_k^{m\otimes} \otimes I_n). \end{aligned} \tag{21}$$

The parameter-dependent matrices $W_{1\rho}$, $W_{2\rho}$ and $W_{3\rho}$ in (17) can be expressed in the forms

$$\begin{aligned} W_{1\rho} &= (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)^T W_{1,k} (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n), \\ W_{2\rho} &= (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)^T W_{2,k} (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_p), \\ W_{3\rho} &= (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n)^T W_{3,k} (\rho_m^{[k]} \otimes \ldots \otimes \rho_1^{[k]} \otimes I_n) \end{aligned} \tag{22}$$

with parameter-independent matrices $\{W_{1,k},\, W_{3,k}\} \in \Re^{k^m n \times k^m n}$ and $\{\bar{W}_{2,k},\, \bar{W}_{2d,k},\, \tilde{W}_{2,k}\} \in \Re^{(k+1)^m n \times (k+1)^m p}$. Then, the following relations can be concluded

$$\begin{aligned} W_{2\rho} C_\rho &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T \bar{W}_{2,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_p), \\ W_{2\rho} C_{d\rho} &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T \bar{W}_{2d,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_p), \\ W_{2\rho} E_{2\rho} &= (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n)^T \tilde{W}_{2,k} (\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_p), \end{aligned} \tag{23}$$

where the parameter-independent matrices $\{W_{1,k}, W_{3,k}\} \in \Re^{k^m n \times k^m n}$ are defined, respectively, as

$$\begin{aligned} \bar{W}_{2,k} &= (\hat{J}_k^{m\otimes} \otimes I_n)^T W_{2,k}\left(\hat{J}_k^{m\otimes} \otimes C_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes C_i\right), \\ \bar{W}_{2d,k} &= (\hat{J}_k^{m\otimes} \otimes I_n)^T W_{2d,k}\left(\hat{J}_k^{m\otimes} \otimes C_{0d} + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes C_{id}\right), \\ \tilde{W}_{2,k} &= (\hat{J}_k^{m\otimes} \otimes I_n)^T W_{2,k}\left(\hat{J}_k^{m\otimes} \otimes E_{02} + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes E_{i2}\right). \end{aligned} \tag{24}$$

Similarly, the parameter-dependent matrices $L_\rho^T L_\rho$ and $L_{d\rho}^T L_{d\rho}$ and the parameter-independent matrix I_s can also be represented, respectively, by

$$\begin{aligned} L_\rho^T L_\rho &= \left(\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n\right)^T \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \bar{L}_k \\ &\quad \times \left(\hat{J}_k^{m\otimes} \otimes I_n\right)\left(\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n\right), \\ L_{d\rho}^T L_{d\rho} &= \left(\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n\right)^T \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \bar{L}_{dk} \\ &\quad \times \left(\hat{J}_k^{m\otimes} \otimes I_n\right)\left(\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_n\right), \\ I_s &= \left(\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_s\right)^T \left(\hat{J}_k^{m\otimes} \otimes I_s\right)^T \bar{I}_k \\ &\quad \times \left(\hat{J}_k^{m\otimes} \otimes I_s\right)\left(\rho_m^{[k+1]} \otimes \ldots \otimes \rho_1^{[k+1]} \otimes I_s\right), \end{aligned} \tag{25}$$

where the certain parameter-independent matrices $\bar{L}_k \in \Re^{k^m n \times k^m n}$, $\bar{L}_{dk} \in \Re^{k^m n \times k^m n}$ and $\bar{I}_k \in \Re^{k^m s \times k^m s}$ are given by

$$\begin{aligned} \bar{L}_k &= \text{Block diagonal } \left(\begin{bmatrix} L_0^T \\ \vdots \\ L_m^T \end{bmatrix} \begin{bmatrix} L_0 & \ldots & L_m \end{bmatrix}, \underbrace{0_n, \ldots, 0_n}_{(k^m - m - 1) \text{ elements}} \right), \\ \bar{L}_{dk} &= \text{Block diagonal } \left(\begin{bmatrix} L_{0d}^T \\ \vdots \\ L_{md}^T \end{bmatrix} \begin{bmatrix} L_{0d} & \ldots & L_{md} \end{bmatrix}, \underbrace{0_n, \ldots, 0_n}_{(k^m - m - 1) \text{ elements}} \right), \\ \bar{I}_k &= \text{Block diagonal } \left(I_s, \underbrace{0_s, \ldots, 0_s}_{(k^m - 1) \text{ elements}} \right). \end{aligned} \tag{26}$$

We are now in the position to state our main results on parameter-dependent robust H_∞ filter design based on LMI approach in the following theorem.

Theorem 4.1 *Let the positive integer $k-1$ as the degree of the PPDQ functions be given. Consider the LPV system (1) with the known time-delay parameter h. For a given performance bound γ, if there exist the set of parameter-independent matrices $\{W_{1,k}, W_{2,k}, W_{3,k}, X_{11,11k}, X_{12,11k}, X_{12,21k}, X_{12,12k}, X_{12,22k}, X_{22,12k}, X_{13,11k}, X_{13,21k}, X_{23,11k}, X_{23,21k}, Y_{11,k}, Y_{12,k}, Y_{21,k}, Y_{22,k}, T_{11,k}, T_{12,k}, T_{21,k}, T_{22,k}, S_{11,k}, S_{12,k}\}$, the set of parameter-independent positive definite matrices $\{P_{1,k}, P_{2,k}, Q_{11,k}, Q_{22,k}, Z_{11,k}, X_{11,11k}, X_{11,22k}, X_{22,11k}, X_{22,22k}, X_{33,k}\}$ and the set of positive definite multipliers $\{\hat{Q}_{i,k}^{(1)}, \ldots, \hat{Q}_{i,k}^{(7)}, \tilde{Q}_{i,k}^{(1)}, \ldots, \tilde{Q}_{i,k}^{(7)}, \bar{Q}_{i,k}^{(1)}, \bar{Q}_{i,k}^{(2)}\}$ for $i = 1, 2, \ldots, m$ to the following LMIs,*

$$\Omega_{m,k} = \begin{bmatrix} \Psi_{11} & X_{11,12k} & X_{12,11k} & X_{12,12k} & X_{13,11k} & Y_{11,k} & Y_{12,k} \\ * & \Psi_{22} & X_{12,21k} & X_{12,22k} & X_{13,21k} & Y_{21,k} & Y_{22,k} \\ * & * & \Psi_{33} & X_{22,12k} & X_{23,11k} & T_{11,k} & T_{12,k} \\ * & * & * & \Psi_{44} & X_{23,21k} & T_{21,k} & T_{22,k} \\ * & * & * & * & \Psi_{55} & S_{11,k} & S_{12,k} \\ * & * & * & * & * & \Psi_{66} & 0 \\ * & * & * & * & * & * & \Psi_{77} \end{bmatrix} \geq 0, \tag{27}$$

$$\Pi_{m,k} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & hS_{2,k}^T & \Sigma_{17} \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} & 0 & \Sigma_{27} \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} & hS_{3d,k}^T & \Sigma_{37} \\ * & * & * & \Sigma_{44} & \Sigma_{45} & 0 & \Sigma_{47} \\ * & * & * & * & \Sigma_{55} & h\Xi_{3,k}^T & h\left(\Xi_{2,k}^T - \tilde{W}_{2,k}^T\right) \\ * & * & * & * & * & \Sigma_{66} & 0 \\ \star & * & * & * & * & * & \Sigma_{77} \end{bmatrix} < 0, \tag{28}$$

$$\Phi_{m,k} = \begin{bmatrix} \Lambda_{11} & Q_{12,k} \\ * & \Lambda_{22} \end{bmatrix} > 0, \tag{29}$$

where

$$\Psi_{11} = X_{11,11k} - \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(1)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)$$
$$+ \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(1)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{22} = X_{11,22k} - \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(2)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)$$
$$+ \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(2)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{33} = X_{22,11k} - \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(3)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)$$
$$+ \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(3)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{44} = X_{22,22k} - \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(4)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)$$
$$+ \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(4)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{55} = X_{33,k} - \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(5)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)$$
$$+ \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(5)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{66} = Z_{11,k} - \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(6)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)$$
$$+ \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(6)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{77} = P_{2,k} - \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(7)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)$$

$$
\begin{aligned}
&+ \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \hat{Q}_{i,k}^{(7)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),\\
\Sigma_{11} &= R_{1,k} + \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(Q_{1,k} + Y_{11,k} + Y_{11,k}^T + hX_{11,11k}\right) \left(\hat{J}_k^{m\otimes} \otimes I_n\right)\\
&+ \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(1)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)\\
&- \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(1)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),\\
\Sigma_{12} &= S_{1,k}^T - \bar{W}_{2,k}^T\\
&+ \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(-W_{1,k}^T + Y_{12,k} + Y_{21,k}^T + Q_{12,k} + hX_{11,12k}\right) \left(\hat{J}_k^{m\otimes} \otimes I_n\right),\\
\Sigma_{13} &= S_{1d,k} + \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(-Y_{11,k} + hX_{12,11k}\right) \left(\hat{J}_k^{m\otimes} \otimes I_n\right),\\
\Sigma_{14} &= \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(-Y_{12,k} + hX_{12,12k}\right) \left(\hat{J}_k^{m\otimes} \otimes I_n\right),\\
\Sigma_{15} &= \Xi_{1,k} + \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(S_{11,k}^T + hX_{13,11k}\right) \left(\hat{J}_k^{m\otimes} \otimes I_s\right),\\
\Sigma_{17} &= h\left(S_{1,k}^T - \bar{W}_{1,k}^T - \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T W_{1,k}^T \left(\hat{J}_k^{m\otimes} \otimes I_n\right)\right),\\
\Sigma_{22} &= \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \Big(W_{1,k} + W_{1,k}^T + \bar{L}_k + Q_{22,k}\\
&+ Y_{22,k} + Q_{22,k}^T + hX_{11,22k}\Big) \left(\hat{J}_k^{m\otimes} \otimes I_n\right)\\
&+ \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(2)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)\\
&- \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(2)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),\\
\Sigma_{23} &= S_{2d,k} - \bar{W}_{2d,k}^T - \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T W_{3,k} \left(\hat{J}_k^{m\otimes} \otimes I_n\right),\\
\Sigma_{24} &= \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(W_{3,k} + \bar{L}_k - Y_{22,k} + hX_{12,22k}\right) \left(\hat{J}_k^{m\otimes} \otimes I_n\right),\\
\Sigma_{25} &= \Xi_{2,k} - \tilde{W}_{2,k} + \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(S_{12,k}^T + hX_{13,21k}\right) \left(\hat{J}_k^{m\otimes} \otimes I_s\right),\\
\Sigma_{27} &= h\left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T W_{1,k}^T \left(\hat{J}_k^{m\otimes} \otimes I_n\right),\\
\Sigma_{33} &= \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(-Q_{11,k} - T_{11,k} - T_{11,k}^T + hX_{22,11k}\right) \left(\hat{J}_k^{m\otimes} \otimes I_n\right)\\
&+ \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(3)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)\\
&- \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(3)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),\\
\Sigma_{34} &= -\left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(Q_{12,k} + T_{12,k} + T_{12,k}^T - hX_{22,12k}\right) \left(\hat{J}_k^{m\otimes} \otimes I_n\right),
\end{aligned}
$$

$$\Sigma_{35} = -\left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(S_{11,k}^T - hX_{23,11k}\right)\left(\hat{J}_k^{m\otimes} \otimes I_s\right),$$

$$\Sigma_{37} = h\left(S_{2d,k}^T - \bar{W}_{2d,k}^T - \left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T W_{3,k}^T \left(\hat{J}_k^{m\otimes} \otimes I_n\right)\right),$$

$$\begin{aligned}\Sigma_{44} = &-\left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(Q_{22,k} + T_{22,k} + T_{22,k}^T - \bar{L}_{d,k} - hX_{22,22k}\right)\left(\hat{J}_k^{m\otimes} \otimes I_n\right)\\ &+ \sum_{i=1}^m \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(4)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)\\ &- \sum_{i=1}^m \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(4)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),\end{aligned}$$

$$\Sigma_{45} = -\left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T \left(S_{12,k}^T - hX_{23,21k}\right)\left(\hat{J}_k^{m\otimes} \otimes I_s\right),$$

$$\Sigma_{47} = h\left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T W_{3,k}^T \left(\hat{J}_k^{m\otimes} \otimes I_n\right),$$

$$\begin{aligned}\Sigma_{55} = &\left(\hat{J}_k^{m\otimes} \otimes I_s\right)^T \left(-\gamma^2 \bar{I}_k + hX_{33,k}\right)\left(\hat{J}_k^{m\otimes} \otimes I_s\right)\\ &+ \sum_{i=1}^m \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(5)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)\\ &- \sum_{i=1}^m \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(5)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),\end{aligned}$$

$$\begin{aligned}\Sigma_{66} = &-h\left(\hat{J}_k^{m\otimes} \otimes I_s\right)^T Z_{11,k} \left(\hat{J}_k^{m\otimes} \otimes I_s\right)\\ &+ \sum_{i=1}^m \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(6)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)\\ &- \sum_{i=1}^m \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(6)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),\end{aligned}$$

$$\begin{aligned}\Sigma_{77} = &-h\left(\hat{J}_k^{m\otimes} \otimes I_n\right)^T P_{2,k} \left(\hat{J}_k^{m\otimes} \otimes I_n\right)\\ &+ \sum_{i=1}^m \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(7)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)\\ &- \sum_{i=1}^m \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \tilde{Q}_{i,k}^{(7)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),\end{aligned}$$

$$\begin{aligned}\Lambda_{11} = &\, Q_{11,k} + \sum_{i=1}^m \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \bar{Q}_{i,k}^{(1)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)\\ &- \sum_{i=1}^m \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right)^T \bar{Q}_{i,k}^{(1)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n}\right),\end{aligned}$$

$$\Lambda_{22} = Q_{22,k} + \sum_{i=1}^m \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)^T \bar{Q}_{i,k}^{(2)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n}\right)$$

$$-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\bar{Q}_{i,k}^{(2)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right).$$

Then the state-space parameter-dependent matrices for the delay-dependent robust H_∞ filter of the type (3) which achieve the asymptotic stability and H_∞ performance, simultaneously, in the sense of Definition 2.1 are given by

$$\begin{bmatrix}F_\rho & G_\rho & F_{d\rho}\end{bmatrix} = P_{2\rho}^{-1}\begin{bmatrix}W_{1\rho} & W_{2\rho} & W_{3\rho}\end{bmatrix}. \tag{30}$$

Notice that the conditions (27)–(29) are sufficient conditions to both asymptotic stability and H_∞ performance in the sense of Definition 2.1. Moreover, Theorem 4.1 gives a sub-optimal solution to the delay-dependent robust H_∞ filtering and this result can be reformulated as an optimal H_∞ filter by solving the following convex optimization problem

$$\min_{\text{subject to (27), (28), and (29) with } \lambda := \gamma^2} \lambda.$$

Remark 4.2 It is observed that the parameter-independent LMIs (27)–(29) are linear in the set of matrices which are calculated independently from the vector ρ.

Remark 4.3 A new set of matrices verifying $\Omega_{m,k+1} \geq 0$, $\Pi_{m,k+1} < 0$ and $\Phi_{m,k+1} > 0$ can be generated, with index $k+1$ instead of k in (27)–(29), respectively. In this case, the solvability of $\Omega_{m,k} \geq 0$, $\Pi_{m,k} < 0$ and $\Phi_{m,k} > 0$ implies the same property for the larger values of the index k.

5 Example

Consider the following state-space matrices for the LPV state-delayed system (case $m = 1$ and $r = 1$)

$$A_0 = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}, \quad A_1 = \begin{bmatrix}0 & 0.2\\ 0 & 0.1\end{bmatrix}, \quad A_{0d} = \begin{bmatrix}0 & 0.1\\ -0.2 & -0.3\end{bmatrix}, \quad A_{1d} = \begin{bmatrix}0.2 & 0\\ 0.1 & 0\end{bmatrix},$$
$$E_{01} = \begin{bmatrix}-0.2\\ -0.2\end{bmatrix}, \quad C_0 = \begin{bmatrix}0 & 1\\ 0.5 & 0\end{bmatrix}, \quad E_{02} = \begin{bmatrix}0\\ 1\end{bmatrix}, \quad L_0 = I_2.$$

Assume that the compact set of the parameter ρ is $\zeta = [-1, 1]$. By considering $k = 2$ and the performance bound $\gamma = 0.9$, the delay-dependent robust H_∞ filter synthesis is solved using the Lmitool toolbox of the Matlab software [14]. By considering the parameter $\rho = 0.4$, the result of simulations for constant delay parameter $h = 0.1$ sec. and a unit step disturbance are shown in Figures 1 and 2. These figures show the plant and filter states trajectory. It is observed that the delay-dependent filter is doing well to estimate the plant states.

6 Conclusion

The delay-dependent robust H_∞ filtering problem for a class of LPV systems with constant delay in the states and outputs has been studied in this paper. By using the Leibniz-Newton formula and a suitable change of variables, some new parameter-dependent delay-dependent stability conditions are established in terms of LMIs such that the filtering process remains asymptotically stable and satisfies a prescribed H_∞

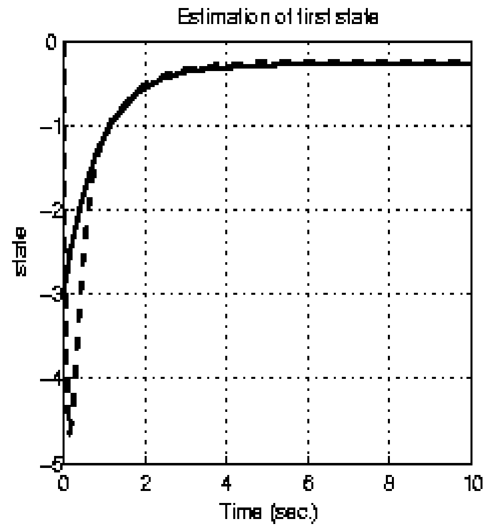

Figure 5.1: Estimation results of the first state (for $h = 0.1$ sec.): real state (solid line), and result of estimation with delay-dependent robust H_∞ filter (dashed line).

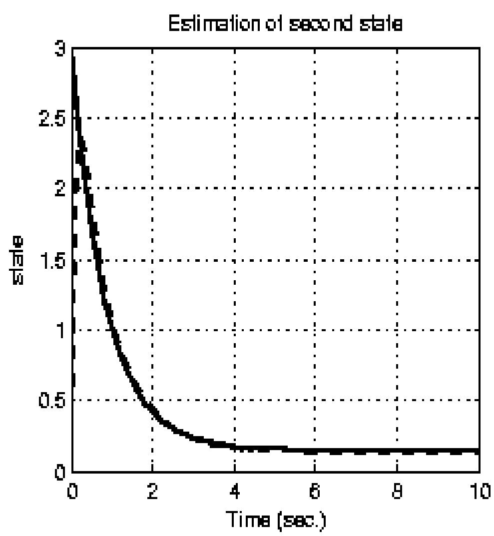

Figure 5.2: Estimation results of the second state (for $h = 0.1$ sec.): real state (solid line), and result of estimation with delay-dependent robust H_∞ filter (dashed line).

performance level. Moreover, using the PPDQ functions and some multiplier matrices, the parameter-independent delay-dependent conditions are developed with high precision under which the desired robust H_∞ filters exist and the explicit expression of these filters is derived. A numerical example has been provided to demonstrate the usefulness of the theory developed.

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