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On a Class of Manifolds for Sliding Mode Control and Observation of Induction Motor

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Abstract: The aim of this paper is to develop a general class of manifolds on which sliding mode flux observation and control of induction motors are achieved. For flux-speed tracking, we consider the case where the sliding surface is formed by the derivative of the output tracking error and a function of this error. For flux observation, the surface is a function of the estimated error. At first, we will derive the properties that must be fulfilled by the above class of manifolds in order to attain the control and observation objective. Then, we design the control law and the observer gains to make the proposed manifolds globally attractive and invariant. Simulations results are given to highlight the performances of the proposed control method.

Keywords: *Induction motor; manifold; sliding mode control; sliding observer; robustness; global stability.*

Mathematics Subject Classification (2000): 93C15, 93D05.

1 Introduction

Today, the developments of electrical machine drive grow more and more in order to follow the increasing need for various fields such as industry, electric cars, actuators, etc. By means of electrical machine drive, we can get high level of productivity in industry and product quality enhancement. The induction motor is the motor of choice in many industrial applications due to its reliability, ruggedness and relatively low cost. Nevertheless, controlling induction motors has been not easy due to significant nonlinear characteristics and the imprecise knowledge of its physical parameters.

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The control of induction motors has attracted much attention in the last decades. The vector control provides decoupling control of torque and flux similar to the control of separately excited DC motor. However, this decoupling is achieved only if the instantaneous rotor flux angle is precisely known [1]. The accuracy of the knowledge of the field rotor position affects greatly the control performances. Meanwhile, this accuracy is related to the method chosen for the position field determination.

The direct field oriented control method, where the position field is known by measuring the rotor flux of motor, is robust against parameter variation due to feedback flux [2]. The manufacturers avoid this method because it requires specially prepared machines in order to install flux sensor that rises the motor cost and decreases its reliability.

In the indirect orientation field control, the position field is deduced from speed rotor and the q component stator current [8]. The latter method needs the exact machine parameters. Hence it is very sensitive to parametric variations. Many works found in the literature over the last decade are devoted to the robust field orientation in order to overcome or to compensate the increasing resistances or saturation effects [9, 10, 11].

Another way to control induction machine is to apply the nonlinear control theory that covers many aspects such as nonlinear feedback linearization, passivity approach and sliding mode control.

The nonlinear feedback linearization allows to make the dynamic of induction machine fully or partially linearized. Its major drawback comes from the fact that it requires relatively complicated differential geometry and the precise value of parameters [4, 12, 13, 14].

The passivity theory is developed for AC machines in [15] and experimental results for induction machines are given in [16]. The main idea behind the passivity based controller design is to reshape the system natural energy and inject the required damping in such a way that the control objective is achieved.

The sliding mode theory is widely applied in the field of electrical machine drive. This success is due to the fact that the design methodology is easy. Moreover, the technical constraints limits are removed, since the theoretical conditions of the sliding mode theory are actually best accomplished in practice: the new electronic power devices allow a high limit of switching frequency, and the high performance DSP ensures a weak computational time.

Furthermore, the sliding mode control of the induction machine allows obtaining excellent properties of robustness against the parametric variation [6, 17, 18, 19, 20]. This advantage is, nevertheless, attained at the expense of large control effort that produces the well known chattering phenomenon.

Beside, the flux machine is not measured but it is estimated through an observer. The problem of estimating flux has been tackled from different point of view. The classical Luenberger observer for flux estimation was first developed in [21, 22]. The extended Kalman filter is used in [23] to estimate both the flux and the rotor resistance. To cope with parameter variations, adaptive versions of the above observers are developed in [24] and [25]. Motivated by the attractive robustness properties of the sliding mode, a variable structure flux observer is proposed in [6, 20, 3].

In this paper, we consider invariant manifold technique to control flux-speed and to estimate rotor flux of induction machine. To this end, we develop a wide class of surfaces and we search the properties that must be fulfilled in order to achieve our control objective. Then, we design the control law (or the observer gain) to make the developed surfaces globally attractive and invariant. Conditions that ensure internal stability as well as the stability of the coupling between the flux observer and the control law are given. Some simulation results involving a $3.7Kw$ induction machine are also proposed.

2 Problem Formulation

Our problem consists in developing a general class of manifolds, for sliding mode control to achieve flux-speed tracking and, for flux observation, in the case of induction motor (the reader is referred to [7] for a general theory on design and control of induction motors). To do so, we give firstly the induction machine model. In the stator reference frame, the state space model of voltage fed induction machine is obtained from Park's model. The state vector is composed of the stator current components (i_α, i_β), rotor flux components $(\varphi_\alpha, \varphi_\beta)$ and rotor rotating pulsation ω_r, whereas a vector control is composed of the stator voltage components $(v_\alpha, v_\beta\ )$ and the external disturbance is represented by the load torque Γ_r. In the sequel, the state vector and the control vector are given respectively by: $x = (i_\alpha, i_\beta, \varphi_\alpha, \varphi_\beta, \omega_r)^T$, and $u = (v_\alpha, v_\beta)^T$. Using these notations, the state space model of voltage fed induction machine takes the form:

$$\begin{aligned}
\dot{x}_1 &= f_1(x) + d_1u_1, & f_1(x) &= -a_1x_1 + b_1x_3 + c_1x_4x_5,\\
\dot{x}_2 &= f_2(x) + d_1u_2, & f_2(x) &= -a_1x_2 + b_1x_4 - c_1x_3x_5,\\
\dot{x}_3 &= f_3(x), & f_3(x) &= a_3x_1 - b_3x_3 - x_4x_5,\\
\dot{x}_4 &= f_4(x), & f_4(x) &= a_3x_2 - b_3x_4 + x_3x_5,\\
\dot{x}_5 &= f_5(x), & f_5(x) &= -a_5x_5 - b_5x_1x_4 + b_5x_2x_3 - c_5\Gamma_r.
\end{aligned} \tag{1}$$

The coefficients $(a_1, \dots, c_5\ )$ are given by $a_1 = \dfrac{1}{\sigma T_s} + \dfrac{1-\sigma}{\sigma T_r}$, $b_1 = \dfrac{(1-\sigma)}{\sigma M T_r}$, $c_1 = \dfrac{(1-\sigma)}{\sigma M}$, $d_1 = \dfrac{1}{\sigma L_s}$, $a_3 = \dfrac{M}{T_r}$, $b_3 = \dfrac{1}{T_r}$, $a_5 = \dfrac{k_f}{J}$, $b_5 = \dfrac{p^2M}{JL_r}$, $c_5 = \dfrac{p}{J}$, $\sigma = 1 - \dfrac{M^2}{L_sL_r}$ where: T_r, T_s are the stator and rotor electric time constant; σ is the leakage coefficient; L_s, L_r are the stator inductance, the rotor inductance; M is the mutual inductance between stator and rotor; k_f is the friction coefficient and Γ_r is the load torque; J is the inertia; p is the number of poles pairs.

Our objective is to control rotor speed ω_r and rotor magnitude flux given by $\phi = x_3^2 + x_4^2$. In the sequel, the flux dynamic is needed so it is given by:

$$\dot{\phi} = f_\phi(x) = -2b_3\phi + 2a_3(x_3x_1 + x_4x_2). \tag{2}$$

Hence, the augmented plant dynamic is as follows:

$$\begin{aligned}
\dot{e}_1 &= f_5(x) - \dot{w}_{ref} \quad \text{with} \quad e_1 = w_r - w_{ref},\\
\dot{e}_2 &= f_\phi(x) - \dot{\phi}_{ref} \quad \text{with} \quad e_2 = \phi - \phi_{ref},\\
\dot{x}_1 &= f_1(x) + d_1u_1,\\
\dot{x}_2 &= f_2(x) + d_1u_2,\\
\dot{x}_3 &= f_3(x),\\
\dot{x}_4 &= f_4(x),\\
\dot{x}_5 &= f_5(x).
\end{aligned} \tag{3}$$

Here ϕ_{ref} , w_{ref} are the desired flux and the desired speed respectively.

To solve our control problem, we will proceed as follows:

Step 1: We characterize a class of manifolds on which flux-speed tracking (respectively flux observation) is achieved.

Step 2: We design the control law u (respectively the observer gains) that makes, the manifolds introduced in step1, attractive and invariant.

3 Design of the Control Manifold

In this section, our goal is to characterize a class of manifolds on which speed and flux tracking is achieved. Recall that, the sliding mode control objective consists in designing a suitable manifold $M(x,t) \in R^m$ defined by $M = \{x \in R^n: \ \Psi(x) = 0\}$; so that the state trajectories of the plant restricted to this manifold have a desired behavior such as tracking, regulation and stability. Then, determine a switching control law, $u(x,t)$, that is able to drive the state trajectory to this manifold and maintain it on $M(x,t)$, once intercepted, for all subsequent time. That is, $u(x,t)$ is determined such that the selected manifold $M(x,t)$ is made attractive and invariant.

Similarly, the basic sliding mode observer design procedure is performed in two steps. Firstly, design an attractive manifold $S_c(y,t) \in R^p$ so that the output estimation error trajectories restricted to $S_c(y,t)$, have a desired stable dynamics. In the second step, determine the observer gain, to stabilize the equivalent dynamic on $S_c(y,t)$.

In [26] the authors give a form of this surface which is a Hurwitz polynomial of the error and its derivative up to $r-1$, where r is the relative degree of the output.

From the fact that the outputs ϕ and ω_r are of relative degree two and in order to obtain static feedback we propose the manifolds $\Psi = (\Psi_1(e_1) \ \ \Psi_2(e_2))^T$ defined in [26]:

$$\begin{aligned} \Psi_1(e_1) &= \{x \in R^5: \quad S_1(e_1) = \dot{e}_1 + \Lambda_1(e_1) = 0\}, \\ \Psi_2(e_2) &= \{x \in R^5: \quad S_2(e_2) = \dot{e}_2 + \Lambda_2(e_2) = 0\} \end{aligned} \tag{4}$$

with $S = (S_1, \ S_2)^T$, and where $\Lambda_1(\cdot)$ and $\Lambda_2(\cdot)$ are any given class C^1 functions whose properties will be derived below. One has the following result:

Proposition 3.1 *Consider the manifold Ψ defined in (4), and assume that $\Lambda_1(\cdot)$ and $\Lambda_2(\cdot)$ are continuous functions such that $e_i\Lambda_i(e_i) > 0 \ \forall e_i \neq 0$ $(i = 1,2)$. Then, on the manifold Ψ the outputs errors e_1, e_2 converge at least asymptotically to zero.*

Proof Due to the form of manifold $\Psi(x)$, one has:

$$\dot{e}_i = -\Lambda_i(e_i), \quad i = 1,2. \tag{5}$$

Let us use the Lyapunov function given by $V_1 = \frac{1}{2}e_1^2$ and $V_2 = \frac{1}{2}e_2^2$. Their derivatives are then:

$$\dot{V}_i = -e_i\Lambda_i(e_i), \quad i = 1,2. \tag{6}$$

In order to make $\dot{V}_1$ and $\dot{V}_2$ negative definite, it is enough that $e_i\Lambda_i(e_i) > 0 \ \forall e_i \neq 0$ $(i = 1,2)$. Hence the output errors e_1 and e_2 are bounded and moreover they tend at least asymptotically to zero. □

Remark 3.1 For example, the functions $\Lambda_1(\cdot)$ and $\Lambda_2(\cdot)$, can be taken as the two following functions and their linear combination with positive real coefficients: e_i^k with k odd natural number, and $\sinh(e_i)$.

Remark 3.2 From the literature, it is noticed that in [20, 21], the proposed sliding surface corresponds to the case of: $\Lambda_i(e_i) = ke_i$ with $k > 0$ and we obtain exponential convergence of the tracking errors e_i.

In the sequel, we show how to design the control signal u such that the selected manifold Ψ is attractive and invariant.

Proposition 3.2 *Consider the manifold $\Psi = (\Psi_1(e_1) \quad \Psi_2(e_2))^T$ defined in (4) and let the control signal u be given by*

$$\begin{aligned} u &= u_e + u_i, \\ u_i &= -A^{-1}(x)M\,\mathrm{sign}(\mathrm{S}), \\ u_e &= -A^{-1}(x)(B(x) + C(x)) \end{aligned} \tag{7}$$

with $m_i > 0$, $i = 1, 2$, where

$$A(x) = \begin{pmatrix} -b_5d_1x_4 & b_5d_1x_3 \\ 2a_3d_1x_3 & 2a_3d_1x_4 \end{pmatrix}, \quad B(x) = \begin{pmatrix} B_1(x) \\ B_2(x) \end{pmatrix}, \quad C(x) = \begin{pmatrix} C_1(x) \\ C_2(x) \end{pmatrix},$$

$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad \mathrm{sign}(S) = \begin{pmatrix} \mathrm{sign}(S_1) \\ \mathrm{sign}(S_2) \end{pmatrix},$$

$$\begin{aligned} B_1(x) &= -a_5f_5(x) + b_5[-x_4f_1(x) - x_1f_4(x) + x_3f_2(x) + x_2f_3(x)] - \ddot{\omega}_{ref}, \\ B_2(x) &= -2b_3f_\phi(x) + 2a_3[x_3f_1(x) + x_1f_3(x) + x_4f_2(x) + x_2f_4(x)] - \ddot{\phi}_{ref}, \\ C_1(x) &= (f_5(x) - \dot{\omega}_{ref})\frac{d\Lambda_1}{de_1}, \\ C_2(x) &= (f_\phi(x) - \dot{\phi}_{ref})\frac{d\Lambda_2}{de_2}, \end{aligned}$$

where $f_i(x)$ for $i = 1, \dots, 5$ are given in (1) while $f_\phi(x)$ is given in (2) and the functions $\Lambda_1(\cdot)$ and $\Lambda_2(\cdot)$ are characterized in Proposition 3.1. Then, Ψ is globally attractive and invariant.

Proof Let us consider the following Lyapunov function candidate $V = \frac{1}{2}S^TS$, its time derivative is then

$$\dot{V} = S^T\dot{S}, \tag{8}$$

where

$$\dot{S} = B(x) + C(x) + A(x)U. \tag{9}$$

With the control law given by

$$U = -A^{-1}(x)[B(x) + C(x) + M\,\mathrm{sign}(S)] \tag{10}$$

the surface dynamic $\dot{S}$ can be rewritten in the form

$$\dot{S} = -M\,\mathrm{sign}(S). \tag{11}$$

With relation (11), the expression (8) takes the form

$$\dot{V} = -m_1S_1\,\mathrm{sign}(S_1) - m_2S_2\,\mathrm{sign}(S_2). \tag{12}$$

In order to make $\dot{V}$ negative $\forall\, S \neq 0$, it is sufficient to take coefficients m_1 and m_2 as

$$m_i > 0, \quad i = 1, 2. \tag{13}$$

This condition makes $(S = 0)$, and hence Ψ is globally attractive. Furthermore since $\dot{S} = 0$, Ψ is invariant. □

Remark 3.3 The determination of the input vector u is possible only if the matrix $A(x)$ has an inverse. Its determinant given by $2a_3b_5d_1(x_3^2 + x_4^2)$ is not null if the rotor flux magnitude is different from zero. The latter condition is verified, since the machine is connected to the supply, and hence the control signal is bounded.

Remark 3.4 The convergence of the output tracking error e_i $(i = 1, 2)$ to zero does not imply that the state vector $x = (x_1, x_2, x_3, x_4, x_5)^T$ of the induction motor remains bounded. However, since $e_1 = x_5 - \omega_{ref}$ and $e_2 = x_3^2 + x_4^2 - \phi_{ref}$ are asymptotically stable with w_{ref} and ϕ_{ref} bounded, one concludes that the states x_5, x_3 and x_4 are bounded. Let $\xi = (x_1, x_2)^T$ and $\eta = (x_3, x_4, x_5)^T$. We have proven that the state η is bounded and we want to prove that ξ remains bounded. From the dynamic equation (1) we can see that, since the coefficient a_1 is positive, the origin of the subsystem $\dot{\xi} = f(\xi, \eta_d)$ is stable for any fixed value η_d of the vector η. One concludes that the state ξ is bounded.

4 Flux Observer Design

In this section, the purpose is to design a current-flux sliding observer based on a general class of manifold S_c. The basic sliding mode observer design procedure is performed in two steps. Firstly, design an attractive manifold $S_c(y, t) \in R^p$ so that the output estimation error trajectories restricted to $S_c(y, t)$, have a desired stable dynamics. In the second step, determine the observer gain, to stabilize the equivalent dynamic on $S_c(y, t)$.

The (x_1, x_2) component current, the rotor speed ω_r and the control input (u_1, u_2) are assumed to be available by measurement. Furthermore, the dynamic of rotor speed ω_r is assumed to be slower than the current and flux dynamics. The observer is considered as a copy of the induction machine electric equations where the speed ω_r is taken as a time varying parameter. In the sequel, $(\hat{x}_1,\ \hat{x}_2)$ denote the observed currents, $(\hat{x}_3,\ \hat{x}_4)$ is the observed flux, (e_{r1}, e_{r2}) is the current observation error and (e_{r3}, e_{r4}) is the flux observation error. Further, we assume that the real flux is bounded as follows: $|x_3| < \rho_3$, $|x_4| < \rho_4$.

We propose an observer constituted by two subsystems, the first one concerns with the stator current observation and is given by:

$$\begin{pmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{pmatrix} = -a_1 \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} + A_0 \begin{pmatrix} \hat{x}_3 \\ \hat{x}_4 \end{pmatrix} + d_1 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \Delta \begin{pmatrix} \mathrm{sign}(S_{c1}) \\ \mathrm{sign}(S_{c2}) \end{pmatrix}, \tag{14}$$

and the second subsystem concerns with the flux observation and is of the form:

$$\begin{pmatrix} \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \end{pmatrix} = a_3 \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} + B_0 \begin{pmatrix} \hat{x}_3 \\ \hat{x}_4 \end{pmatrix} + K \begin{pmatrix} \mathrm{sign}(S_{c1}) \\ \mathrm{sign}(S_{c2}) \end{pmatrix}, \tag{15}$$

where the matrices A_0 and B_0 are given by

$$A_0 = \begin{pmatrix} b_1 & c_1\omega_r \\ -c_1\omega_r & b_1 \end{pmatrix}, \quad B_0 = \begin{pmatrix} -b_3 & -\omega_r \\ \omega_r & -b_3 \end{pmatrix}, \tag{16}$$

and the gain matrices Δ and K are taken as:

$$\Delta = \begin{pmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{pmatrix}, \quad K = \begin{pmatrix} k_1 & k_2 \\ k_3 & k_4 \end{pmatrix}. \tag{17}$$

Consider the sliding surfaces error $Sc = (Sc_1, Sc_2)^T$ defined by:

$$\begin{cases} S_{c1} = \Theta_1(e_{r1}) & \text{with} \;\; e_{r1} = x_1 - \hat{x}_1, \\ S_{c2} = \Theta_2(e_{r2}) & \text{with} \;\; e_{r2} = x_2 - \hat{x}_2. \end{cases} \tag{18}$$

Here $\Theta_1(x)$ and $\Theta_2(x)$ are class C^1 functions characterized by some properties, which will be derived later.

Proposition 4.1 *For the first subsystem (14), if the following conditions are fulfilled:*

(i) $\Theta_1(x)$ *and* $\Theta_2(x)$ *are strictly increasing function satisfying:* $\Theta_i(x) = 0$ *if and only if* $x = 0$ *for* $i = 1, 2$*;*

(ii) *the coefficients* δ_1 *and* δ_2 *satisfy*

$$\begin{aligned} \delta_1 &> a_1|e_{r1}| + b_1(|\hat{x}_3| + \rho_3) + c_1\omega_r(|\hat{x}_4| + \rho_4), \\ \delta_2 &> a_1|e_{r2}| + c_1\omega_r(|\hat{x}_3| + \rho_3) + b_1(|\hat{x}_4| + \rho_4). \end{aligned} \tag{19}$$

Then, the manifold $\Psi_c = \{x \in R^5 : \; S_c = 0\}$ *is made globally attractive and invariant, moreover the observation errors* e_{r1} *and* e_{r2} *converge at least asymptotically to zero value.*

Proof On the manifold Ψ_c one has:

$$S_{ci} = 0 \quad \text{or} \quad \Theta_i(e_{ri}) = 0, \quad i = 1, 2, \tag{20}$$

when $\Theta_1(x)$ and $\Theta_2(x)$ are chosen among functions that take zero value only at the origin $x = 0$, the condition (20) leads to $e_{r1} = e_{r2} = 0$.

Let us take the Lyapunov function $V_c = \frac{1}{2}S_c^T S_c$ with $S_c = (S_{c1}, \, S_{c2})^T$ and its derivative is

$$\dot{V}_c = S_c^T \dot{S}_c, \tag{21}$$

where

$$\dot{S}_c = \begin{pmatrix} \dot{\Theta}_1(e_{r1}) \\ \dot{\Theta}_2(e_{r2}) \end{pmatrix} = \begin{pmatrix} \dot{e}r_1 \dfrac{d\Theta_1(e_{r1})}{de_1} \\ \dot{e}r_2 \dfrac{d\Theta_2(e_{r2})}{de_2} \end{pmatrix}. \tag{22}$$

From the fact that the current error dynamics $(\dot{e}r_1, \, \dot{e}r_2)$ is given by

$$\begin{pmatrix} \dot{e}r_1 \\ \dot{e}r_2 \end{pmatrix} = -a_1 \begin{pmatrix} e_{r1} \\ e_{r2} \end{pmatrix} + A_0 \begin{pmatrix} e_{r3} \\ e_{r4} \end{pmatrix} - \Delta \begin{pmatrix} \text{sign}(S_{c1}) \\ \text{sign}(S_{c2}) \end{pmatrix} \tag{23}$$

the sliding surface dynamic $(\dot{S}_{c1}, \, \dot{S}_{c2})$ becomes

$$\dot{S}_{c1} = P_1(S_{c1}) \frac{d\Theta_1(e_{r1})}{de_{r1}}, \tag{24}$$

$$\dot{S}_{c2} = P_2(S_{c2}) \frac{d\Theta_2(e_{r2})}{de_{r2}}, \tag{25}$$

where

$$P_1(S_{c1}) = [-a_1 e_{r1} + b_1 e_{r3} + c_1 \omega_r e_{r4} - \delta_1 \operatorname{sign}(S_{c1})],$$
$$P_2(S_{c2}) = [-a_1 e_{r2} - c_1 \omega_r e_{r3} + b_1 e_{r4} - \delta_2 \operatorname{sign}(S_{c2})],$$

and hence $\dot{V}$ can take the form

$$\dot{V}_c = P_1(S_{c1})\Theta_1(e_{r1})\frac{d\Theta_1(e_{r1})}{de_{r1}} + P_2(S_{c2})\Theta_2(e_{r2})\frac{d\Theta_2(e_{r2})}{de_{r2}}. \tag{26}$$

The latter can be written as

$$\dot{V}_c = P_1(\Theta_1)\Theta_1(e_{r1})\frac{d\Theta_1(e_{r1})}{de_{r1}} + P_2(\Theta_2)\Theta_2(e_{r2})\frac{d\Theta_2(e_{r2})}{de_{r2}}. \tag{27}$$

In order to make $\dot{V}_c$ negative $\forall\, S_c \neq 0$, it is sufficient that the terms $\dfrac{d\Theta_1}{de_{r1}}$ and $\dfrac{d\Theta_2}{de_{r2}}$ must be positive $\forall\, e_i \neq 0$ $(i = 1, 2)$ and the gains δ_1 and δ_2 are taken as

$$\delta_1 > Max\{|-a_1 e_{r1} + b_1 e_{r3} + c_1 \omega_r e_{r4}|\} = a_1|e_{r1}| + b_1(|\hat{x}_3| + \rho_3) + c_1\omega_r(|\hat{x}_4| + \rho_4), \tag{28}$$

$$\delta_2 > Max\{|-a_1 e_{r2} - c_1 \omega_r e_{r3} + b_1 e_{r4}|\} = a_1|e_{r2}| + b_1(|\hat{x}_4| + \rho_4) + c_1\omega_r(|\hat{x}_3| + \rho_3) \tag{29}$$

Hence, the manifold Ψ_c is globally attractive and the observation errors e_{r1} and e_{r2} converge at least asymptotically to zero value. □

Remark 4.1 For example, the function Θ_i $(i = 1, 2)$ can be taken as the two following functions and their linear combination with positive real coefficients: e_i^k with k odd natural number, and $\sinh(e_{ri})$;

Remark 4.2 From the literature, it is noticed that in [20, 21], the proposed sliding surface corresponds to the case of: $\Theta_i = e_{ri}$ with $i = (1, 2) > 0$.

When the first subsystem is in sliding mode, the gain matrix K is determined in order to make the flux observation errors converge exponentially to zero. One has

Proposition 4.2 *If the first subsystem (14) satisfies Proposition 4.1 and with the gain matrix K chosen as*

$$K = \left[B_0 + \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix}\right]\left[(A_0)^{-1}\begin{pmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{pmatrix}\right] \tag{30}$$

with $q_1 > 0$ and $q_2 > 0$ then, the flux observation errors (e_{r3}, e_{r4}) are uniformly exponentially stable.

Proof When the first subsystem (14) is in sliding mode ($Sc \equiv \dot{S}c \equiv 0$), then $e_{r1} = e_{r2} = \dot{e}r_1 = \dot{e}r_2 = 0$, and the terms $\operatorname{sign}(Sc_1)$, $\operatorname{sign}(Sc_2)$ are equivalent to:

$$\begin{pmatrix} \operatorname{sign}(S_{c1}) \\ \operatorname{sign}(S_{c2}) \end{pmatrix} \equiv \begin{pmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{pmatrix}^{-1} A_0 \begin{pmatrix} e_{r3} \\ e_{r4} \end{pmatrix}. \tag{31}$$

As consequence, the second subsystem dynamic (15) is reduced to

$$\begin{pmatrix} \dot{e}r_3 \\ \dot{e}r_4 \end{pmatrix} = \left[B_0 - K \begin{pmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{pmatrix}^{-1} A_0 \right] \begin{pmatrix} e_{r3} \\ e_{r4} \end{pmatrix} \tag{32}$$

with the gain matrix K given by

$$K = \left[B_0 + \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \right] \left[(A_0)^{-1} \begin{pmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{pmatrix} \right]. \tag{33}$$

The observation error dynamic e_{r3} and e_{r4} become

$$\begin{pmatrix} \dot{e}r_3 \\ \dot{e}r_4 \end{pmatrix} = - \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \begin{pmatrix} e_{r3} \\ e_{r4} \end{pmatrix}. \tag{34}$$

From expression (34) it appears clearly that the flux observation errors e_{r3} and e_{r4} converge uniformly exponentially to zero. $\square$

As the flux components are not available by measure, we must use the observed flux in the implementation of the control law (7). Besides, the convergence of the flux observation errors (e_{r3}, e_{r4}) and the controlled output errors (e_1, e_2), defined in (3), to zero does not imply that these variables will tend to zero when the observed flux is used instead of the real flux in the control law (7). This is so, because the separation principle is no longer valid for nonlinear systems. However, since the flux observation errors are uniformly exponentially stable, a sufficient condition for the global stability of the overall system resulting from the association of the control law (7) with the flux observer is given in [27]. This condition is that functions $\Lambda_1(\cdot)$ and $\Lambda_2(\cdot)$ must be chosen such that the control law (7) ensures global exponential stability of the controlled output errors (e_1, e_2). One possible choice of such functions is $\Lambda_i(e_i) = ke_i$ with $i = 1, 2$.

5 Simulation Results

The three phase induction machine under test is characterized by $P = 3.7\,Kw, 220/380, 8.54/14.8\,A$, $M = 0.048\,H$, $Ls = 0.17\,H$, $Lr = 0.015\,H$, $Ts = 0.151\,s$, $Tr = 0.136\,s$, $J = 0.135\,mN/rds^{-2}$, $K_f = 0.0018\,mN/rds^{-1}$. The chattering effect, due to sliding terms contained in input control, is largely attenuated using the function sign designed by the following relation:

$$\begin{cases} \operatorname{sign}(s) = s/\varepsilon & \text{if } \ |s| \le \varepsilon, \\ \operatorname{sign}(s) = 1 & \text{if } \ s > \varepsilon, \\ \operatorname{sign}(s) = -1 & \text{if } \ s < \varepsilon. \end{cases}$$

With a view to illustrate the method, we use the following surfaces:

(i) For flux-speed tracking: $\Lambda_1(e_1) = \sinh(e_1)$ and $\Lambda_2(e_2) = \sinh(e_2)$.

(ii) For flux observation $\Theta_1(e_{r1}) = \lambda_1 e_{r1} + \sinh(e_{r1})$ and $\Theta_2(e_{r2}) = \lambda_2 e_{r2} + \sinh(e_{r2})$.

Figures 5.1 and 5.2 give the machine responses in tracking regime (for both $\omega_{ref} > 0$ and $\omega_{ref} < 0$). It appears clearly that the flux machine and speed track their references with a good accuracy. Moreover, the initial stator peak currents are attenuated by

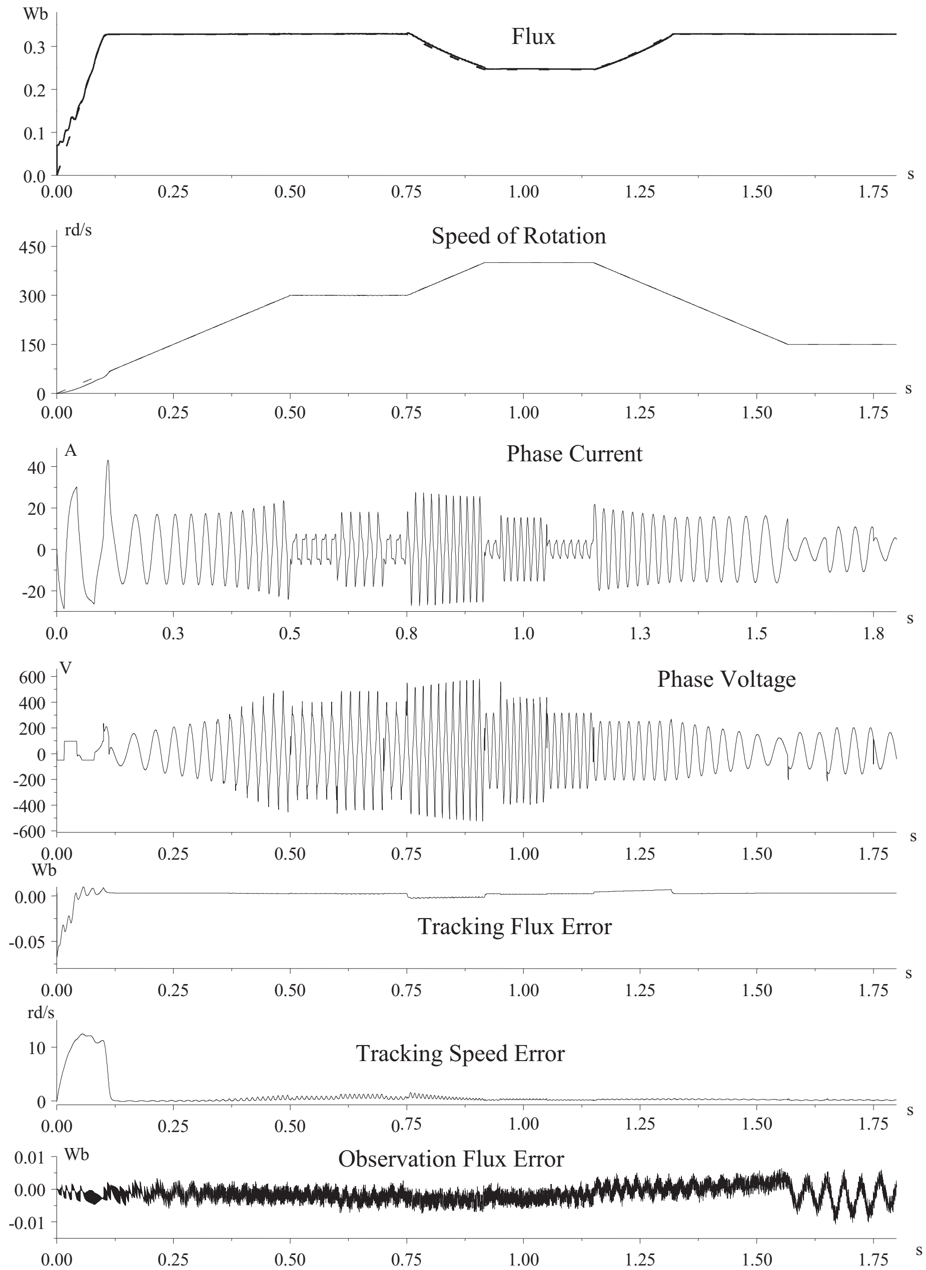

Figure 5.1: Induction machine responses in tracking regime for positive reference speed with the disturbances applied during only 0.1 s respectively at time t=0.6 s, 0.95 and 1.65 s (solid line for outputs; dashed line for references).

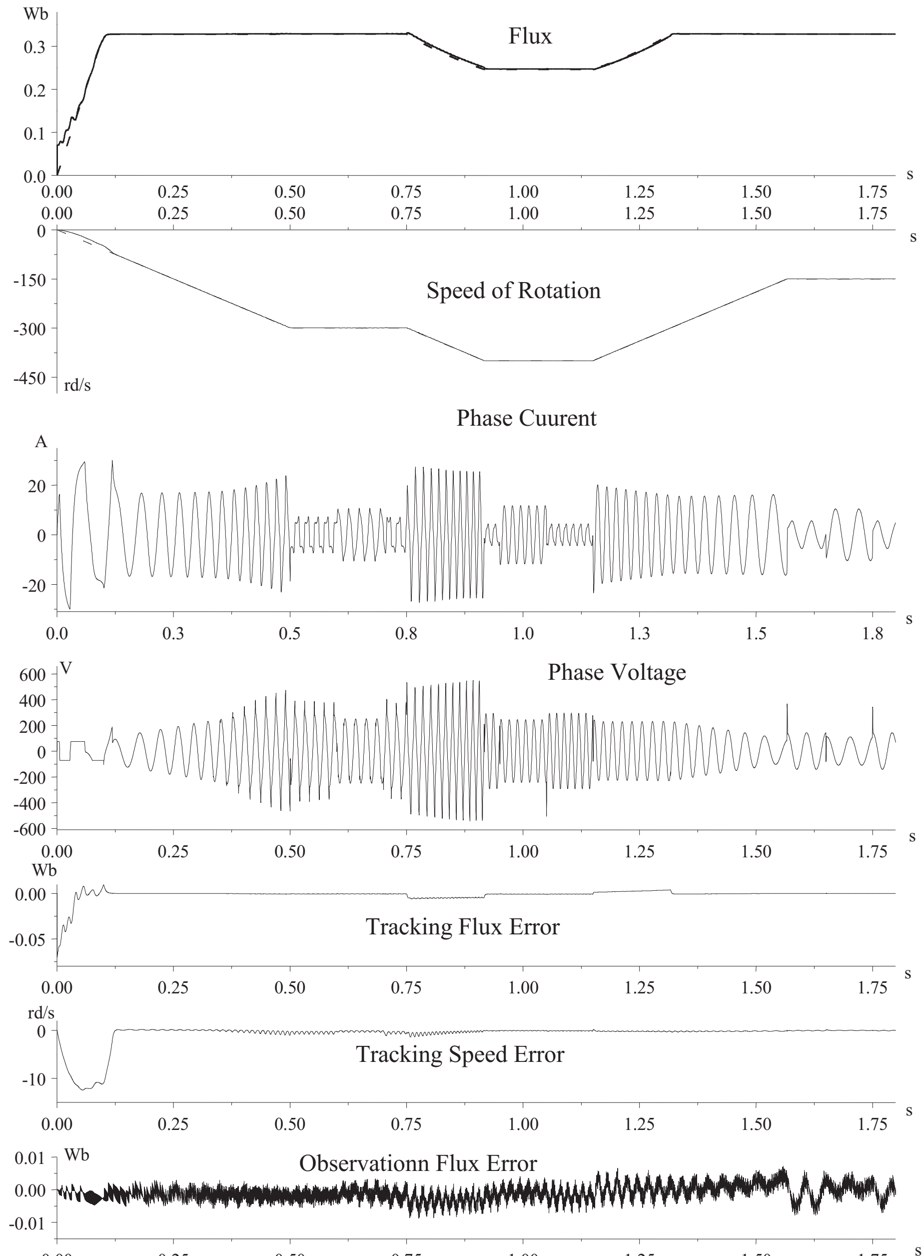

Figure 5.2: Induction machine responses in tracking regime for negative reference speed with the disturbances applied during only 0.1 s respectively at time t=0.6 s, 0.95 and 1.65 s (solid line for outputs; dashed line for references).

reducing the control inputs only in the beginning of the transient stage (for time $t \leq 0.1s$). This reduction affects the tracking speed during this interval of time. An appreciable flux tracking error (around 2%) is obtained due to an important threshold value used in function sign. In order to maintain the voltage in admissible range when the speed reference ω_{ref} grows up to nominal value $n = 300\, rd/s$, the reference flux ϕ_{ref} is reduced down to the nominal flux ϕ_n as: $\phi_{ref} = \phi_n \omega_n / \omega_{ref}$.

The machine flux tracks the desired value with a good accuracy in all speed range. Moreover, the estimated flux provided by the observer is sensibly the same as the flux machine (the error flux is around 0.001) independently of the speed value.

Further, it is noted that the speed and flux tracking and the estimated flux reveal a good robustness against disturbances represented by parametric variations and nominal load torque occurring at the same time. These disturbances are applied during $0.1s$ respectively at the time $t = 0.85\, s$, $1.35\, s$ and $t = 2.6\, s$. The robustness tests are performed for the parameter variations around nominal values as the all rotor resistances increase by an amount of 100% and, all inductances decrease by an amount of 50%. In spite of the occurring disturbances, the voltage phase value remains admissible.

6 Conclusion

In this paper, a general class of manifolds for sliding mode observation and control of induction machine, is developed. Firstly, the properties of sliding surfaces, ensuring the tracking flux-speed and observation flux, are derived. In the case of flux-speed tracking, we have studied the case when the derivative of the error control and a function of this error form the sliding surface. It has been demonstrated that this function must be odd and its derivative must be even function vanishing only at the origin. In the case of flux observation, this surface is a function of the estimated error. In later case, it has been proved that this function must be odd and it takes zero value at the origin and its derivative must be continuous even for the function taking zero value only at the origin. The simulation results have allowed obtaining the flux-speed tracking and flux observation with good accuracy. Moreover, the behavior of tracking against disturbances represented by the application of the nominal load torque in the presence of increasing rotor resistances (by an amount of 100%) and decreasing inductances (by an amount of 50%) reveals the high robustness level.

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