Nonlinear Dynamics and Systems Theory, **8** (2) (2008) 195–204

Output Feedback Passive Control of Neutral Systems with Time-Varying Delays in State and Control Input

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Received: October 5, 2007; Revised: March 18, 2008

Abstract: This paper is concerned with the passive control problem of neutral systems with time-varying delays. Time-varying delays are assumed to appear in both the state and the control input. A state feedback passive controller and an output feedback passive controller for neutral systems with time-varying delays in state and control input are presented. Through modifying algebraic Riccati equation, we can construct controllers which depend on the maximum value of the time derivative of time-varying delays. A numerical example is also given to illustrate the effectiveness of the proposed design method.

Keywords: *Neutral system; output feedback passive control; time-varying delays; Riccati equation.*

Mathematics Subject Classification (2000): 34K40, 93C23.

1 Introduction

The theory of neutral delay-differential systems, which contain delays both in its state and in the derivatives of its states, is of both theoretical and practical interest. For example, functional differential equations of neutral type are the natural models of fluctuations of voltage and current in problems arising in transmission lines [1]. Also, the neutral systems often appear in the study of automatic control, population dynamics, and vibrating masses attached to an elastic bar. Recently, considerable attention has first been focused on the stability analysis of various neutral differential systems [2-10]. And there are

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authors pay attention to the oscillation of parabolic equations of neutral type [11-13] and H_∞ control of neutral type [14].

The passivity theory intimately related to the circuit analysis methods [15,16] has received a lot of attention from the control community since the 70s (see [17-23], to cite only a few). On the other hand, many efforts have been devoted to the study of output feedback control of uncertain systems [24-26]. However, to the best of our knowledge, few authors pay attentions to study the output feedback passive control of neutral systems with time-varying delays.

In this paper, we shall discuss the passive control problem of neutral systems with time-varying delays. A state feedback passive controller and an output feedback passive controller which render the closed-loop system to be quadratic stable and passive for neutral systems with time-varying delays in state and control input are presented.

The layout of this paper is as follows. In Section 2, the problem to be studied is stated and some preliminaries are presented. The asymptotical stability and passivity of neutral system condition is derived in Section 3. A state feedback passive controller and an output feedback passive controller for neutral systems with time-varying delays in state and control input are proposed in Section 4 and Section 5, respectively. In Section 6, a numerical example is given to demonstrate the effectiveness of the theoretical results. And finally, conclusions are drawn in Section 7.

Notation and fact. In the sequel, we denote A^T and A^{-1} the transpose and the inverse of any square matrix A. We use $A > 0$ ($A < 0$) to denote a positive- (negative-) definite matrix A; and I is used to denote the $n \times n$ identity matrix. $\mathbf{L}_2[0,\infty]$ is the space of integrable function vector over $[0,\infty]$. $\mathbf{R}^n$ denotes the n-dimensional Euclidean space. The symbol "$\star$" within a matrix represents the symmetric term of the matrix.

Fact 1 (Schur complement). Given constant matrices $\Omega_1, \Omega_2, \Omega_3$, where $\Omega_1 = \Omega_1^T$ and $0 < \Omega_2 = \Omega_2^T$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if

$$\begin{pmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{pmatrix} < 0.$$

2 System Description and Preliminaries

In this paper, we consider a class of neutral functional differential equation (NFDE) described as follows:

$$\begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t-\tau_1(t)) + A_2 \dot{x}(t-\tau_2(t)) + B_1 w(t) + B_2 u(t-\tau_3(t)), \\ z(t) = C_1 x(t) + D_1 u(t) + D_{11} w(t), \\ y(t) = C_2 x(t) + D_2 w(t), \\ x(t) = \phi(t), t \geq 0. \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state; $u(t) \in R^m$ is the control input with $u(t) = 0$ for $t < 0$; $y(t) \in R^q$ is the output measurement; $w(t) \in R^p$ is the square-integrable disturbance input; $z(t) \in R^p$ is the controlled output; $\phi(t)$ are continuous functions defined on $(-\infty, 0]$. A_0, A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2, D_{11} are given constant matrices with appropriate dimensions and $\tau_1(t)$, $\tau_2(t)$ and $\tau_3(t)$ are arbitrary differentiable function satisfying

$$\begin{cases} 0 \leq \tau_1(t) < \infty, \ 0 \leq \tau_2(t) < \infty, \ 0 \leq \tau_3(t) < \infty, \\ \dot{\tau_1}(t) \leq \sigma_1 < 1, \ \dot{\tau_2}(t) \leq \sigma_2 < 1, \ \dot{\tau_3}(t) \leq \sigma_3 < 1. \end{cases} \tag{2}$$

Our problem is to establish the passive control for the system (1) to determine the conditions. First, we introduce the following definition of passivity.

Definition 2.1 The dynamical system (1) is called passive if

$$\int_0^\infty w^T(t)z(t)dt > \beta, \ \ \forall w \in L_2[0,\infty), \tag{3}$$

where β is some constant which depends on the initial condition of the system. In addition, the system is said to be strictly passive (SP) if it is passive and $D_{11} + D_{11}^T > 0$.

3 Asymptotical Stability and Passivity of Neutral System

Now, we consider a class of neutral system with time-varying delays described by:

$$\begin{cases} \dot{x}(t) = A_0x(t) + A_1x(t-\tau_1(t)) + A_2\dot{x}(t-\tau_2(t)) + A_3x(t-\tau_3(t)) + B_1w(t), \\ z(t) = C_1x(t) + D_{11}w(t), \\ y(t) = C_2x(t) + D_2w(t), \\ x(t) = \phi(t), \ \ t \geq 0. \end{cases} \tag{4}$$

Our first result establishes the passive control of the time-varying delay system (4).

Theorem 3.1 *Consider a state-delay neutral system (4), if there exist positive definite matrices P and Q which satisfy the following algebraic Riccati inequality (ARI):*

$$A_0^TP + PA_0 + 2Q + (1-\sigma_1)^{-1}PA_1Q^{-1}A_1^TP + (1-\sigma_2)^{-1}PA_2Q^{-1}A_2^TP$$
$$+(1-\sigma_3)^{-1}PA_3Q^{-1}A_3^TP + (PB_1 - C_1^T)(D_{11}+D_{11}^T)^{-1}(B_1^TP - C_1) + M < 0, \tag{5}$$

where

$$\begin{aligned} M \ = \ & A_0^TQA_0 + A_0^TQA_1 + A_0^TQA_2 + A_0^TQA_3 + A_0^TQB_1 \\ & +A_1^TQA_0 + A_1^TQA_1 + A_1^TQA_2 + A_1^TQA_3 + A_1^TQB_1 \\ & +A_2^TQA_0 + A_2^TQA_1 + A_2^TQA_2 + A_2^TQA_3 + A_2^TQB_1 \\ & +A_3^TQA_0 + A_3^TQA_1 + A_3^TQA_2 + A_3^TQA_3 + A_3^TQB_1 \\ & +B_1^TQA_0 + B_1^TQA_1 + B_1^TQA_2 + B_1^TQA_3 + B_1^TQB_1, \end{aligned}$$

or equivalently satisfying the linear matrix inequality (LMI):

$$\begin{pmatrix} A_0^TP + PA_0 + 2Q & PA_1 & PA_2 & PA_3 & PB_1 - C_1^T & A_0^T \\ \star & -(1-\sigma_1)Q & 0 & 0 & 0 & A_1^T \\ \star & \star & -(1-\sigma_2)Q & 0 & 0 & A_2^T \\ \star & \star & \star & -(1-\sigma_3)Q & 0 & A_3^T \\ \star & \star & \star & \star & -(D_{11}+D_{11}^T) & B_1^T \\ \star & \star & \star & \star & \star & -Q^{-1} \end{pmatrix} < 0. \tag{6}$$

Then the systems (4) is asymptotically stable and passive for all time-varying state delays $\tau_1(t)$, $\tau_2(t)$ and $\tau_3(t)$.

Proof Define a Lyapunov functional V(x(t)) as follows:

$$\begin{aligned} V(x(t)) &= x^T(t)Px(t) + \int_{t-\tau_1(t)}^{t} x^T(s)Qx(s)ds \\ &\quad + \int_{t-\tau_2(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds + \int_{t-\tau_3(t)}^{t} x^T(s)Qx(s)ds. \end{aligned} \tag{7}$$

Calculating the derivative of the Lyapunov functional V(x(t)) along the solution of (4), it follows that

$$\begin{aligned} \dot{V}(x(t)) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ &\quad + x^T(t)Qx(t) - (1-\dot{\tau}_1(t))x^T(t-\tau_1(t))Qx(t-\tau_1(t)) \\ &\quad + \dot{x}^T(t)Q\dot{x}(t) - (1-\dot{\tau}_2(t))\dot{x}^T(t-\tau_2(t))Q\dot{x}(t-\tau_2(t)) \\ &\quad + x^T(t)Qx(t) - (1-\dot{\tau}_3(t))x^T(t-\tau_3(t))Qx(t-\tau_3(t)) \\ &\le \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ &\quad + 2x^T(t)Qx(t) - (1-\sigma_1)x^T(t-\tau_1(t))Qx(t-\tau_1(t)) \\ &\quad + \dot{x}^T(t)Q\dot{x}(t) - (1-\sigma_2)\dot{x}^T(t-\tau_2(t))Q\dot{x}(t-\tau_2(t)) \\ &\quad - (1-\dot{\tau}_3(t))x^T(t-\tau_3(t))Qx(t-\tau_3(t)) \\ &= x^T(t)\Big(A_0^TP + PA_0 + 2Q\Big)x(t) \\ &\quad + 2x^T(t)PA_1x(t-\tau_1(t)) + 2x^T(t)PA_2\dot{x}(t-\tau_2(t)) \\ &\quad + 2x^T(t)PA_3x(t-\tau_3(t)) + 2x^T(t)PB_1w(t) \\ &\quad - (1-\sigma_1)x^T(t-\tau_1(t))Qx(t-\tau_1(t)) \\ &\quad + \dot{x}^T(t)Q\dot{x}(t) - (1-\sigma_2)\dot{x}^T(t-\tau_2(t))Q\dot{x}(t-\tau_2(t)) \\ &\quad - (1-\sigma_3)x^T(t-\tau_3(t))Qx(t-\tau_3(t)). \end{aligned}$$

So we can obtain that

$$\begin{aligned} \dot{V}(x(t)) - 2z^T(t)w(t) &= x^T(t)(A_0^TP + PA_0 + 2Q)x(t) \\ &\quad + 2x^T(t)PA_1x(t-\tau_1(t)) + 2x^T(t)PA_2\dot{x}(t-\tau_2(t)) \\ &\quad + 2x^T(t)PA_3x(t-\tau_3(t)) + 2x^T(t)(PB_1 - C_1^T)w(t) \\ &\quad - w^T(t)(D_{11} + D_{11}^T)w(t) + \dot{x}^T(t)Q\dot{x}(t) \\ &\quad - (1-\sigma_1)x^T(t-\tau_1(t))Qx(t-\tau_1(t)) \\ &\quad - (1-\sigma_2)\dot{x}^T(t-\tau_2(t))Q\dot{x}(t-\tau_2(t)) \\ &\quad - (1-\sigma_3)x^T(t-\tau_3(t))Qx(t-\tau_3(t)) \\ &= \eta^T(t)\Omega\eta(t), \end{aligned} \tag{8}$$

where

$$\eta(t) = \Big[x(t)\ x(t-\tau_1(t))\ \dot{x}(t-\tau_2(t))\ x(t-\tau_3(t))\ w(t)\Big]^T,$$

$$\Omega = \left(\begin{array}{ccc} A_0^TP + PA_0 + 2Q & PA_1 & PA_2 \\ \star & -(1-\sigma_1)Q & 0 \\ \star & \star & -(1-\sigma_2)Q \\ \star & \star & \star \\ \star & \star & \star \end{array} \right.$$

$$\begin{matrix} PA_3 & PB_1 - C_1^T \\ 0 & 0 \\ 0 & 0 \\ -(1-\sigma_3)Q & 0 \\ \star & -(D_{11}+D_{11}^T) \end{pmatrix} + \begin{pmatrix} A_0 \\ A_1^T \\ A_2^T \\ A_3^T \\ B_1^T \end{pmatrix} Q\begin{pmatrix} A_0 & A_1 & A_2 & A_3 & B_1 \end{pmatrix}.$$

From Schur complement, it easily follows that (5) and (6) hold. Hence,

$$\dot{V}(x(t)) \le 2z^T(t)w(t). \tag{9}$$

Integrating (9) from t_0 to t_1, we have

$$\int_{t_0}^{t_1} z^T(t)w(t)dt > \frac{1}{2}\Big[V(x(t_1)) - V(x(t_0))\Big].$$

Since $V(x(t)) > 0$ for $x \neq 0$ and $V(x(t)) = 0$ for $x = 0$, it follows that as $t_0 = 0$ and $t_1 \to \infty$ that the system (4) with $w = 0$ is asymptotically stable and passive. □

Remark 3.1 In this section, we provide a method of solving the synthesis problem for neutral systems with time-varying delays. In Section 4 and Section 5, a state feedback passive controller and an output feedback passive controller for neutral systems with time-varying delays in state and control input are proposed.

4 State-Feedback Passive Controller

On the basis of Theorem 1, we now want to construct the state feedback controller

$$u(t) = Kx(t), \tag{10}$$

such that the input-state-delay neutral system (1) is asymptotically stable and passive. Then the transformed systems become

$$\begin{cases} \dot{x}(t) = A_0x(t) + A_1x(t-\tau_1(t)) + A_2\dot{x}(t-\tau_2(t)) + B_1w(t) + B_2Kx(t-\tau_3(t)), \\ z(t) = (C_1 + D_1K)x(t) + D_{11}w(t), \\ y(t) = C_2x(t) + D_2w(t), \\ x(t) = \phi(t), \ \ t \ge 0. \end{cases} \tag{11}$$

Theorem 4.1 *Consider a state-delay neutral system (11), if there exist positive definite matrices P which satisfy the following inequality:*

$$\begin{pmatrix} A_0^TP + PA_0 + 2Q & PA_1 & PA_2 \\ \star & -(1-\sigma_1)Q & 0 \\ \star & \star & -(1-\sigma_2)Q \\ \star & \star & \star \\ \star & \star & \star \\ \star & \star & \star \end{matrix}$$

$$\begin{matrix} PB_2K & PB_1 - (C_1+D_1K)^T & A_0^T \\ 0 & 0 & A_1^T \\ 0 & 0 & A_2^T \\ -(1-\sigma_3)Q & 0 & A_3^T \\ \star & -(D_{11}+D_{11}^T) & B_1^T \\ \star & \star & -Q^{-1} \end{pmatrix} < 0, \tag{12}$$

and

$$D_1K = B_1^TP - C_1, \tag{13}$$

then the system (11) is passive by the state-feedback passive controller (10).

Proof The closed form of (11) is similar to (4). Therefore, by Theorem 3.1, given positive definite matrix Q, if there exits positive definite symmetric matrix P which satisfies the following inequality

$$\begin{pmatrix} A_0^TP + PA_0 + 2Q & PA_1 & PA_2 & PB_2K & PB_1 - (C_1 + D_1K)^T & A_0^T \\ \star & -(1-\sigma_1)Q & 0 & 0 & 0 & A_1^T \\ \star & \star & -(1-\sigma_2)Q & 0 & 0 & A_2^T \\ \star & \star & \star & -(1-\sigma_3)Q & 0 & A_3^T \\ \star & \star & \star & \star & -(D_{11} + D_{11}^T) & B_1^T \\ \star & \star & \star & \star & \star & -Q^{-1} \end{pmatrix} < 0. \tag{14}$$

Since there are two unknown matrices P, K to be solved in (14), we can let the matrices P, K in (14) satisfy the following two conditions at the same time

$$\begin{pmatrix} A_0^TP + PA_0 + 2Q & PA_1 & PA_2 & PB_2K & PB_1 & A_0^T \\ \star & -(1-\sigma_1)Q & 0 & 0 & 0 & A_1^T \\ \star & \star & -(1-\sigma_2)Q & 0 & 0 & A_2^T \\ \star & \star & \star & -(1-\sigma_3)Q & 0 & K^TB_2^T \\ \star & \star & \star & \star & -(D_{11} + D_{11}^T) & B_1^T \\ \star & \star & \star & \star & \star & -Q^{-1} \end{pmatrix} < 0, \tag{15}$$

$$\begin{aligned} &-PB_1(D_{11} + D_{11}^T)^{-1}(C_1 + D_1K) - (C_1 + D_1K)^T)(D_{11} + D_{11}^T)^{-1}B_1^TP \\ &+(C_1 + D_1K)^T(D_{11} + D_{11}^T)^{-1}(C_1 + D_1K) < 0. \end{aligned} \tag{16}$$

Then the controller (10) can make system (11) be asymptotically stable and passive. From (16), we observe that if we choose

$$D_1K = B_1^TP - C_1, \tag{17}$$

then the inequality (16) is satisfied. In order to satisfied (15), we let

$$A_0^TP + PA_0 + 2Q + (1-\sigma_1)^{-1}PA_1Q^{-1}A_1^TP + (1-\sigma_2)^{-1}PA_2Q^{-1}A_2^TP$$

$$+(1-\sigma_3)^{-1}PA_3Q^{-1}A_3^TP + (PB_1 - C_1^T)(D_{11} + D_{11}^T)^{-1}(B_1^TP - C_1) + M = -Q. \quad (18)$$

Therefore, we can say that the P in (15) is the solution satisfying the following modified algebraic Riccati equation shown as follows:

$$A_0^TP + PA_0 + 3Q + (1-\sigma_1)^{-1}PA_1Q^{-1}A_1^TP + (1-\sigma_2)^{-1}PA_2Q^{-1}A_2^TP$$

$$+(1-\sigma_3)^{-1}PA_3Q^{-1}A_3^TP + (PB_1 - C_1^T)(D_{11} + D_{11}^T)^{-1}(B_1^TP - C_1) + M = 0. \quad (19)$$

The existence of solution K in (17) can be seen in the following:

(i) If D_1 is square matrix and $det(D_1) \neq 0$, the unique solution $K = D_1^{-1}(B_1^TP - C_1)$ is presented.

(ii) Suppose the size of D_1 is $n \times m$ $(n > m)$ and $rank[D_1 \;\; B_1^TP - C_1] = r$. Then if $r = m$, the unique solution K in (17) exists; if $r < m$, there are many solutions; if $r > m$ and $det(D_1^TD_1) \neq 0$, a least square approximation solution of K in (17) is shown as follows:

$$K = (D_1^TD_1)^{-1}D_1^T(B_1^TP - C_1). \quad (20)$$

□

5 Output Feedback Passive Controller

When state variable are not available for the feedback, it is necessary to construct a output feedback passive controller. If the state in (1) is not available, we propose the following dynamic output feedback controller in order to stabilize system (1):

$$\begin{cases} \dot{\eta}(t) = G\eta(t) + Ly(t), \\ u(t) = K\eta(t), \;\; \eta(0) = 0, \end{cases} \quad (21)$$

where $\eta(t) \in R^n$ is the controller state vector, and G, L, K are gain matrices with appropriate dimensions to be determined later. Applying this controller (21) to system (1) results in the closed-loop system

$$\begin{cases} \dot{\overline{x}}(t) = \overline{A}_0\overline{x}(t) + \overline{A}_1\overline{x}(t-\tau_1(t)) + \overline{A}_2\dot{\overline{x}}(t-\tau_2(t)) + \overline{A}_3\overline{x}(t-\tau_3(t)) + \overline{B}_1w(t), \\ z(t) = \overline{C}_1\overline{x}(t) + D_{11}w(t), \end{cases} \quad (22)$$

where

$$\overline{x}(t) = \begin{pmatrix} x(t) \\ \eta(t) \end{pmatrix}, \; \overline{A}_0 = \begin{pmatrix} A_0 & 0 \\ LC_2 & G \end{pmatrix}, \; \overline{A}_1 = \begin{pmatrix} A_1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$\overline{A}_2 = \begin{pmatrix} A_2 & 0 \\ 0 & 0 \end{pmatrix}, \; \overline{A}_3 = \begin{pmatrix} 0 & B_2K \\ 0 & 0 \end{pmatrix}, \; \overline{B}_1 = \begin{pmatrix} B_1 \\ LD_2 \end{pmatrix}, \; \overline{C}_1 = \begin{pmatrix} C_1 & D_1K \end{pmatrix}.$$

Theorem 5.1 *For a given symmetric positive Q, if there exist positive definite matrices P and gain matrices G, L, K such that the following linear matrix inequality (LMI):*

$$\left(\begin{array}{ccccccccccc}
A_0^T P + PA_0 + 2Q & C_2^T L^T & PA_1 & 0 & PA_2 & 0 & 0 & PB_2K & PB_1 - C^T & A_0^T & C_2^T L^T \\
LC_2 & G^T + G + 2Q & 0 & 0 & 0 & 0 & 0 & 0 & LD_2 - K^T D_1^T & 0 & G^T \\
\star & \star & (\sigma_1 - 1)Q & 0 & 0 & 0 & 0 & 0 & 0 & A_1^T & 0 \\
\star & \star & \star & (\sigma_1 - 1)Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & (\sigma_2 - 1)Q & 0 & 0 & 0 & 0 & A_2^T & 0 \\
\star & \star & \star & \star & \star & (\sigma_2 - 1)Q & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & (\sigma_3 - 1)Q & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & (\sigma_3 - 1)Q & 0 & K^T B_2^T & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & -(D_{11} + D_{11}^T) & B_1^T & D_2^T L^T \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & -Q^{-1} & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & 0 & -Q^{-1}
\end{array}\right) < 0, \tag{23}$$

then the time-varying input-state-delay neutral system (1) is passive by the output feedback passive controller (21).

Proof Define positive symmetric matrices $\overline{P} > 0$ and $\overline{Q} > 0$ by

$$\overline{P} = \begin{pmatrix} P & 0 \\ 0 & I \end{pmatrix}, \quad \overline{Q} = \begin{pmatrix} Q & 0 \\ 0 & Q \end{pmatrix}.$$

Similar to the proof of Theorem 1, we can easily get that if the following LMI

$$\begin{pmatrix}
\overline{A}_0^T \overline{P} + \overline{PA}_0 + 2\overline{Q} & \overline{PA}_1 & \overline{PA}_2 & \overline{PA}_3 & \overline{PB}_1 - \overline{C}_1^T & \overline{A}_0^T \\
\star & (\sigma_1 - 1)\overline{Q} & 0 & 0 & 0 & \overline{A}_1^T \\
\star & \star & (\sigma_2 - 1)\overline{Q} & 0 & 0 & \overline{A}_2^T \\
\star & \star & \star & (\sigma_3 - 1)\overline{Q} & 0 & \overline{A}_3^T \\
\star & \star & \star & \star & -D_{11} - D_{11}^T & \overline{B}_1^T \\
\star & \star & \star & \star & \star & -\overline{Q}^{-1}
\end{pmatrix} < 0, \tag{24}$$

holds, then the time-varying input-state-delay neutral system (1) is passive by the output feedback passive controller (21).

Now, substitute the expressions of $\overline{A}_0, \overline{A}_1, \overline{A}_2, \overline{A}_3, \overline{B}_1, \overline{C}_1, \overline{P}, \overline{Q}$ into (24), it easily follows that (23) holds. This completes the proof. □

6 An Illustrative Example

In this section, the expanded theoretical results are illustrated through a numerical example. Consider the differential system of neutral type (1) under supposing

$$A_0 = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}, \ A_1 = \begin{pmatrix} 0 & 0 \\ 0.2 & 0.1 \end{pmatrix}, \ A_2 = \begin{pmatrix} 0 & 0 \\ 0.3 & 0.2 \end{pmatrix}, \ B_1 = \begin{pmatrix} 0 \\ 0.1 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ C_1 = \begin{pmatrix} 1 & 1 \end{pmatrix}, \ C_2 = \begin{pmatrix} 1 & 1 \end{pmatrix}, \ D_1 = 1, \ D_2 = 1, \ D_{11} = 1,$$

$$\tau_1(t) = 2.0 + 0.3\sin(t), \ \ \tau_2(t) = 3.5 + 0.4\cos(t), \ \ \tau_3(t) = 4.0 + 0.2\sin(t).$$

Hence we have $\sigma_1 = 0.3, \sigma_2 = 0.4, \sigma_3 = 0.2$. In order to solve the solution simply, we select $Q = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}$. So using MATLAB LMI Toolbox we solve the condition (12) and (13) and obtain that

$$K = \begin{pmatrix} -0.9427 & -0.8289 \end{pmatrix}, \quad P = \begin{pmatrix} 1.4530 & 0.5734 \\ 0.5734 & 1.7115 \end{pmatrix} > 0.$$

Hence, the system (1) is passive by the state-feedback passive controller (10).

7 Conclusions

In this paper, the passivity analysis and passive controllers' designs for the neutral systems with time-varying delays in state and control input are investigated by the Lyapunov functional method. The results are presented in terms of LMIs or Riccati equation, which can be solved easily by using the effective interior-point algorithm [24]. A numerical example is worked through to illustrate the effectiveness of results.

Acknowledgements

This work was supported by NSFC (No. 60674026), the Jiangsu Provincial Natural Science Foundation of China (No. BK2007016), the Jiangsu Provincial Program for Postgraduate Scientific Innovative Research of Jiangnan University (No. CX07B_116z) and PIRTJiangnan.

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