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The Connection Between Boundedness and Periodicity in Nonlinear Functional Neutral Dynamic Equations on a Time Scale

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Abstract: Let $\mathbb{T}$ be a time scale that is unbounded above. We use a direct mapping and then utilize a Krasnosel'skiĭ fixed point theorem to show the existence of solutions of the nonlinear functional neutral dynamic system with delay

$$x^{\Delta}(t) = f(t, x(t), x^{\Delta}(t - h(t))) + g(t, x(t), x(t - h(t))),\ t, t - h(t) \in \mathbb{T}.$$

Then, we consider a special form of the above mentioned system and use the contraction mapping principle and show the existence of a uniform bound on all solutions and then conclude the existence of a unique periodic solution. Finally, the connection between the boundedness of solutions and the existence of periodic solutions leads us to the extension of Massera's theorem to functional differential equations on general periodic time scales.

Keywords: *connection between boundedness and periodic solutions; existence; functional; neutral; time scale.*

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