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PERSONAGE IN SCIENCE

Academician N.N. Bogoliubov

(*to the 100th Birthday Anniversary*)

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This paper is dedicated to the memory of N.N. Bogoliubov in recognition of the significance of his efforts in the development of nonlinear mechanics and theoretical physics, his remarkable and versatile genius, as well as the novelty and depth of his contribution to the world science.

N.N. BOGOLIUBOV was born on August 21, 1909 in the city of Nizhny Novgorod. He grew up as a prodigy. At the age of only 13, Bogoliubov participated in a seminar led by Academician N.M. Krylov, a widely recognized scientist and teacher. In 1924, at the age of 15, Bogoliubov wrote his first scholarly work, "On the behavior of solutions to linear differential equations at infinity". Between 1925 and 1951, Bogoliubov was an employee in the mathematical physics division of the Ukrainian Academy of Sciences. During a period of his collaboration with Academician N.M. Krylov, Bogoliubov conducted fundamental research in the area of boundary value problems, approximation theory of differential equations, dynamical systems, and direct methods of variational calculus.

During those years, Bogoliubov created a new direction in the theory of uniform, almost periodic functions. Thereby he established a close link to the general behavior of linear combinations of arbitrary bounded functions. In 1930, one of Bogoliubov's first works was awarded the A. Merlani Prize by the Bologna's Academy of Sciences. In the same year, at the age of 21, with no formal thesis defense, Bogoliubov earned an honorable Habilitation degree in Mathematics from the Presidium of Ukrainian Academy of Sciences.

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The progress in science and technology created new avenues in telecommunication and electrical engineering, as well as mechanics of complex oscillating systems and aerospace. So, from 1932, Bogoliubov, together with his supervisor and mentor, N.M. Krylov, began developing an entirely new branch of mathematical physics — the theory of nonlinear oscillations, which they called "nonlinear mechanics". Their first work in this new direction was dealt with the theory of oscillations in power machinery and mechanical systems. The research in nonlinear mechanics was carried out in two directions: developing new asymptotic integration techniques of nonlinear equations of motion in oscillatory systems and laying a foundation for these methods based on measure theory.

Having overcome copious difficulties, Krylov and Bogoliubov extended the tools of perturbation theory to more general nonconservative systems and they created new and well established asymptotic methods of nonlinear mechanics. Unlike the popular Van der Pol's method, the corresponding solutions could be obtained not only in the first but in higher approximations as well. These methods became very useful in the studies of both periodic and quasiperiodic oscillatory processes. Moreover, they met practical needs in terms of simplicity and transparency of associated numerical algorithms.

Krylov and Bogoliubov quickly applied their asymptotic methods to many open and crucial problems. Among them, they obtained second approximation formulas for the frequency of stationary vibrations in electrical generators, which could estimate the overtone effect on stability of frequency. Furthermore, the results had an impact on the studies of resonances of frequency scaling and internal resonances in the systems with many degrees of freedom. A primal attention in resonance theory was paid to applications of nonlinear elements for controlling resonance in mechanical engineering. The asymptotic methods were employed to solve problems on aircraft longitudinal stability, vibrations and stability of rods, diverse frame structures and other engineering constructions.

The general measure theory in nonlinear mechanics developed by Bogoliubov and Krylov was a driving force in further development of the theory of dynamical systems. It also explained such properties of stationary motions as recurrence, i.e. strong stability in the sense of Poisson. Applying Lyapunov–Poincare and Poincare–Denjoy theory of trajectories on torus they studied the nature of the exact stationary solution near a proximate solution for a small parameter value and established existence and stability theorems for quasiperiodic solutions.

No wonder that the results of this research have become classical in modern theory of dynamical systems. Furthermore, the development of efficient methods of asymptotic integration for a wide class of nonlinear equations was due to Bogoliubov and Krylov's fundamental results. Bogoliubov also created new mathematical tools to study the behavior of general nonconservative systems with small parameter, which explained the nature of the stationary solution near a proximate solution.

Krylov's and Bogoliubov's studies on resonances in nonlinear oscillations are of especial importance, as so are their studies of the related phenomena of synchronization, demultiplication and diminishing of oscillation amplitudes in resonance under the presence of nonlinear elements in oscillatory system. Another sound result was their prediction of a possibility to observe a new phenomenon, called by them an anomalous perturbation, later on confirmed in practice. This phenomenon states that an equilibrium point, stable in the traditional sense, looses its stability under the effect of small sinusoidal perturbing forces.

The averaging method formulated and developed by Bogoliubov in the context of standard form equations contained in its essence a solution to the following two problems.

Firstly, it is the establishment of conditions under which the norm of the difference between the solutions of an exact and the associated averaged systems, for sufficiently small parameter values, remains arbitrarily large on a finite interval. Secondly, it is the establishment of a relationship between various properties of the solutions to exact and averaged equations on a finite interval.

As early as in 1945, Bogoliubov proved a fundamental existence theorem on main properties of a single-parametric integral manifold of a system of nonlinear differential equations in the standard form. He investigated periodic and quasi-periodic solutions on a one-dimensional manifold. This laid a foundation for a new method of nonlinear mechanics — the "method of integral manifolds".

In 1950, Bogoliubov developed a perturbation method in nonlinear mechanics, which he applied to a pendulum problem with a vibrating pivot point. In this problem, Bogoliubov was the first to prove that any unstable upper position of the pendulum can be made stable by means of a high vibration frequency of the pivot point. This breakthrough laid foundation to the theory of stability raise of elastic systems by vibrations.

Bogoliubov also obtained key results for systems of differential equations with a rapidly switching phase. Here the construction of solutions to averaged equations was rendered by separation of slow and rapid motions.

In 1963, Bogoliubov presented a new idea on application of accelerated convergence techniques to nonlinear mechanics. As early as in 1934, Bogoliubov, jointly with Krylov, developed a mixture of various changes of variables. Its application to the method of integral manifolds solved an existence problem of multi-frequent conditionally periodic solutions of nonlinear oscillatory systems, not only in the asymptotic, but also in a strict sense. When establishing this theory, Bogoliubov combined the method of integral manifolds with the iteration method. The latter has been proposed and used by A.N. Kolmogorov and V.I. Arnold for Hamiltonian systems by that time. This combined method gave rise to yet another method of accelerated convergence in nonlinear mechanics, which allowed Bogoliubov efficiently exclude the effect of small denominators that occur when using the change of variables.

Bogoliubov's ideas and methods expressed by him during his lectures in the international workshop in nonlinear mechanics (that took place in 1963 in Kanev) gave rise to their further development and applications to many vital problems in nonlinear mechanics. Among them, the problems of reducibility of a nonlinear system to a linear one with constant coefficients, reducibility of linear differential equations with quasi-periodic coefficients, as well as the problem of the behavior of integral curves in a vicinity of analytic and smooth manifolds.

The creative ideas and fundamental results of Bogoliubov in nonlinear mechanics laid the foundation to the global research in such areas as general mechanics, continuum mechanics, celestial mechanics, mechanics of rigid bodies and gyroscopic systems, motion stability, control theory, regulation and stabilization, mechanics of space flights, oscillations of mechanical systems, aero- and rocket construction, mathematical ecology, as well as other branches of science and engineering.

The word "nonlinear mechanics" has entered the scientific lexicon and is being widely used by mechanical and electrical engineers who are involved in the construction of systems with small perturbations. It is also being used by mathematicians who deal with differential equations containing small additive perturbation terms. The use of "nonlinear mechanics" has been further extended to include other mathematical disciplines such as nonlinear analysis and nonlinear dynamics.

The methods developed by Bogoliubov for the investigation of dynamical systems opened new avenues to problems of classical statistical physics. As early as in 1945, when studying the impact of a random force on a harmonic oscillator, Bogoliubov developed and applied for the first time an idea about the time hierarchy in the statistical theory of irreversible processes.

Chaining of recursive equations method proposed by Bogoliubov in 1946 for the distribution function of complexes of one, two, and more particles proved to be most efficient in modern statistical mechanics of processes in their equilibrium or transient state.

Discarding Ludwig Boltzmann's hypothesis on molecular chaos and having suggested a new idea of using boundary conditions that reduce correlation, Bogoliubov arrived at a method which allowed to include higher order terms of expansion in density powers. The same methods Bogoliubov utilized in statistical quantum mechanics in 1946–1948. Here he proposed a version of a secondary quantization with numerous forthcoming applications. He also developed a generalized method of self-conformed field presently referred to as the Hartry–Fock–Bogoliubov's method.

Bogoliubov's name is inseparable with the appearance of modern theory of nonideal quantum macrosystems. His scientific dissertation on such significant physical phenomena as superfluidity (1946) and superconductivity (1957) was a core element of this theory.

Bogoliubov constructed an appropriate mathematical apparatus based on a special canonic transformation of birth-death operators now widely known as Bogoliubov's (u, v)-transformation. This transformation is extensively applied in theoretical physics, in particular, in recent works on quantum theory of gravitation and the theory of nonideal Bose-condensate in magneto-optical traps.

Further development of superconductivity theory as superfluidity of the Fermi systems led Bogoliubov to the discovery of a new principal phenomenon — the superfluidity of nuclear substance. The notion of superfluidity of nuclear substance formed the basis for the modern theory of nucleus.

When studying a stabilization problem of condensate in nonideal systems, Bogoliubov developed the method of quasi-mean (1961). The latter turned out to be a universal tool for the investigation of systems whose main state is unstable under small perturbations.

In the fiftieth, Bogoliubov formulated quantum field theory with new causality condition. This condition is widely known today as the Bogoliubov's microcausality condition. The axiomatic theory of perturbations and in quantum field theory created by Bogoliubov was based on the disperse matrix and it reshaped the development of this theory to the present day. Bogoliubov proved a theorem stating that the disperse matrix in all orders of perturbation theory is well defined by the conditions of relativistic invariance, spectrality, unitarity and causality up to quasi-local operators. It yielded a source of ultra-violet divergences of the disperse matrix and gave rise to the method of their successive elimination. This method was called the R-operation (1955).

Bogoliubov was among the first scientists who had dealt with axiomatic quantum field theory (neither making any assumption on the weakness of interactions nor using perturbation theory). Furthermore, he only partially modified the existing system of axioms in perturbation theory by augmenting it with the stability condition on vacuum and single-particle state and reformulating the causality condition (1956). Bogoliubov used this to establish expressions for pion-nucleolus dispersion. The latter in turn required the development of mathematical tool of analytical continuation for generalized functions of many complex variables. His famous "edge of the wedge" theorem was formulated and proved in 1956 among his purely mathematical results and today this theorem is named after him. His work in quantum field theory gave rise to the new direction — the theory of strong interactions.

The scope of Bogoliubov's scientific accomplishments is not limited by the areas cited herein. It also includes a fundamental research on the theory of plasma and on kinetic equations which are of great practical significance.

Among his most profound creations was the idea of spontaneous violation of symmetry, initially proposed in the framework of statistical mechanics when developing superfluidity theory in 1948. It was a core part of the theory of weak electrical interaction, various versions of the theory of Great Unification. It also laid foundation to the most significant research on elementary particles, the theory of nucleus, and theory of phase transitions in early Universe models.

For many years Bogoliubov has carried out an enormous work on training young scientists. Being the department head at Kiev and then Moscow University he systematically gave lectures which were received with great interest. He also presented talks in England, Belgium, Bulgaria, Hungary, Italy, India, Poland, the USA, Finland, Germany, Yugoslavia, Japan and many other countries. Each one of them was a major scientific event.

Another Bogoliubov's distinct achievement was his creation of several scientific schools. During his employment in the Ukraine he established a school of mathematical physics and nonlinear mechanics in Kiev and then — the schools of elementary particles (theoretical physics) in Moscow and Dubna. Many well-known scientists proudly and respectfully regard Bogoliubov as their teacher.

Academician Bogoliubov was a dedicated organizer of science in the former Soviet Union. He was a member of the Presidium and the Academician-Secretary of mathematics division of the USSR Academy of Sciences exerting a beneficial influence in promoting the development of research in mathematics and physics in the Ukraine. Over 25 years did he head the largest International Scientific Center — the Joint Institute of Nuclear Research in Dubna, and was the founder and the first director of the Institute of Theoretical Physics of the Ukrainian Academy of Sciences. He was the founding chief editor of the two world renowned journals: "Theoretical and Mathematical Physics" and "Physics of Elementary Particles and Atom Nucleus".

Despite his deep involvement in teaching and an immense scholarly work, Bogoliubov was a dedicated volunteer in public work and political life. He was a representative of the Supreme Soviet Parliament and a member of the Pagoush piece movement of scientists. The Government of the former USSR rightfully recognized Bogoliubov's scientific and public achievements and awarded him with two Gold Star Medals and two titles of the prestigious Social Labor Hero and five Lenin's Medals of Honor. He was also highly decorated with many other prestigious awards and medals.

Bogoliubov was an Honorary Member of several foreign Academies and various scientific societies. He earned an Honorary Doctorate of numerous foreign Universities and he was a Laureate of prestigious awards and medals.

Bogoliubov is regarded a triune personality in science. He was a great mathematician, physicist and a pioneer in mechanics. He mastered fine problems of mathematical modeling and became a ground breaker of new horizons in modern mathematical physics. Finally, he was well-acquainted with both theoretical and practical needs of mechanics and technology.

Monographs and Books by N.N. Bogoliubov in the area of nonlinear mechanics

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