Nonlinear Dynamics and Systems Theory, 9 (2) (2009) 117–145

Novel Qualitative Methods of Nonlinear Mechanics and their Application to the Analysis of Multifrequency Oscillations, Stability, and Control Problems ◇

A.M. Kovalev [1], A.A. Martynyuk [2*], O.A. Boichuk [3], A.G. Mazko [3], R.I. Petryshyn [4], V.Yu. Slyusarchuk [5], A.L. Zuyev [1], and V.I. Slyn'ko [2].

[1] *Institute of Applied Mathematics and Mechanics, National Academy of Sciences of Ukraine, R. Luxemburg Str., 74, Donetsk, 83114, Ukraine*
[2] *Institute of Mechanics, National Academy of Sciences of Ukraine, Nesterov Str., 3, Kyiv, 03057, MSP-680, Ukraine*
[3] *Institute of Mathematics, National Academy of Sciences of Ukraine, Tereshchenkivska Str., 3, 01601 Kyiv-4, Ukraine*
[4] *Yuriy Fedkovych Chernivtsi National University, Kotsiubynskogo Str., 2, 58012, Chernivtsi, Ukraine*
[5] *National University of Water Management and Nature Resources Use, Soborna Str., 11, 33000 Rivne, Ukraine*

Received: December 1, 2008; Revised: March 15, 2009

Abstract: The method of oriented manifolds is developed to study geometric properties of the sets of trajectories of nonlinear differential systems with control. This method is conceptually connected with the classical methods of Lyapunov, Poincaré, and Levi–Civita and is a natural extension and development of results of the Donetsk school of mechanics. In terms of the method of oriented manifolds, sufficient conditions for stabilizability of nonlinear control systems are established.

A new method for stability investigation of nonlinear differential systems of perturbed motions is created on the basis of the concept of matrix-valued Lyapunov functions. This method is generalized for the systems with impulse action and aftereffect, differential equations with explosive right-hand sides and hybrid systems.

New conditions of practical stability of motion for nonlinear systems with impulse action are established on the basis of two auxiliary Lyapunov functions and the condition of exponential stability for linear impulse systems in a Hilbert space.

◇ Series of works honoured with the State Prize of Ukraine in the Field of Science and Technology in 2008.

* Corresponding author: anmart@stability.kiev.ua

General theory of the Fredholm boundary-value problems is constructed for systems of functional-differential equations, a classification of resonance boundary-value problems is elaborated, efficient coefficient criteria of existence of solutions are obtained and bifurcation and branching conditions for solutions to such problems are established.

New matrix methods are developed for the analysis of stability, localization of spectrum and representation of solutions of arbitrary order linear differential and difference systems. The methods of comparison and robust stability analysis are worked out for nonlinear dynamic systems in partially ordered space.

The averaging technique and the method of integral manifolds are developed for nonlinear resonance oscillating systems with slowly varying frequencies. New exact error estimations are established for the averaging technique in the initial and boundary-value problems for multifrequency systems and systems with impulse action.

New statements on stability and instability of linear approach to solutions of evolutionary equations in a Banach space are made. Absolute stability conditions are established for systems with aftereffect. In particular, a process of aircraft undercarriage galloping is studied at landing on the ground airfield with constant velocity. Also, stability conditions are established for the metal cutting process at turning behind a track with constant angular velocity of spindle rotation.

Keywords: *stability; robust stability; practical stability of motion; initial and boundary-value problems; differential and difference systems; systems with impulse action and aftereffect; comparison principle; Lyapunov functions; matrix equation; generalized Lyapunov equation; cone inequality.*

Mathematics Subject Classification (2000): 15A04, 15A18, 15A22, 15A24,15A42, 15A48, 34K06, 34K11,34K20, 37N15, 93A15, 93B05, 93B07, 93B18, 93B25, 93B55, 93C10, 93C35, 93D09, 93D15, 93D20, 93D30.