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Decoupled-natural-dynamics Model for the Relative Motion of two Spacecraft without and with J2 Perturbation

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Abstract: This paper presents the analytical steps for decoupling the natural dynamics representing the relative motion of two spacecraft flying in close orbits, both without and with the inclusion of the J2 perturbation. Linear mathematical models with constant coefficients are available in literature for representing such dynamics. In both cases two modes can be highlighted through the eigenvalue analysis of the state matrix: a double integrator, representing the secular part of the spacecraft relative motion, and a harmonic part, related to the typical oscillations present in spacecraft relative dynamics. In this work we introduce a rigorous two-step state vector transformation, based on a Jordan form, in order to decouple the two modes and be able to focus on either of them independently. The obtained results give a deep insight to the control designer, allowing for easy stabilization of the two spacecraft relative dynamics, i.e. canceling out the double integrator mode, which implies a constant drift taking the two spacecraft apart. On the other hand, one could desire an immediate control on the harmonic part of the dynamics, which is here made possible thanks to the decoupled form of the final equations. Furthermore, the obtained decoupled equations of motion present an analytical solution when only along-track control is applied to the spacecraft. This solution is here presented. The phase planes behavior for the controlled cases is reported.

Keywords: *spacecraft relative motion; linear dynamics transformation; Jordan form.*

Mathematics Subject Classification (2000): 37N05, 70F10.

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1 Nomenclature

α = Free parameter in generalized eigenvector calculation	J_2 = Second order harmonic of Earth gravitational potential field (Earth flattening) [108263×10^{-8}, [1]]
a = First non zero and non unity value in the state matrix	λ = Vector of the eigenvalues of A
A = State matrix	LVLH = Local Vertical Local Horizontal
A' = State matrix after first transformation	ω = Reference orbit angular velocity
$\hat{A}$ = State matrix decoupled (after second transformation)	r_{ref} = Reference orbit radius
β = First free parameter in second transformation matrix T_2	R_e = Earth mean radius [$6378.1363\,km$, [1]]
b = Second non zero and non unity value in the state matrix	T = Transformation matrix
B = Control distribution matrix	T_1 = First transformation matrix
$\hat{B}$ = Control distribution matrix after transformations	T_2 = Second transformation matrix
γ = Second free parameter in second transformation matrix T_2	t = Time
i = Complex unity	u = Control vector
i_{ref} = Reference orbit inclination	$w_i,\ i = 1, .., 4$ = Eigenvectors of A
	x = Spacecraft relative state vector in LVLH frame
	$x,\ y$ = Spacecraft relative position components in LVLH frame
	z = Transformed spacecraft relative state vector
	$(...)_0$ = Initial value ($t = 0$)

2 Introduction

A formal state vector transformation is presented in order to separate the two modes characterizing the relative motion between a chaser spacecraft and a target spacecraft in circular orbit, for both the well known unperturbed Hill–Clohessy–Wiltshire [2] model and the more recent Schweighart–Sedwick [3] model which includes the J_2 perturbation are used. Only the in-plane part of the relative motion is here considered, being the out-of-plane dynamics decoupled.

Our work is built upon the work of Leonard [4] who separates the dynamic of the Hill–Clohessy–Wiltshire model by averaging the evolution in time of the state variables, without developing a formal state transformation.

In particular, we employ a two-steps transformation into a Jordan form [5, 6] and then into a new decoupled-natural-dynamics form by using a chain of generalized eigenvectors in order to cope with the defectiveness of the state matrix. We obtain two transformed system models (for the cases with and without J_2) with the natural dynamics decoupled into a double integrator and a harmonic oscillator. The present work embodies the results of Leonard ([4], moving-ellipse formulation of Hill–Clohessy–Wilshire model) as a particular case.

The obtained results add further insight to the description of spacecraft relative motion, and, in particular, enables the control designer to focus on either one of two critical goals regarding the stabilization of the chaser's motion with respect to the target: namely, either the stabilization into a closed elliptical relative orbit or into a separate circular orbit with respect to the Earth center.

Furthermore, we perform the analytical integration of the transformed dynamics by considering only along-track thrust (as proposed in recent literature to simplify mission design, [7]–[9]).

The decoupled dynamics here obtained, and in particular the analytical nature of the obtained results, have been used by Bevilacqua and Romano [10, 12] for developing a completely analytical differential drag controller for multiple spacecraft assembly.

The paper is organized as follows: Section 3 introduces the linear models without and with J_2 perturbation. Section 4 is dedicated to the state vector transformations. Section 5 gives the analytical solution for the time evolution of the state vector for the case of constant along-track control. Finally Section 6 concludes the paper.

3 Spacecraft Relative Motion Dynamics

The in-plane part of the motion of a chaser spacecraft with respect to a target spacecraft in circular orbit can be represented by the following general equation, encompassing both the Hill–Clohessy–Wiltshire [2] unperturbed model and the Schweighart–Sedwick [3] model which includes J_2 perturbation

$$\dot{x} = A\,x + Bu, \quad x = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}, \quad A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ b & 0 & 0 & a \\ 0 & 0 & -a & 0 \end{pmatrix}, \tag{3.1}$$

where x is the "R-bar" axis, pointing from the Earth's center to the LVLH frame's origin at the target spacecraft, y is the "V-bar" axis in the direction of the velocity of the target along a circular orbit.

For the Hill-Clohessy–Wiltshire model it is

$$a = 2\omega, \quad b = 3\omega^2. \tag{3.2}$$

For the Schweighart-Sedwick model it is

$$a = 2\omega\,c, \quad b = \left(5c^2 - 2\right)\omega^2, \tag{3.3}$$

where the coefficient c is given by

$$c = \sqrt{1+s}, \quad s = \frac{3J_2R_e^2}{8r_{ref}^2}\left(1 + 3cos2i_{ref}\right). \tag{3.4}$$

The following substantial difference exists between the Hill–Clohessy–Wiltshire model and the Schweighart–Sedwick model: while the state vector of the former model describes the chaser's position and velocity with respect to either a target spacecraft or a reference point in circular orbit, the state vector of the latter model describes the chaser's position and velocity only with respect to a target spacecraft. Indeed, in the Schweighart–Sedwick case, the evolution of the state of the chaser with respect to a reference point in circular orbit is described by a more complicated expression, due to the J_2 perturbation [3].

It is immediate to see that, if we neglect the J_2 perturbation, the Schweighart–Sedwick equations reduce to the Hill–Clohessy–Wiltshire equations. Furthermore, we underline the fact that the condition $b < a^2$ holds for both models. In particular, while this is immediately obvious for the Hill–Clohessy–Wiltshire case, for the Schweighart–Sedwick model it translates onto the following condition for the variable s

$$\left(5c^2 - 2\right)\omega^2 < 4\omega^2 c^2 \rightarrow |s| < 1 \tag{3.5}$$

which is always true, because $\max\left(|s|\right) = \dfrac{3J_2R_e^2}{2r_{ref}^2} \leq \dfrac{3J_2}{2} = 1.624 \cdot 10^{-3}$.

4 State Vector Transformation

The eigenvalues of the state matrix A in (3.1) are

$$\lambda = \left[0, 0, \sqrt{b - a^2}, -\sqrt{b - a^2},\right]^T. \tag{4.1}$$

Being $b < a^2$, the third and fourth eigenvalues in (4.1) are complex conjugated.

By observing (4.1), it is clear that a double integrator and a harmonic oscillator are the two modes composing the natural dynamics.

Only the following three independent eigenvectors exist for the matrix A

$$w_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \, w_3 = \begin{bmatrix} -\dfrac{\sqrt{b-a^2}}{a} \\ 1 \\ \dfrac{a^2 - b}{a} \\ \sqrt{b-a^2} \end{bmatrix}, \, w_4 = \begin{bmatrix} \dfrac{\sqrt{b-a^2}}{a} \\ 1 \\ \dfrac{a^2 - b}{a} \\ -\sqrt{b-a^2} \end{bmatrix}, \tag{4.2}$$

where w_1 corresponds to the two multiple zero eigenvalues (Eq. (4.1)), and w_3 and w_4 correspond to the two complex conjugated eigenvalues. Since the state matrix A is defective (there are only three independent eigenvectors for the system which is of fourth order), it cannot be diagonalized. As an alternative to diagonalization, we look for a similarity transformation aiming to possibly represent the system with the state matrix in the following form

$$\hat{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\Omega^2 & 0 \end{pmatrix}. \tag{4.3}$$

This new form of the system matrix, inspired by the developments of [4], is useful because it decouples the natural dynamics into a double integrator and a harmonic oscillator. In (4.3), Ω represents the frequency of the harmonic oscillator.

As a first step of the transformation, we build a transformation of A into the modified-diagonal form (or Jordan form, see [5],[6]). Let us write

$$x = T_1 z', \tag{4.4}$$

where z' is the corresponding new state. The transformation matrix T_1 is obtained as follows

$$T_1 = \begin{pmatrix} w_1 & w_2 & w_3 & w_4 \end{pmatrix}, \tag{4.5}$$

where w_2 is the generalized eigenvector found by solving the following "Jordan chain" equation ([5])

$$(A - \lambda(1) I) w_2 = w_1 \to A w_2 = w_1 \to A^2 w_2 = A w_1, \tag{4.6}$$

where $\lambda(1) = 0$, from (4.1), leading to

$$w_2 = \left[-a/b, \alpha, 0, 1 \right]^T, \tag{4.7}$$

where α is an arbitrary complex parameter which is obtained form the "Jordan chain" procedure and can be conveniently chosen, as shown in the following.

The transformation of Eq. (4.5) results in the following Jordan-form

$$A' = T_1^{-1} A T_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{b-a^2} & 0 \\ 0 & 0 & 0 & -\sqrt{b-a^2} \end{pmatrix}. \tag{4.8}$$

As a second step of the transformation of the system matrix toward the desired form of Eq. (4.3), we use the following complex transformation matrix

$$T_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\beta\sqrt{a^2-b} & i\beta \\ 0 & 0 & \gamma\sqrt{a^2-b} & i\gamma \end{pmatrix}, \tag{4.9}$$

where β and γ are arbitrary complex parameters which can be conveniently selected, as explained later.

The final expression for the state matrix is calculated as

$$\hat{A} = T_2^{-1} A' T_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & b-a^2 & 0 \end{pmatrix}. \tag{4.10}$$

This transformed system matrix is indeed in the desired form of Eq. (4.3) with $\Omega = \sqrt{a^2 - b}$.

The overall transformation is given by

$$x = Tz, T = T_2 T_1 = \begin{pmatrix} 0 & -\dfrac{a}{b} & i\dfrac{\left(a^2-b\right)(\beta+\gamma)}{a} & \dfrac{(\beta-\gamma)\sqrt{a^2-b}}{a} \\ 1 & \alpha & -(\beta-\gamma)\sqrt{a^2-b} & i(\beta+\gamma) \\ 0 & 0 & \dfrac{(\beta-\gamma)\sqrt{\left(a^2-b\right)^3}}{a} & i\dfrac{\left(a^2-b\right)(\beta+\gamma)}{a} \\ 0 & 1 & -i\left(a^2-b\right)(\beta+\gamma) & -(\beta-\gamma)\sqrt{a^2-b} \end{pmatrix}. \tag{4.11}$$

5 Analytical Solution of the Transformed Equations in Case of Constant Along-Track Control

We here focus the attention on the case of a single control thrust acting along the y axis. In this case, the initial and transformed control distribution matrices are

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \hat{B} = T^{-1}B = \begin{bmatrix} \dfrac{\alpha b}{a^2 - b} \\ -\dfrac{b}{a^2 - b} \\ \dfrac{1}{4}\dfrac{i(\beta+\gamma)a^2}{\beta\gamma(a^2-b)^2} \\ -\dfrac{1}{4}\dfrac{(\beta-\gamma)a^2}{\beta\gamma(a^2-b)^{\frac{3}{2}}} \end{bmatrix}. \tag{5.1}$$

In order to have a control distribution matrix with real values, α, $\dfrac{i(\beta+\gamma)}{\beta\gamma}$ and $\dfrac{(\beta-\gamma)}{\beta\gamma}$ must all be real. The last two conditions are satisfied only if $\gamma = -\beta$, yielding to (5.2)

$$\hat{B} = \begin{bmatrix} \dfrac{\alpha b}{a^2 - b} \\ -\dfrac{b}{a^2 - b} \\ -\dfrac{1}{2}\dfrac{Im(\beta)\,a^2}{\|\beta\|^2(a^2-b)^2} \\ \dfrac{1}{2}\dfrac{Re(\beta)a^2}{\|\beta\|^2(a^2-b)^{\frac{3}{2}}} \end{bmatrix}. \tag{5.2}$$

At this stage, looking at (5.2), we are able to impose convenient values for the arbitrary parameters α and β. We choose those values to be $\alpha = 0$, $\beta = -\dfrac{1}{a}$. Therefore, the matrices in (4.11) and (5.2) become

$$T = T_2T_1 = \begin{pmatrix} 0 & -\dfrac{a}{b} & 0 & -\dfrac{2\sqrt{a^2-b}}{a^2} \\ 1 & 0 & \dfrac{2\sqrt{a^2-b}}{a} & 0 \\ 0 & 0 & -\dfrac{2\sqrt{(a^2-b)^3}}{a^2} & 0 \\ 0 & 1 & 0 & \dfrac{2\sqrt{a^2-b}}{a} \end{pmatrix}, \hat{B} = T^{-1}B = \begin{bmatrix} 0 \\ -\dfrac{b}{a^2-b} \\ 0 \\ \dfrac{a^3}{2(a^2-b)^{\frac{3}{2}}} \end{bmatrix}. \tag{5.3}$$

The expressions of (5.3) are expanded in the Appendix as functions of ω and c. Moreover, we have

$$x = Tz = \begin{bmatrix} -\dfrac{a^3 z_2 + 2b\sqrt{a^2-b}z_4}{a^2 b} \\ \dfrac{az_1 + 2\sqrt{a^2-b}z_3}{a} \\ -\dfrac{2\sqrt{a^2-b}z_3}{a^2} \\ \dfrac{az_2 + 2\sqrt{a^2-b}z_4}{a} \end{bmatrix}, \quad z = T^{-1}x = \begin{bmatrix} \dfrac{a^2 y - by - a\dot{x}}{a^2-b} \\ -\dfrac{b\,(ax+\dot{y})}{a^2-b} \\ -\dfrac{a^2\dot{x}}{2\,(a^2-b)^{\frac{3}{2}}} \\ -\dfrac{a^2\,(bx+a\dot{y})}{2\,(a^2-b)^{\frac{3}{2}}} \end{bmatrix}. \tag{5.4}$$

In particular, for the Hill–Clohessy–Wiltshire dynamic model, the transformed system with control along y is obtained by substituting the values of a and b given in (3.2) into (4.10) and (5.3)

$$\hat{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega^2 & 0 \end{pmatrix}, \quad \hat{B} = \begin{bmatrix} 0 \\ -3 \\ 0 \\ 4 \end{bmatrix}. \tag{5.5}$$

Eq. (5.5), corresponding to our new state $\begin{bmatrix} z_1 & z_2 & z_3 & z_4 \end{bmatrix}^T$, reproduces the results of Leonard ([4], where the state, in Leonard's notation, is $\begin{bmatrix} \bar{y} & \dot{\bar{y}} & \beta & \dot{\beta} \end{bmatrix}^T$, with β having a different meaning with respect to our notation).

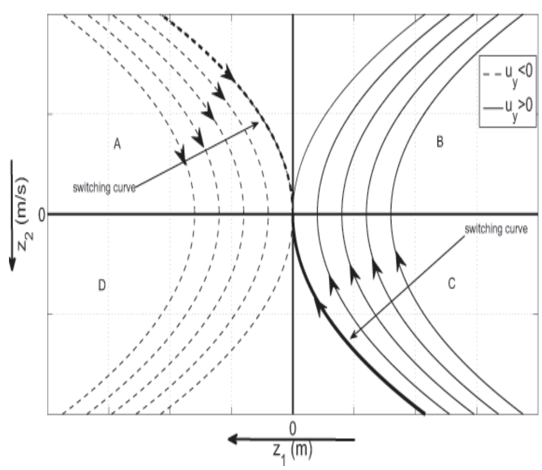

Figure 5.1: Qualitative shape of the curves on the phase plane of the double integrator subsystem (z_1 vs. z_2) in case of constant thrust along the y axis for both the Hill–Clohessy–Wiltshire and the Schweighart–Sedwick models.

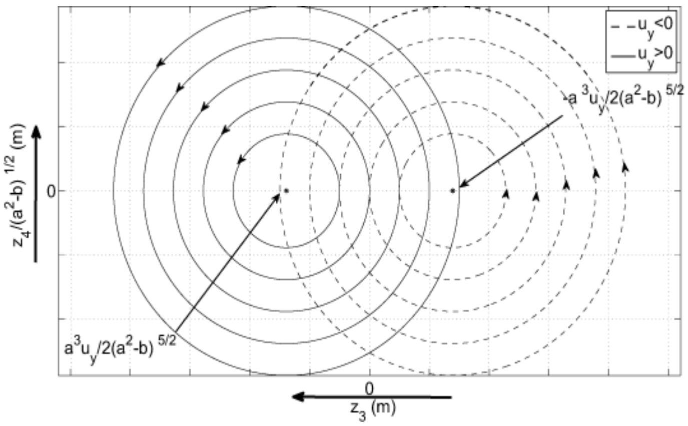

Figure 5.2: Qualitative shape of the curves on the phase plane of the harmonic oscillator subsystem (z_3 vs. z_4) in case of constant thrust along the y axis for both the Hill–Clohessy–Wiltshire and the Schweighart–Sedwick models.

Analytical integration of the transformed dynamics, taking into account only a constant controlling thrust along y, leads to

$$
\begin{gathered}
z_1 = -\frac{b}{a^2-b}u_y\frac{t^2}{2} + z_{2_0}t + z_{1_0}, \quad z_2 = -\frac{b}{a^2-b}u_y t + z_{2_0}, \\
z_3 = \left(z_{3_0} - \frac{a^3 u_y}{2\left(a^2-b\right)^{\frac{5}{2}}}\right)\cos\left[\left(\sqrt{a^2-b}\right)t\right] + \frac{z_{4_0}}{\sqrt{a^2-b}}\sin\left[\left(\sqrt{a^2-b}\right)t\right] + \frac{a^3 u_y}{2\left(a^2-b\right)^{\frac{5}{2}}}, \\
z_4 = z_{4_0}\cos\left[\left(\sqrt{a^2-b}\right)t\right] - \sqrt{a^2-b}\left(z_{3_0} - \frac{a^3 u_y}{2\left(a^2-b\right)^{\frac{5}{2}}}\right)\sin\left[\left(\sqrt{a^2-b}\right)t\right].
\end{gathered}
\tag{5.6}
$$

The assumption of continuous constant thrust reflects the state of the art for space thrusters, where only a regime value for the control is available [12]. Figure 5.1 and Figure 5.2 show the phase planes for the two types of forced motion (the forced double integrator represented by state variables z_1 and z_2, and the forced harmonic oscillator represented by state variables z_3 and z_4) with either positive or negative constant control along y. Arrows are indicating the paths directions according to the sign of the control. The curves on phase plane z_1 vs. z_2 are parabolas with symmetry about the z_2 axis for both the Hill–Clohessy–Wiltshire and the Schweighart–Sedwick models (only the curvature changes in the two cases, being in particular equal to $-\frac{3\omega^2}{8u_y}$ for the Hill–Clohessy–Wiltshire model and $-\frac{a^2-b}{2bu_y}$ for the Schweighart–Sedwick model). The curves on the phase plane $z_3\frac{z_4}{\sqrt{a^2-b}}$ are circles for both the Hill–Clohessy–Wiltshire and Schweighart-Sedwick models. The z_3 coordinates for the centers of the circles in Figure 5.2 are given by

$$
\pm\frac{a^3 u_y}{2\left(a^2-b\right)^{\frac{5}{2}}} \tag{5.7}
$$

as calculated through the analytical solution in (5.6). The position of those centers is positive or negative according to the control sign.

Eq. (5.6) also gives the state vector evolution for coasting (control off) phases, by simply imposing $u_y = 0$. In particular, when the control is off, a drift parallel to the z_1 axis is experienced in the z_1 vs. z_2 phase plane, whose direction is related to the sign of z_2 (see Eq. (5.6)), while the circles in Figure 5.2 simply evolve around the origin. Again, the phase planes reproduce the results of [4] when the values for Hill–Clohessy–Wiltshire equations are used for a and b.

Eq. (5.4) and Eq. (5.6) together show how the spacecraft relative motion can be seen as an oscillation, represented by the states z_3 and z_4, around a virtual point, whose evolution is given by z_1 and z_2 in (5.6).

6 Conclusion

We developed a linear transformation of both the Hill–Clohessy–Wiltshire model for spacecraft relative motion nearby a circular orbit and the more recent Schweighart–Sedwick including the J2 effect. The proposed transformation highlights the superposition of double integrator and harmonic oscillator modes. Previous results in literature, regarding the traveling-ellipse formulation of the Hill–Clohessy–Wiltshire equations are included as a particular case of our state vector transformation. In particular we give analytical solution and a description of the phase planes when only along-track control is used. The achieved dynamic separation via state transformation allows the control designer to focus directly on either one of two critical goals regarding the stabilization of the chaser's motion with respect to the target: namely, either the stabilization into a closed elliptical relative orbit or into a separate circular orbit with respect to the Earth center.

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7 APPENDIX

Substitution of (3.3) into (5.3) leads to

$$T = \begin{pmatrix} 0 & -2\dfrac{c}{\omega\left(5c^2-2\right)} & 0 & \dfrac{i\sqrt{\omega^2\left(c^2-2\right)}}{2\omega^2c^2} \\ 1 & 0 & \dfrac{-i\sqrt{\omega^2\left(c^2-2\right)}}{\omega c} & 0 \\ 0 & 0 & \dfrac{i\left(c^2-2\right)\sqrt{\omega^2\left(c^2-2\right)}}{2c^2} & 0 \\ 0 & 1 & 0 & \dfrac{i\sqrt{\omega^2\left(c^2-2\right)}}{\omega c} \end{pmatrix},$$

$$\hat{B} = \begin{bmatrix} 0 \\ -\frac{\left(5c^2-2\right)\omega^2}{4\omega^2c^2-\left(5c^2-2\right)\omega^2} \\ 0 \\ \frac{-4i\omega^3c^3}{\left(\left(5c^2-2\right)\omega^2-4\omega^2c^2\right)^{\frac{3}{2}}} \end{bmatrix}. \tag{7.1}$$

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