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On the Absolute Stabilization of Dynamical-Delay Systems

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Abstract: In this paper we deal with the problem of absolute stabilization for Lur'e systems with time-varying delay in a range. An appropriate Lyapunov-Krasovskii functional is proposed to investigate the delay-range-dependant stabilization problem.The time-varying delay is assumed to belong to an interval and no restriction on its derivative is needed. Some relaxation matrices are introduced, which allow the delay to be a fast time-varying function. Furthermore, a numerical example is given to prove effectiveness of our main result.

Keywords: *time-varying delay system; absolute stability; Lur'e system; LMI; $\mathcal{S}$-procedure; Shur complement; Lyapunov–Krasovskii functional.*

Mathematics Subject Classification (2000): 93D05, 93D09, 93D15, 34D23.

1 Introduction

During the last two decades, considerable attention has been devoted to the problem of delay-dependent stability analysis and controller design for time-delay systems. For the recent progress, the reader is referred to [10, 11, 19, 27, 33, 37]. It is well known that the choice of an appropriate Lyapunov–Krasovskii functional (LKF) is crucial for deriving stability criteria and for obtaining a solution to various control problems.

We shall note that studies of stability of time-delay systems have grown steadily. Indeed, since 1940 all the results were delay independent see for examples [3, 9, 15, 20, 22, 29, 30]. But, the problem is that when the time-delay is small, these results are often overly conservative, especially, they are not applicable to closed-loop systems which are open-loop unstable and are stabilized using delayed inputs. That's why, many efforts were sacrificed to provide delay-dependant stability criteria.

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Since the introduction of absolute stability by Lur'e (1957), the absolute stability problem of nonlinear control systems with a fixed matrix in the linear part of the system and one or multiple uncertain nonlinearities satisfying the sector constraints has been the subject of many researches see [2, 18, 22, 25, 28, 34].

From the practical point of view and since in general the delay is not known, it is worth considering it as time-varying [5, 32, 35, 24]. For this object, one is interested in conditions that constrain the upper and lower bounds of the delay and the upper bound of the first derivative of the time-varying delay.

To the best of our knowledge, for the case where only the upper and lower bounds of the interval time-varying delay are precisely known and the lower bound of the delay is greater than zero, there is no result available for stability for such kinds of systems. It should also be mentioned that even for the case where the lower bound of the time-varying delay is zero and without considering the derivative of the time-varying delay, there are few works available in the existing literature [7, 13, 6] using Lyapunov–Krasovskii functional approach.

For this reason we are motivated to provide new stabilization criterion, in order to improve those in which some useful terms are ignored, when estimating the upper bound of the derivative of Lyapunov functional [8, 11].

Those resulting criteria are applicable to both fast and slow time-varying delay, in contrast with previous works in which the upper bound of the first derivative of the time-varying delay was either restricted to one or completely neglected, see [13, 31, 36]. It is important to mention that this became possible since the free matrices M_1 and M_2 of the proposition provide some extra freedom in their selection.

The stabilization criterion is formulated in the form of Linear Matrix Inequality (LMI). Moreover, we give an example to show the applicability of our main result.

Notation: Throughout this paper, $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^n$ denotes the n dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. I is the identity matrix. The set $\mathcal{C}_{n,\tau_M} := \mathcal{C}([-\tau_M, 0], \mathbb{R}^n)$ is the space of continuous functions mapping the interval $[-\tau_M, 0]$ to $\mathbb{R}^n$. The notation $A > 0$ is that the matrix A is positive definite.

2 Stabilization of Nonlinear Delay System

Consider the following time-varying-delay control system

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + A_1x(t-\tau(t)) + B\omega(t) + Gu(t), \\ y(t) &= C_0x(t) + C_1x(t-\tau(t)), \\ \omega(t) &= -\psi(t, y(t)), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $y(t) \in \mathbb{R}^p$ is the measured output, and the nonlinear function $\varphi(.,.) : \mathbb{R}_+ \times \mathbb{R}^p \to \mathbb{R}^p$ is assumed to be continuous and belongs to sector $[0, K]$, i.e $\varphi(.,.)$ satisfies

$$\varphi^\top(t, y)\,[\varphi(t, y) - Ky] \leq 0,\ \forall (t, y) \in \mathbb{R}_+ \times \mathbb{R}^p, \tag{2}$$

where K is a positive definite matrix. The matrices A_0, A_1, B, G, C_0, and C_1 are real matrices with appropriate dimensions. The time delay $\tau(t)$ is a time-varying continuous function that satisfies

$$0 \leq \tau_m \leq \tau(t) < \tau_M \ \text{ and } \ \dot{\tau}(t) < \mu, \tag{3}$$

where τ_m, τ_M and μ are known constant reals.

Note that τ_m may not be equal to 0. The initial condition of 1 is given by

$$x(t) = \phi(t),\ t \in [-\tau_M, 0],\ \phi \in \mathcal{C}_{n,\tau_M}.$$

It is assumed that the right-hand side of (1) is continuous and satisfies enough smoothness conditions to ensure the existence and uniqueness of the solution through every initial condition ϕ.

The closed-loop system with the state control feedback

$$u(t) = \tilde{K}x(t) \tag{4}$$

is given by

$$\dot{x}(t) = \left(A_0 + G\tilde{K}\right)x(t) + A_1 x(t-\tau(t)) + B\omega(t). \tag{5}$$

We first introduce the following definition.

Definition 2.1 The system (1) is said to be absolutely stabilizable in the sector $[0, K]$ if there exists a control $u(t) = Nx(t)$ such that the closed-loop system (5) is globally uniformly asymptotically stable for any nonlinear function $\varphi(t, y(t))$ satisfying (2).

The development of the work in this paper requires the following lemma which can be found in [36].

Lemma 2.1 *Let $x(t) \in \mathbb{R}^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any matrices $M_1, M_2 \in \mathbb{R}^{n\times n}$ and $X = X^\top > 0$, and a scalar function $\tau := \tau(t) \geq 0$:*

$$-\int_{t-\tau(t)}^{t} \dot{x}^\top(s)X\dot{x}(s)ds \quad \leq \quad \xi^\top(t)\Upsilon\xi(t) + \tau(t)\xi^\top(t)\Gamma^\top X^{-1}\Gamma\xi(t), \tag{6}$$

where

$$\Upsilon := \begin{bmatrix} M_1^\top + M_1 & -M_1^\top + M_2 \\ * & -M_2^\top - M_2 \end{bmatrix},\ \Gamma^\top := \begin{bmatrix} M_1^\top \\ M_2^\top \end{bmatrix},\ \xi(t) := \begin{bmatrix} x(t) \\ x(t-\tau(t)) \end{bmatrix}.$$

The following theorem gives a sufficient condition for stabilization of the system by means a state feedback when the nonlinearity $\psi(t, y)$ belongs to the sector $[0, K]$.

Theorem 2.1 *For given scalars $0 \leq \underline{\tau_m < \tau_M,}\ \lambda_i,\ \alpha_{\underline{i},}\ \underline{\beta_i} \in \mathbb{R},\ \underline{i} = 1, 2\underline{,}$ if there exist a scalar $\epsilon > 0$, positive definite matrices $\overline{P} > 0,\ \overline{Q_1} > 0,\ \overline{Q_2} > 0,\ \overline{Q_3} > 0,\ \overline{R_1} > 0,\ \overline{R2} > 0,\ \overline{R_3} > 0$, and a matrix $Y \in \mathbb{R}^{r\times n}$ such that the LMI*

$$\Xi_2 = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & 0 & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & 0 & \Xi_{110} & \Xi_{111} \\
* & \Xi_{22} & 0 & 0 & \Xi_{25} & \Xi_{26} & 0 & \Xi_{28} & 0 & \Xi_{210} & \Xi_{211} \\
* & * & \Xi_{33} & \Xi_{34} & 0 & 0 & 0 & 0 & \Xi_{39} & 0 & 0 \\
* & * & * & \Xi_{44} & 0 & 0 & \Xi_{47} & 0 & \Xi_{49} & 0 & 0 \\
* & * & * & * & -2\varepsilon.I & \tau_M B^T & 0 & \Xi_{58} & 0 & \tau_M B^T & 0 \\
* & * & * & * & * & -\tau_M\overline{R_1} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\tau_M\overline{R_1} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Xi_{88} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Xi_{99} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\tau_M\overline{R_3} & 0 \\
* & * & * & * & * & * & * & * & * & * & -\tau_M\overline{R_3}
\end{bmatrix} < 0, \tag{7}$$

where

$\Xi_{11} = \overline{P}(A_0 + (\lambda_1 + \alpha_1)I)^\top + (A_0 + (\lambda_1 + \alpha_1)I)\overline{P} + GY + Y^\top G^\top + \overline{Q_1} + \overline{Q_2} + \overline{Q_3},$
$\Xi_{12} = A_1\overline{P} + (\alpha_2 - \alpha_1)\overline{P},$
$\Xi_{14} = (\lambda_2 - \lambda_1)\overline{P},$
$\Xi_{15} = B - \epsilon\overline{P}C_0^\top K,$
$\Xi_{16} = \tau_M\overline{P}A_0^\top + \tau_M Y^\top G^\top,$
$\Xi_{17} = \lambda_1\tau_M\overline{R_1},$
$\Xi_{18} = (\tau_M - \tau_m)\overline{P}A_0^\top + (\tau_M - \tau_m)Y^\top G^\top,$
$\Xi_{110} = \tau_M\overline{P}A_0^\top + \tau_M Y^\top G^\top,$
$\Xi_{111} = \alpha_1\tau_M\overline{R_3},$
$\Xi_{22} = -(1-\mu)\overline{Q_3} - 2\alpha_2\overline{P},$
$\Xi_{25} = -\epsilon\overline{P}C_1^\top K,$
$\Xi_{26} = \tau_M\overline{P}A_1^\top,$
$\Xi_{28} = (\tau_M - \tau_m)\overline{P}A_1^\top,$
$\Xi_{210} = \tau_M\overline{P}A_1^\top,$
$\Xi_{211} = \alpha_2\tau_M\overline{R_3},$
$\Xi_{33} = -\overline{Q_1} + 2\beta_1\overline{P},$
$\Xi_{34} = (\beta_2 - \beta_1)\overline{P},$
$\Xi_{39} = \beta_1(\tau_M - \tau_m)\overline{R_2},$
$\Xi_{44} = -\overline{Q_2} - 2(\lambda_2 + \beta_1)\overline{P},$
$\Xi_{47} = \lambda_2\tau_M\overline{R_1},$
$\Xi_{49} = \beta_2(\tau_M - \tau_m)\overline{R_2},$
$\Xi_{58} = (\tau_M - \tau_m)B^T,$
$\Xi_{88} = -(\tau_M - \tau_m)\overline{R_2},$
$\Xi_{99} = -(\tau_M - \tau_m)\overline{R_2},$

holds. Then the origin of the controlled system (1) is stabilized by the linear state feedback (4), where

$$\tilde{K} = Y\overline{P}^{-1}.$$

Proof Let $0 \le \tau_m < \tau_M$, λ_1, λ_2, α_1, α_2, β_1 and β_2 be fixed reals. Suppose that there exist a scalar $\epsilon > 0$, positive definite matrices $\overline{P} > 0$, $\overline{Q_1} > 0$, $\overline{Q_2} > 0$, $\overline{Q_3} > 0$, $\overline{R_1} > 0$, $\overline{R_2} > 0$, $\overline{R_3} > 0$, and a matrix $Y \in \mathbb{R}^{r\times n}$ such that the LMI (7) is satisfied. Let as denote by Ξ_2' the new matrix obtained after making these changes in the matrix Ξ_2 :
$\overline{P}^{-1} = P,\ \overline{P}^{-1}\ \overline{Q_1}\ \overline{P}^{-1} = Q_1,\ \overline{P}^{-1}\ \overline{Q_2}\ \overline{P}^{-1} = Q_2,\ \overline{P}^{-1}\ \overline{Q_3}\ \overline{P}^{-1} = Q_3,\ \overline{R_1}^{-1} = R_1,\ \overline{R_2}^{-1} = R_2,\ \overline{R_3}^{-1} = R_3,\ \tilde{K}P^{-1} = Y,\ M_i = \lambda_i P,\ N_i = \beta_i P,\ S_i = \alpha_i P,\ i = 1,2.$
Then the LMI (7) is equivalent to the feasibility of the following LMI

$$T^\top\,\Xi_2'\,T = \Xi_1 < 0, \tag{8}$$

where $T = \mathrm{diag}\{P,\ P,\ P,\ P,\ I,\ R_1,\ R_1,\ R_2,\ R_2,\ R_3,\ R_3\}$,

$$\Xi_1 = \begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 & \Xi_{14} & \Xi_{15} & \Xi_{16} & \tau_M M_1^T & \Xi_{18} & 0 & \Xi_{110} & \tau_M S_1^T \\ * & \Xi_{22} & 0 & 0 & \Xi_{25} & \tau_M A_1^T R_1 & 0 & \Xi_{28} & 0 & \tau_M A_1^T R_3 & \tau_M S_2^T \\ * & * & \Xi_{33} & \Xi_{34} & 0 & 0 & 0 & 0 & \Xi_{39} & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & \tau_M M_2^T & 0 & \Xi_{49} & 0 & 0 \\ * & * & * & * & \Xi_{55} & \tau_M B^T R_1 & 0 & \Xi_{58} & 0 & \tau_M B^T R_3 & 0 \\ * & * & * & * & * & -\tau_M R_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\tau_M R_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Xi_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\tau_M R_3 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\tau_M R_3 \end{bmatrix} < 0,$$

where

$$\begin{aligned}
\Xi_{11} &= (A_0 + G\tilde{K})^\top P + P(A_0 + G\tilde{K}) + Q_1 + Q_2 + Q_3 + M_1^\top + M_1 + S_1^\top + S_1, \\
\Xi_{12} &= PA_1 - S_1^\top + S_2, \\
\Xi_{14} &= -M_1^\top + M_2, \\
\Xi_{15} &= PB - \epsilon\, C_0^\top K, \\
\Xi_{16} &= \tau_M (A_0 + G\tilde{K})^T R_1, \\
\Xi_{110} &= \tau_M (A_0 + G\tilde{K})^T R_3, \\
\Xi_{22} &= -(1-\mu) Q_3 - S_2^\top - S_2, \\
\Xi_{25} &= -\epsilon\, C_1^\top K, \\
\Xi_{33} &= -Q_1 + N_1^\top + N_1, \\
\Xi_{34} &= -N_1^\top + N_2, \\
\Xi_{44} &= -Q_2 - M_2^\top - M_2 - N_2^\top - N_2, \\
\Xi_{55} &= -2\epsilon\, I, \\
\Xi_{18} &= (\tau_M - \tau_m)(A_0 + G\tilde{K})^T R_2, \\
\Xi_{28} &= (\tau_M - \tau_m) A_1^T R_2, \\
\Xi_{58} &= (\tau_M - \tau_m) B^T R_2, \\
\Xi_{88} &= -(\tau_M - \tau_m) R_2, \\
\Xi_{39} &= (\tau_M - \tau_m) N_1^T, \\
\Xi_{49} &= (\tau_M - \tau_m) N_2^T, \\
\Xi_{99} &= -(\tau_M - \tau_m) R_2.
\end{aligned}$$

Next let us consider the Lyapunov–Krasovskii functional candidate

$$\begin{aligned}
V(t, x_t) &= x^\top(t) P x(t) + \int_{t-\tau_m}^{t} x^\top(s) Q_1 x(s) ds + \int_{t-\tau_M}^{t} x^\top(s) Q_2 x(s) ds \\
&\quad + \int_{t-\tau(t)}^{t} x^\top(s) Q_3 x(s) ds + \int_{-\tau_M}^{0} \int_{t+\theta}^{t} \dot{x}^\top(s) R_1 \dot{x}(s) ds d\theta \\
&\quad + \int_{-\tau_M}^{-\tau_m} \int_{t+\theta}^{t} \dot{x}^\top(s) R_2 \dot{x}(s) ds d\theta \\
&\quad + \int_{-\tau(t)}^{0} \int_{t+\theta}^{t} \dot{x}^\top(s) R_3 \dot{x}(s) ds d\theta.
\end{aligned}$$

Recall that matrices $P,\ Q_i,\ R_i,\ i = 1, 2, 3$ are positive definite as well as the matrices $\overline{P},\ \overline{Q_i},\ \overline{R_i},\ i = 1, 2, 3$. Then the derivative of V along the trajectories of system (1) is given by

$$\begin{aligned}\dot{V}(t,x_t) &= 2\dot{x}^\top(t)Px(t)+x^\top(t)Q_1x(t)-x^\top(t-\tau_m)Q_1x(t-\tau_m)\\ &+x^\top(t)Q_2x(t)-x^\top(t-\tau_M)Q_2x(t-\tau_M)\\ &+x^\top(t)Q_3x(t)-(1-\dot{\tau}(t))x^\top(t-\tau(t))Q_3x(t-\tau(t))\\ &+\tau_M\dot{x}^\top(t)R_1\dot{x}(t)-\int_{t-\tau_M}^{t}\dot{x}^\top(s)R_1\dot{x}(s)ds\\ &+(\tau_M-\tau_m)\dot{x}^\top(t)R_2\dot{x}(t)-\int_{t-\tau_M}^{t}-\tau_m\dot{x}^\top(s)R_2\dot{x}(s)ds\\ &+\tau(t)\dot{x}^\top(t)R_3\dot{x}(t)-\int_{t-\tau(t)}^{t}\dot{x}^\top(s)R_3\dot{x}(s)ds. \qquad (9)\end{aligned}$$

Using (3) and applying the integral inequality (4) to the right-hand side of (9), we obtain

$$\begin{aligned}\dot{V}(t,x_t) &\leq 2\dot{x}^\top(t)Px(t)+x^\top(t)[Q_1+Q_2+Q_3]x(t)-x^\top(t-\tau_m)Q_1x(t-\tau_m)\\ &-x^\top(t-\tau_M)Q_2x(t-\tau_M)-(1-\mu)x^\top(t-\tau(t))Q_3x(t-\tau(t))\\ &+\dot{x}^\top(t)[\tau_MR_1+(\tau_M-\tau_m)R_2+\tau_MR_3]\dot{x}(t)\\ &+\xi_1^\top(t)\Upsilon_1\xi_1(t)+\tau_M\xi_1^\top(t)\Gamma_1^\top R_1^{-1}\Gamma_1\xi_1(t)\\ &+\xi_2^\top(t)\Upsilon_2\xi_2(t)+(\tau_M-\tau_m)\xi_2^\top(t)\Gamma_2^\top R_2^{-1}\Gamma_2\xi_2(t)\\ &+\xi_3^\top(t)\Upsilon_3\xi_3(t)+\tau_M\xi_3^\top(t)\Gamma_3^\top R_3^{-1}\Gamma_3\xi_3(t)\end{aligned}$$

with

$$\xi_1(t)=\begin{bmatrix} x(t)\\ x(t-\tau_M)\end{bmatrix},\Gamma_1^\top=\begin{bmatrix} M_1^\top\\ M_2^\top\end{bmatrix},\Upsilon_1=\begin{bmatrix} M_1^\top+M_1 & -M_1^\top+M_2\\ * & -M_2^\top-M_2\end{bmatrix},$$

$$\xi_2(t)=\begin{bmatrix} x(t-\tau_m)\\ x(t-\tau_M)\end{bmatrix},\Gamma_2^\top=\begin{bmatrix} N_1^\top\\ N_2^\top\end{bmatrix},\Upsilon_2=\begin{bmatrix} N_1^\top+N_1 & -N_1^\top+N_2\\ * & -N_2^\top-N_2\end{bmatrix},$$

$$\xi_3(t)=\begin{bmatrix} x(t)\\ x(t-\tau(t))\end{bmatrix},\Gamma_3^\top=\begin{bmatrix} S_1^\top\\ S_2^\top\end{bmatrix},\Upsilon_3=\begin{bmatrix} S_1^\top+S_1 & -S_1^\top+S_2\\ * & -S_2^\top-S_2\end{bmatrix}.$$

Rearranging the terms of the right-hand side yields:

$$\dot{V}(t)\leq\eta^\top(t)\;\Pi\;\eta(t), \qquad (10)$$

where

$$\Pi:=\begin{bmatrix}\Pi_{11} & \Pi_{12} & 0 & \Pi_{14} & \Pi_{15}\\ * & \Pi_{22} & 0 & 0 & \Pi_{25}\\ * & * & \Pi_{33} & \Pi_{34} & 0\\ * & * & * & \Pi_{44} & 0\\ * & * & * & * & \Pi_{55}\end{bmatrix},\quad \eta(t):=\begin{bmatrix} x(t)\\ x(t-\tau(t))\\ x(t-\tau_m)\\ x(t-\tau_M)\\ \omega(t)\end{bmatrix}$$

with

$$
\begin{aligned}
\Pi_{11} &= (A_0+G\tilde{K})^\top P + P(A_0+G\tilde{K}) + Q_1+Q_2+Q_3+ \\
&\quad \tau_M(A_0+G\tilde{K})^\top R_1(A_0+G\tilde{K}) + (\tau_M-\tau_m)(A_0+G\tilde{K})^\top R_2(A_0+G\tilde{K}) + \\
&\quad \tau_M(A_0+G\tilde{K})^\top R_3(A_0+G\tilde{K}) + M_1^\top + M_1 + \tau_M M_1^\top R_1^{-1}M_1 \\
&\quad +\tau_M S_1^\top R_3^{-1}S_1 + S_1^\top + S_1, \\
\Pi_{12} &= PA_1 + \tau_M(A_0+G\tilde{K})^\top R_1A_1 + (\tau_M-\tau_m)(A_0+G\tilde{K})^\top R_2A_1 + \\
&\quad \tau_M(A_0+G\tilde{K})^\top R_3A_1 - S_1^\top + S_2 + \tau_M S_1R_3^{-1}S_2, \\
\Pi_{14} &= -M_1^\top + M_2 + \tau_M M_1^\top R_1^{-1}M_2, \\
\Pi_{15} &= PB + \tau_M(A_0+G\tilde{K})^\top R_1B + (\tau_M-\tau_m)(A_0+G\tilde{K})^\top R_2B + \\
&\quad \tau_M(A_0+G\tilde{K})^\top R_3B, \\
\Pi_{22} &= -(1-\mu)Q_3 - S_2^\top - S_2 + \tau_M A_1^\top R_1A_1 + (\tau_M-\tau_m)A_1^\top R_2A_1 + \tau_M A_1^\top R_3A_1 \\
&\quad +\tau_M S_2^\top R_3^{-1}S_2, \\
\Pi_{25} &= \tau_M A_1^\top R_1B + (\tau_M-\tau_m)A_1^\top R_2B + \tau_M A_1^\top R_3B, \\
\Pi_{33} &= -Q_1 + N_1^\top + N_1 + (\tau_M-\tau_m)N_1^\top R_2^{-1}N_1, \\
\Pi_{34} &= -N_1^\top + N_2 + (\tau_M-\tau_m)N_1^\top R_2^{-1}N_1, \\
\Pi_{44} &= -Q_2 - M_2^\top - M_2 + \tau_M M_2^\top R_1^{-1}M_2 + (\tau_M-\tau_m)N_2^\top R_2^{-1}N_2 - N_2^\top - N_2, \\
\Pi_{55} &= \tau_M B^\top R_1B + (\tau_M-\tau_m)B^\top R_2B + \tau_M B^\top R_3B.
\end{aligned}
$$

A sufficient condition for asymptotic stability of the system (1) is to show that

$$\dot{V}(t) \le \eta^\top(t)\Pi\eta(t) < 0 \tag{11}$$

for all $\eta(t)\neq 0$. Then using (8) and Shur Complement we can see that the LMI (8) is equivalent to the following:

$$
\Sigma = \begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} \\
* & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} \\
* & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} \\
* & * & * & \Sigma_{44} & \Sigma_{45} \\
* & * & * & * & \Sigma_{55}
\end{bmatrix} < 0
$$

with $\Sigma_{ij}=\Pi_{ij}$, $(i,j=1,2,3,4)$, $\Sigma_{15}=\Pi_{15}-\epsilon\, C_0^\top K$, $\Sigma_{25}=\Pi_{25}-\epsilon\, C_1^\top K$, $\Sigma_{35}=\Pi_{35}$, $\Sigma_{45}=\Pi_{45}$, $\Sigma_{55}=\Pi_{55}-2\epsilon\, I$. On the other hand, by using the S-procedure and (2) we have

$$\eta^\top(t)\,\Sigma\,\eta(t) = \eta^\top(t)\,\Pi\,\eta(t) - 2\epsilon\,\omega^\top(t)\omega(t) - 2\epsilon\,\omega^\top(t)\,[KC_0x(t)+KC_1x(t-\tau)] < 0 \tag{12}$$

for all $\eta(t)\neq 0$. This completes the proof. □

Example 2.1 Consider the time delay system (1) with the nonlinear function satisfying (2) with

$$
A_0 = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix},\ A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},\ G = \begin{bmatrix} -1 & -1 \end{bmatrix},
$$

$$
C_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ C_1 = \begin{bmatrix} 0 & -1 \\ -1 & -2 \end{bmatrix},\ K = \begin{bmatrix} 0.005 & 0 \\ 0 & 0.005 \end{bmatrix}. \tag{13}
$$

Let the extra parameters be fixed to:

$$\tau_m = 10^{-4},\ \tau_M = 0.088,\ \mu = 0.01,\ \lambda_1 = -1,\ \lambda_2 = -1.2,$$
$$\alpha_1 = -0.2,\ \alpha_2 = 0,\ \beta_1 = 0,\ \beta_2 = 0,$$

then by Theorem 1, we have $\epsilon = 0.6809$ and

$$\overline{Q_1} = \begin{bmatrix} 0.9955 & 0.1991 \\ 0.1991 & 0.9955 \end{bmatrix},\ \overline{Q_2} = \begin{bmatrix} 2.4678 & -0.5239 \\ -0.5239 & 2.0589 \end{bmatrix},\ \overline{Q_3} = \begin{bmatrix} 1.1643 & 0.0690 \\ 0.0690 & 1.1153 \end{bmatrix},$$

$$\overline{R_1} = \begin{bmatrix} 4.2714 & 0.4606 \\ 0.4606 & 4.1847 \end{bmatrix},\ \overline{R_2} = \begin{bmatrix} 13.6586 & -0.0274 \\ -0.0274 & 13.6260 \end{bmatrix},\ \overline{R_3} = \begin{bmatrix} 13.1236 & -0.0336 \\ -0.0336 & 13.0996 \end{bmatrix},$$

$$\overline{P} = \begin{bmatrix} 0.4781 & -0.3380 \\ -0.3380 & 0.3095 \end{bmatrix},\ Y = \begin{bmatrix} 1.1971 & 1.6605 \end{bmatrix},\ \tilde{K} = \begin{bmatrix} 27.6255 & 35.5345 \end{bmatrix}.$$

3 Conclusion

The problem of absolute stabilization of a class of time-varying delay systems with sector-bounded nonlinearity have been considered. New delay-dependant stabilization criterion with sector condition has been proposed. A new result is given and illustrated by numerical example, treated with Matlab, in order to show effectiveness of the main result. This criterion has been formulated in the form of linear matrix inequalities (LMI).

References

[1] Ben Abdallah, A. Ben Hamed, B. and Chaabane, M. Absolute Stability and Application to Design of Observer-Based Controller for Nonlinear Time-Delay Systems. *Asian Journal of Control* **9**(3) (2007) 362–371.

[2] Aizerman, M.A. and Gantmacher, F.R. *Absolute stability of regulator systems.* CA: Holden-Day, San Fransisco, 1964.

[3] Bliman, P.A. Lyapunov–Krasovskii functionals and frequency domain: delay-independent absolute stability criteria for delay systems. *Int. Journal of Robust and Nonlinear Control* **11** (8) (2001) 771–788.

[4] Cao, Y. and Cui, B.T. On the absolute stabilization of dynamical-delay systems. *Nonlinear Dynamics and Systems Theory* **8** (3) (2008) 287–298.

[5] Chen, Y. Xue, A. Lu, R. and Zhou, S. On robustly exponential stability of uncertain neutral systems with time-varying delays and nonlinear perturbations. *Nonlinear Analysis* **68** (2008) 2464–2470.

[6] Fridman, E. Descriptor discretized Lyapunov functional method: Analysis and Design. *IEEE Trans. Automat. Control* **51** (5) (2006) 890–897.

[7] Fridman, E. and Shaked, U. An improved stability and H_∞ control: Constant and time-varying delay. *IEEE Trans. Automat. Control* **47** (2002) 1931–1937.

[8] Fridman, E. and Shaked, U. Delay-dependant stabilization method for linear time-delay systems. *Int. Journal of Control* **76** (2003) 48–60.

[9] Gan, Z.X. and Ge, W.G. Lyapunov functional for multiple delay general Lur'e control systems with multiple non-linearities. *J. Math. Anal. Appl.* **259** (2001) 596–608.

[10] Gu, K. Kharitonov, V.L. and Chen, J. *Stability of time delay systems.* Boston: Birkauser, 2003.

[11] Gu, K. and Niculescu, S.I. Additional dynamics in transformed time-delay systems. In: *Proceedings of the 39th IEEE conference on decision and control* (2001) 2805–2810.

[12] Hale, J.K. and Lunel, S.M.V. *Introduction to functional differential equations. Applied Mathematical Sciences.* Springer-Verlag, 1993.

[13] Han, Q.L. Absolute stability of time-delay systems with sector-bounded nonlinearity. *Automatica* **41** (2005) 2171–2176.

[14] Han, Q.L. and Jiang, X. Delay-dependant robust stability for uncertain linear systems with interval time-varying delay. *Automatica* **42** (2006) 1059–1065.

[15] He, Y. and Wu, M. Absolute stability for multiple delay general Lur'e control systems with multiple nonlinearities. *Journal of Computational and Applied Mathematics* **159** (2003) 241–248.

[16] He, Y. Wu, M. Wang, Q.G. and Lin, C. Delay-range-dependent stability for systems with time-varying delay. *Automatica* (2006), doi:10.1016/j.automatica. (2006.08.015)

[17] Jiang, X. and Han, Q.L. On H_∞ control for linear systems with interval time-varying delay. *Automatica* **41** (2005) 2099–2106.

[18] H.K.Khalil. *Nonlinear systems*, 2nd Ed. Macmillan Publishing Company, 1992.

[19] Kuang, J. Tian, H. and Mitsui, T. Asymptotic and numerical stability of systems of neutral differential equations with many delays. *Journal of Computational and Applied Mathematics* **223** (2) (2009) 614–625.

[20] Li, X.J. On the absolute stability of systems with time lags. *Chinese Mathematics* (4) (1963) 609–626.

[21] Li, X.J. and De Souza, C.E. Delay-dependent robust stability and stabilization of uncertain linear delay systems: A linear matrix inequality approach. *IEEE Trans. Automat. Control* **42** (1997) 1144–1148.

[22] Liao, X.X. *Absolute stability of nonlinear control systems.* Sciences Press, Beijing, 1993.

[23] Lou, X.Y. and Cui, B.T. Output Feedback Passive Control of Neutral Systems with Time-Varying Delays in State and Control Input. *Nonlinear Dynamics and Systems Theory* **8** (2) (2008) 195–204.

[24] Lou, X.Y. and Cui, B.T. Robust Stability for Nonlinear Uncertain Neural Networks with Delay. *Nonlinear Dynamics and Systems Theory* **7** (4) (2007) 369–378.

[25] Lur'e, A.I. *Some nonlinear problems in the theory of automatic control.* London, H.M. Stationery Office, 1957.

[26] Niculescu. Trofino, S.A. Dion, J.M. and Duguard, L. Delay dependent stability of linear systems with delayed state: an LMI approach. *In: IEEE conference on decision and control. New Orleans.* (1995) 1495–1496.

[27] Peng, C. and Tian, Y.C. Delay-dependent robust stability criteria for uncertain systems with interval time-varying delay. *Journal of Computational and Applied Mathematics* **214** (2) (2008) 480–494.

[28] Popov, V.M. *Hyperstability of control systems.* New York, NY: Springer, 1973.

[29] Popov, V.M. and Halanay, A. About stability of nonlinear controlled systems with delay. *Automation and Remote Control* **23** (1962) 849–851.

[30] A.Somolines. Stability of Lurie type functional equations. *Journal of Differential Equations.* **23** (1977) 191–199.

[31] Wu, M. He, Y. She, J.H. and Liu, G.P. Delay-dependent criteria for robust stability of time-varying delay systems. *Automatica* **40** (2004) 1435–1439.

[32] Yangling.Wang. Global exponential stability analysis of bidirectional associative memory neural networks with time-varying delays. *Nonlinear Analysis:* Real World Applications (2008) doi:10.1016/j. nonrwa. (2008.01.023).

[33] Lijuan Wang. Asymptotic behavior of solutions to a system of differential equations with state-dependent delays, to appear in *Journal of Computational and Applied Mathematics*.

[34] Yakubovich, V.A., Leonov, G.A. and Gelig, A.Kh. *Stability of stationnary sets in control systems with discontinuous nonlinerities*. World Scientific, Singapore, 2004.

[35] Yan, H. Huang, X. Zhang, H. and Wang.M. Delay-dependent robust stability criteria of uncertain stochastic systems with time-varying delay. *Chaos, Solitons and Fractals* (2008), doi:10.1016/j. chaos. 2007.09.049.

[36] Zhang, X.M. Wu, M. She, J.H. and He, Y. Delay-dependent stabilization of linear systems with time-varying state and input delays. *Automatica* **41** (2005) 1405–1412.

[37] Zidong, W. James, L. and Xiaohui, L. Stabilization of a Class of Stochastic Nonlinear Time-Delay Systems. *Nonlinear Dynamics and Systems Theory* **4**(3) (2004) 357–368.