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Application of Passivity Based Control for Partial Stabilization

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Abstract: In this paper, the problem of partial stabilization is considered for nonlinear control systems and a general approach for partial stabilization is proposed. In this approach, by introducing the notion of *partially passive systems*, some theorems for partial stabilization are developed. For this purpose, the nonlinear system is divided into two subsystems based on stability properties of system's states. The reduced control input vector (the vector that includes components of input vector appearing in the first subsystem), is designed based on the new passivity based control theorems, in such a way to guarantee asymptotic stability of the nonlinear system with respect to the first part of states vector.

Keywords: *nonlinear systems; partial stability; partial passivity; partial control.*

Mathematics Subject Classification (2000): 34D20, 37N35, 70K99, 74H55, 93C10, 93D15.

1 Introduction

For many of engineering problems, application of Lyapunov stability is required [1]– [3]. However, there are other physical systems like inertial navigation systems, spacecraft stabilization, electromagnetic, adaptive stabilization, guidance, etc. [4]– [12], where partial stability is necessary. In the mentioned applications, while the plant may be unstable in the standard sense, it is partially and not totally asymptotically stable. It means that naturally the plant is stable with respect to just some -and not all- of the state variables. For example, consider the equation of motion for the slider-crank mechanism depicted in Figure 1 [8]:

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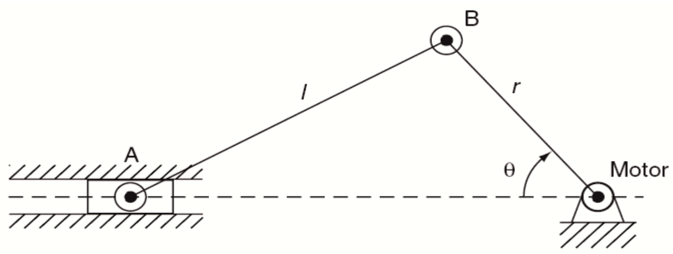

Figure 1: Slider-crank mechanism [8].

$$m(\theta(t))\ddot{\theta}(t) + c(\theta(t))\dot{\theta}^2(t) = u(t),$$
$$\theta(0) = \theta_0, \quad dot\theta(0) = \dot{\theta}_0, \quad t \geq 0,$$

where

$$m(\theta) = m_B r^2 + m_A r^2 \left(\sin\theta + \frac{r\cos\theta\sin\theta}{\sqrt{l^2 - r^2\sin^2\theta}}\right)^2,$$

$$c(\theta) = m_A r^2 \left(\sin\theta + \frac{r\cos\theta\sin\theta}{\sqrt{l^2 - r^2\sin^2\theta}}\right)\left(\cos\theta + r\frac{l^2(1-2\sin^2\theta) + r^2\sin^4\theta}{(l^2 - r^2\sin^2\theta)^{3/2}}\right)$$

and m_A and m_B are point masses, r and l are the lengths of the rods, and $u(.)$ is the control torque applied by the motor. Suppose that a feedback control law in the form of $u(.) = k(\theta, \dot{\theta})$ should be designed in a way that the angular velocity becomes constant; that is, $\dot{\theta}(t) \to \Omega$ as $t \to \infty$ where $\Omega > 0$. This implies that $\theta(t) = \Omega t \to \infty$ as $t \to \infty$. In addition, the angular position θ may not be disregarded. It is because $m(\theta)$ and $c(\theta)$ are functions of θ, and $\sin\theta$ does not converge to a limit. Consequently, it is obvious that the slider-crank mechanism is unstable in the standard sense; however, it is partially asymptotically stabilizable with respect to $\dot{\theta}$ [8].

In spite of variety of research papers in the ground of partial stability applications, there are only few papers in partial control design and advantages of partial control are not fully recognized. Furthermore, most of papers do not propose a general framework to design a partially stabilizing controller for nonlinear systems. In [6], the design of a partial controller is done for an Euler dynamical system. The references [5, 7] deal with several types of partial stabilization and control problems, such as permanent rotations of a rigid body, relative equilibrium of a satellite, stationary motions of a gimbaled gyroscope. Application of partial stabilization to achieve chaos synchronization is investigated in [10, 11].

In this article, a general approach for partial control design is proposed. This approach provides the possibility to transform the control problem into a simpler one by reducing the control input variables. For this purpose, the state vector of the system is separated into two parts and accordingly the nonlinear dynamical system is divided into two subsystems. The subsystems, hereafter, are referred to as the *first* and the *second* subsystems. The reduced control input vector (the vector that includes components of input vector which appear in the *first* subsystem) is designed based on new concept of passivity, i.e., *partial passivity* in such a way to guarantee asymptotic stability of the nonlinear system with respect to the first part of state vector.

The concept of passivity and its application in stability have been widely studied in many books and papers [13]– [18]. In this paper, introducing the notion of *partially passive systems*, a new approach for partial stabilization is developed.

The remainder of this paper is arranged as follows. First, the preliminaries on partial stability/control are given in Section 2. In Section 3, the theorems for partial control design are presented and explained in detail. Finally, conclusions are made in Section 4.

2 Preliminaries

In this section, the definitions and notations of partial stability are introduced. Consider a nonlinear system in the form:

$$\dot{x} = f(x), \qquad x(t_0) = x_0, \tag{1}$$

where $x \in R^n$ is the state vector. Let vectors x_1 and x_2 denote the partitions of the state vector, respectively. Therefore, $x = (x_1^T, x_2^T)^T$ where $x_1 \in R^{n_1}$, $x_2 \in R^{n_2}$ and $n_1 + n_2 = n$. As a result, the nonlinear system (1) can be divided into two subsystems (the *first* and the *second* subsystems) as follows:

$$\begin{aligned} \dot{x}_1(t) &= F_1(x_1(t), x_2(t)), \qquad x_1(t_0) = x_{10}, \\ \dot{x}_2(t) &= F_2(x_1(t), x_2(t)), \qquad x_2(t_0) = x_{20}, \end{aligned} \tag{2}$$

where $x_1 \in D \subseteq R^{n_1}$, D is an open set including the origin, $x_2 \in R^{n_2}$ and $F_1 : D \times R^{n_2} \to R^{n_1}$ is such that for every $x_2 \in R^{n_2}$, $F_1(0, x_2) = 0$ and $F_1(., x_2)$ is locally Lipschitz in x_1. Also, $F_2 : D \times R^{n_2} \to R^{n_2}$ is such that for every $x_1 \in D$, $F_2(x_1, .)$ is locally Lipschitz in x_2, and $I_{x_0} = [0, \tau_{x_0}), 0 < \tau_{x_0} \leq \infty$ is the maximal interval of existence of solution $(x_1(t), x_2(t))$ of (2) $\forall t \in I_{x_0}$. Under these structures, the existence and uniqueness of solution is ensured. Stability of the dynamical system (2) with respect to x_1 can be defined as follows [8]:

Definition 2.1 The nonlinear system (2) is Lyapunov stable with respect to x_1 if for every $\epsilon > 0$ and $x_{20} \in R^{n_2}$, there exists $\delta(\epsilon, x_{20}) > 0$ such that $\|x_{10}\| < \delta$ implies $\|x_1(t)\| < \epsilon$ for all $t \geq 0$. This system is asymptotically stable with respect to x_1, if it is Lyapunov stable with respect to x_1 and for every $x_{20} \in R^{n_2}$, there exists $\delta = \delta(x_{20}) > 0$ such that $\|x_{10}\| < \delta$ implies $\lim_{t\to\infty} x_1(t) = 0$.

Now, in order to analyze partial stability, the following results are taken from [8].

Theorem 2.1 *Nonlinear dynamical system* (2) *is asymptotically stable with respect to x_1 if there exist a continuously differentiable function $V : D \times R^{n_2} \to R$ and class K functions, $\alpha(.)$ and $\gamma(.)$, such that:*

$$V(0, x_2) = 0, \qquad x_2 \in R^{n_2}, \tag{3}$$

$$\alpha(\|x_1\|) \leq V(x_1, x_2), \qquad (x_1, x_2) \in D \times R^{n_2}, \tag{4}$$

$$\frac{\partial V(x_1, x_2)}{\partial x_1} F_1(x_1, x_2) + \frac{\partial V(x_1, x_2)}{\partial x_2} F_2(x_1, x_2) \leq -\gamma(\|x\|), \qquad (x_1, x_2) \in D \times R^{n_2}. \tag{5}$$

Proof See [8]. □

Corollary 2.1 [8] *Consider the nonlinear dynamical system* (2). *If there exist a positive definite, continuously differentiable function* $V : D \to R$ *and a class* K *function* $\gamma(.)$, *such that:*

$$\frac{\partial V(x_1)}{\partial x_1} F_1(x_1, x_2) \leq -\gamma(\|x\|), \qquad (x_1, x_2) \in D \times R^{n_2}, \tag{6}$$

then the nonlinear system (2) *is asymptotically stable with respect to* x_1.

Now, consider the following autonomous nonlinear control system:

$$\begin{aligned} \dot{x}_1(t) &= F_1(x_1, x_2, u(x_1, x_2)), \qquad x_1(t_0) = x_{10}, \\ \dot{x}_2(t) &= F_2(x_1, x_2, u(x_1, x_2)), \qquad x_2(t_0) = x_{20}, \end{aligned} \tag{7}$$

where $u \in R^m$ and $F_1 : D \times R^{n_2} \times R^m \to R^{n_1}$ is such that for every $x_2 \in R^{n_2}$, $F_1(0, x_2, 0) = 0$ and also $F_1(., x_2, .)$ is locally Lipschitz in x_1 and u. Also $F_2 : D \times R^{n_2} \times R^m \to R^{n_2}$ is such that for every $x_1 \in D$, $F_2(x_1, ., .)$ is locally Lipschitz in x_2 and u. These assumptions guarantee the local existence and uniqueness of the solution of the differential equations (7).

Definition 2.2 The nonlinear control system (7) is said to be asymptotically stabilizable with respect to x_1 if there exists some admissible feedback control law $u = k(x_1, x_2)$, which makes system (7) asymptotically stable with respect to x_1.

3 An Approach for Partial Control Design

Suppose that $\dot{x}_1$-*equation* in (7) is affine with respect to control input (the second subsystem may have the general dynamical form):

$$\begin{aligned} \dot{x}_1(t) &= f_1(x_1, x_2) + \sum_{i=1}^{m} g_{1i}(x_1, x_2) u_i, \\ \dot{x}_2(t) &= F_2(x_1, x_2, u), \end{aligned} \tag{8}$$

where u_i is the i^{th} component of input vector u. Also, g_{1i}, for $i = 1, 2, ..., m$ are the vectors which belong to R^{n_1}. Let us define:

$$r = \text{number of } (g_{1i} \neq 0)_{i=1,\ldots,m},$$

where r indicates the number of control components of input vector which appear in $\dot{x}_1$-*equation*. Thus $0 \leq r \leq m$. Now, with respect to the value of r, two cases may be considered.

3.1 Case 1: $r \neq 0$.

By augmenting the r nonzero vectors g_{1i} in a matrix, i.e., G_1, the nonlinear control system (8) can be rewritten as follows:

$$\begin{aligned} \dot{x}_1(t) &= f_1(x_1, x_2) + G_1(x_1, x_2) u_r, \\ \dot{x}_2(t) &= F_2(x_1, x_2, u), \end{aligned} \tag{9}$$

where $u_r \in R^r$ is the reduced version of control input vector u, that contains r control variables appearing in $\dot{x}_1$-*equation*, $G_1(x_1, x_2)$ is a $n_1 \times r$ matrix where its columns are the r nonzero vectors g_{1i}. In this case, the task is to find an appropriate u_r, which guarantees partial stabilization of nonlinear system (9) with respect to x_1. Indeed, instead of design u, we design u_r to achieve partial stability and this approach lead to simplifying the controller design. Before this, some definitions about the new concept of passivity, i.e., partial passivity are introduced.

Definition 3.1 Consider the system (9) with output function (10):

$$y_r = h(x_1, x_2), \tag{10}$$

where $y_r \in R^r$ and h is a continuous function. The system (9)-(10) is partially passive (with respect to input u_r and output y_r) if there exists a continuously differentiable positive semi definite function $V : D \to R$ (called partially storage function) such that

$$u_r^T y_r \geq \dot{V}(x_1), \qquad (x_1, x_2, u_r) \in D \times R^{n_2} \times R^r. \tag{11}$$

Remark 3.1 It is important to note the difference between passive systems which have been proposed in literature and partially passive systems which is introduced in this paper. For this purpose, the definition of passive systems is taken from [13]. Consider the following nonlinear system

$$\dot{x} = f(x, u),$$

$$y = H(x),$$

where $x \in R^n$, $y, u \in R^n$, f is locally Lipshitz in (x, u) and H is continuous. The above system is passive with respect to input u and output y if there exists a continuously differentiable positive semidefinite function $V(x)$ (storage function) such that

$$u^T y \geq \dot{V}(x), \qquad (x, u) \in R^n \times R^m.$$

In Definition 3.1, by dividing the state vector x into two parts x_1 and x_2, the passivity concept only with respect to the first subsystem, i.e., $\dot{x}_1$-*equation* is considered (partial passivity). Also, the partial storage function (in Definition 3.1) is only function of a part of states, i.e., x_1, while the storage function in definition of passive systems is function of all states, i.e., x. In what follows some new lemma and theorems are proposed for partially passive systems.

Lemma 3.1 *Consider the nonlinear system* (9)*. Suppose there exists a positive definite, continuously differentiable function $V : D \to R$ such that:*

$$\frac{\partial V(x_1)^T}{\partial x_1} f_1(x_1, x_2) \leq 0, \qquad (x_1, x_2) \in D \times R^{n_2}. \tag{12}$$

Take virtual output y_r as

$$y_r = h(x_1, x_2) = \frac{\partial V(x_1)^T}{\partial x_1} G_1(x_1, x_2). \tag{13}$$

Then the system (9)-(13) *is partially passive with respect to input u_r and output y_r.*

Proof Consider the following statement

$$u_r^T y_r - \frac{\partial V(x_1)^T}{\partial x_1}(f_1(x_1,x_2)+G_1(x_1,x_2)u_r) = u_r^T h - \frac{\partial V(x_1)^T}{\partial x_1} f_1(x_1,x_2) - h^T u_r. \quad (14)$$

Since $u_r, y_r \in R^r$, thus $u_r^T h = h^T u_r$ are scalar terms. Therefore,

$$u_r^T y_r - \frac{\partial V(x_1)^T}{\partial x_1}(f_1(x_1,x_2)+G_1(x_1,x_2)u_r) = -\frac{\partial V(x_1)^T}{\partial x_1} f_1(x_1,x_2), \quad (15)$$

where according to assumption (12), $\frac{\partial V(x_1)^T}{\partial x_1} f_1(x_1,x_2) \le 0$, therefore,

$$u_r^T y_r - \frac{\partial V(x_1)^T}{\partial x_1}(f_1(x_1,x_2)+G_1(x_1,x_2)u_r) \ge 0. \quad (16)$$

Consequently,

$$u_r^T y_r \ge \frac{\partial V(x_1)^T}{\partial x_1}(f_1(x_1,x_2)+G_1(x_1,x_2)u_r) = \dot{V}(x_1). \quad (17)$$

Hence, $u_r^T y_r \ge \dot{V}(x_1)$. Thus, by using the function $V(x_1)$ as the partial storage function candidate, the system is partially passive with respect to the input u_r and the output y_r (according to Definition 3.1). □

Theorem 3.1 *Consider the nonlinear dynamical system* (9)*. Suppose there exist a positive definite, continuously differentiable function* $V(x_1) : D \to R$ *and a class* K *function* $\gamma(.)$ *such that:*

$$\frac{\partial V(x_1)^T}{\partial x_1} f_1(x_1,x_2) \le -\gamma(\|x_1\|), \qquad (x_1,x_2) \in D \times R^{n_2}. \quad (18)$$

Then the state feedback control law (19)*, makes the system* (9) *asymptotically stable with respect to* x_1*.*

$$u_r = -\varphi(h(x_1,x_2)), \quad (19)$$

where $h(x_1,x_2) = \frac{\partial V(x_1)^T}{\partial x_1} G_1(x_1,x_2)$ *and* φ *is any smooth mapping such that* $\varphi(0) = 0$ *and* $h^T\varphi(h) > 0$ *for all* $h \neq 0$ *(It reads a function belonging to the first-third quadrant sector).*

Proof Let us define the virtual output function as follow

$$y_r = h(x_1,x_2) = \frac{\partial V(x_1)^T}{\partial x_1} G_1(x_1,x_2). \quad (20)$$

The derivative of $V(x_1)$ satisfies:

$$\begin{aligned} \dot{V}(x_1) &= \frac{\partial V(x_1)^T}{\partial x_1}\dot{x}_1 \\ &= \frac{\partial V(x_1)^T}{\partial x_1}(f_1(x_1,x_2)+G_1(x_1,x_2)u_r). \end{aligned} \quad (21)$$

Using (18)

$$\begin{aligned}\dot{V}(x_1) &\leq -\gamma\left(\|x_1\|\right) + \frac{\partial V(x_1)^T}{\partial x_1} G_1(x_1, x_2) u_r \\ &= -\gamma\left(\|x_1\|\right) + y_r^T u_r. \end{aligned} \tag{22}$$

Take,

$$u_r = -\varphi(y_r) \tag{23}$$

Therefore, $y_r^T u_r = -y_r^T \varphi(y_r) \leq 0$. As a result,

$$\dot{V}(x_1) \leq -\gamma\left(\|x_1\|\right). \tag{24}$$

Thus, according to Corollary 2.1, the control law (19) makes the nonlinear system (9) asymptotically stable with respect to x_1. $\square$

Remark 3.2 There is great freedom in choosing φ which makes the possibility for u_r to satisfy some constraints. For instance, if u_r is constrained to $|u_{ri}| \leq k_i$ for $1 \leq i \leq r$, then $\varphi_i(y_r)$ can be chosen as $\varphi_i(y_r) = k_i sat(y_{ri})$ or $\varphi_i(y_r) = (2k_i/\pi)\tan^{-1}(y_{ri})$ (where u_{ri}, φ_i and y_{ri} are the i^{th} component of u_r, φ and y_r, respectively).

Remark 3.3 Consider the system (9). If condition (18) was not satisfied, by taking $u_r = \alpha(x_1, x_2) + \beta(x_1, x_2) v_r$, the appropriate functions α and β may be found such that condition (18) be satisfied for $f_{1new} = f_1 + G_1\alpha$. Then, the control law $v_r = -\varphi(h_1)$ may be designed for partial stabilization (where $h_1 = \frac{\partial V(x_1)}{\partial x_1}^T G_{1new} = \frac{\partial V(x_1)}{\partial x_1}^T G_1\beta$)

3.2 Case 2: $r = 0$.

It means that there is no component of control input vector in $\dot{x}_1$-*equation*. Therefore, the nonlinear system (8) can be rewritten as follows:

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2), \\ \dot{x}_2 &= F_2(x_1, x_2, u). \end{aligned} \tag{25}$$

In this case, the task is to find an appropriate u; which guarantees partial stabilization of the closed-loop system. Suppose that system (25) has the following structure,

$$\begin{aligned}\dot{x}_1 &= f_1(x_1) + G_1(x_1) x_2, \\ \dot{x}_2 &= f_2(x_1, x_2) + G_2(x_1, x_2) u. \end{aligned} \tag{26}$$

This system may be viewed as a cascade connection of two subsystems where x_2 is to be viewed as an input for the *first* subsystem. The system (26) is in the regular form. Assume that x_2 and u both belong to R^m (in other words, $n_2 = m$) and $G_2(x_1, x_2) \in R^{m\times m}$ is a nonsingular matrix. This assumption is not so restrictive and many design methods, which are based on regular forms, e.g., backstepping or sliding mode techniques use such an assumption [13, 14]. In this case, the task is to find an appropriate u; which guarantees partial stabilization of the closed-loop system.

Theorem 3.2 *Consider the nonlinear dynamical system* (26). *Suppose there exist a positive definite, continuously differentiable function $V : D \to R$ and a class K function $\gamma(.)$ such that*

$$\frac{\partial V(x_1)^T}{\partial x_1} f_1(x_1) \leq -\gamma\left(\|x_1\|\right). \tag{27}$$

Then the state feedback control law (28), *makes the closed-loop nonlinear system* (26) *asymptotically stable with respect to* x_1 *and*

$$u = G_2^{-1}[-\frac{\partial \varphi(y)}{\partial y}\dot{y} - f_2(x_1, x_2)], \tag{28}$$

where $y = \frac{V(x_1)^T}{\partial x_1} G_1(x_1)$ *and* φ *is any locally Lipschitz function such that* $\varphi(0) = 0$ *and* $y^T\varphi(y) > 0$ *for all* $y \neq 0$.

Proof The derivative of $V(x_1)$ is given by

$$\begin{aligned}\dot{V}(x_1) &= \frac{\partial V(x_1)^T}{\partial x_1}\dot{x}_1 \\ &= \frac{\partial V(x_1)^T}{\partial x_1}(f_1(x_1) + G_1(x_1)x_2).\end{aligned} \tag{29}$$

Using (27), we have

$$\dot{V}(x_1) \leq -\gamma\left(\|x_1\|\right) + \frac{\partial V(x_1)^T}{\partial x_1} G_1(x_1)x_2. \tag{30}$$

Take,

$$y = \frac{\partial V(x_1)^T}{\partial x_1} G_1(x_1) \tag{31}$$

and

$$x_2 = -\varphi(y). \tag{32}$$

Then

$$\dot{V}(x_1) \leq -\gamma\left(\|x_1\|\right) + y^T x_2. \tag{33}$$

Since $y^T x_2 = -y^T\varphi(y) \leq 0$, thus $\dot{V}(x_1) \leq -\gamma\left(\|x_1\|\right)$ and according to Corollary 2.1, partial stabilization with respect to x_1 is achieved. Also,

$$\dot{x}_2 = -\frac{\partial \varphi(y)}{\partial y}\dot{y}. \tag{34}$$

In addition, from $\dot{x}_2$-*equation*, one has

$$\dot{x}_2 = f_2(x_1, x_2) + G_2(x_1, x_2)u. \tag{35}$$

Therefore, combination of (34) and (35) results in:

$$u = G_2^{-1}[-\frac{\partial \varphi}{\partial y}\dot{y} - f_2(x_1, x_2)]. \tag{36}$$

This feedback law guarantees partial stabilization of the closed-loop system. □

3.3 Design example

Consider the following system

$$\begin{aligned} \dot{z}_1 &= \frac{z_1^2 z_2}{z_3^2} + z_1 u_1, \\ \dot{z}_2 &= -z_2(1+z_1^2) + z_3 u_2, \\ \dot{z}_3 &= -z_2^2 \sin(z_3) - z_3, \end{aligned} \tag{37}$$

where $z_1, z_2 \in R$ and $z_3 \in [-\pi\ \pi]$. By separating the states into $x_1 = [z_2\ z_3]^T$ and $x_2 = z_1$, one has: $r = 1$ and $u_r = u_2$. The task is to design u_r according to Theorem 3.1 to achieve asymptotic stability with respect to x_1. For this purpose, first the condition (18) should be checked. By choosing $V(x_1) = \frac{1}{2}x_1^T x_1 = \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2$, one has:

$$\begin{aligned} \frac{\partial V(x_1)^T}{\partial x_1} f_1(x_1, x_2) &= [z_2\ z_3] \begin{bmatrix} -z_2(1+z_1^2) \\ -z_2^2 \sin(z_3) - z_3 \end{bmatrix} \\ &= -z_2^2(1+z_1^2) - z_2^2 z_3 \sin(z_3) - z_3^2 \\ &= -z_2^2 - z_2^2 z_1^2 - z_2^2 z_3 \sin(z_3) - z_3^2 \\ &\leq -z_2^2 - z_3^2. \end{aligned} \tag{38}$$

Therefore, condition (18) is satisfied for $\gamma(\|x_1\|) = x_1^T x_1 = z_2^2 + z_3^2$. Now, by choosing h as,

$$\begin{aligned} h(x_1, x_2) &= \frac{\partial V(x_1)^T}{\partial x_1} G_1(x_1, x_2) \\ &= [z_2\ z_3] \begin{bmatrix} z_3 \\ 0 \end{bmatrix} \\ &= z_2 z_3. \end{aligned} \tag{39}$$

Then, the reduced input vector may be designed as

$$u_r = -\varphi(z_2 z_3), \tag{40}$$

where φ is any locally Lipschitz function such that $\varphi(0) = 0$ and $h^T\varphi(h) > 0$ for all $h \neq 0$. For example, by choosing $\varphi(h) = h$, then $u_r = -z_2 z_3$ which guarantees partial stabilization of system (37) with respect to x_1.

4 Conclusion

In this paper, a new approach for partial stabilization of nonlinear systems was proposed and it was shown that in this approach the controller synthesis can be simplified by reducing its variables. The reduced input vector was designed based on new introduced partial passivity concept. In the proposed design method, a virtual output with the same dimension as the reduced input vector was designed such that the nonlinear system was partially passive with respect to the reduced input vector and the virtual output vector. Then, the feedback law was designed as a first-third quadrant sector function of virtual output vector and it was shown that this law guarantees partial stabilization of the nonlinear system.

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