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A Decentralized Stabilization Approach of a Class of Nonlinear Polynomial Interconnected Systems Application for a Large Scale Power System

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Abstract: This paper presents a new approach dealing with the decentralized control of non linear interconnected systems. The key of this work is, on one hand, the description of the nonlinear systems using the Kronecker product notations which allow important manipulations, and on the other hand the use of the Lyapunov's direct method of stability analysis, associated with a quadratic function. The proposed approach is then applied to an industrial process: a three-machine-based interconnected power system, to improve its decentralized stabilization.

Keywords: *nonlinear systems; interconnected systems; decentralized stabilization; Kronecker product; power systems.*

Mathematics Subject Classification (2010): 93A15, 93D15.

1 Introduction

In recent years, modern control methods have found their way into decentralized design of interconnected large scale nonlinear systems, leading to a wide variety of new concepts and results ([2]- [4], [18], [23]).

Decentralized control aims mainly to carry out a feedback control for each subsystem using only its local state variables.

The decentralized control law implementation is more feasible and more economical than a centralized control being dependant on the whole state variables for each subsystem local control. This kind of control is very important for the power systems which

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are generally large scale, interconnected and highly nonlinear systems. Centralized control for the large scale power system is usually impractical: first, because it requires an intensive exchange of information between many sub-systems that are geographically located in different and, generally distant areas; and second for lack of computing capacity. Consequently, a decentralized nonlinear controller, for which the development is based only on local information and measurements, is often preferable in power industry applications. A wide variety of properties for the decentralized control of power systems are extensively studied in the literature and different design approaches are proposed accordingly ([2], [8], [11], [21], [22], [24]).

It is essential to verify that the collection of these decentralized local controls should obviously guarantee the stability of the global interconnected system.

Analysis of decentralized stability properties of large scale systems has been the motivation of many works over the past twenty years ([3], [5], [9], [16]- [18]).

Power system stability has been recognized as an important problem for secure system operation. Many major blackouts caused by power system instability have illustrated the importance of this phenomenon. Historically, transient instability has been the dominant stability problem for most systems and also the focus of much of the power industry's attention related to system instability ([8], [11]- [15]). It is mainly interested in the maintenance of synchronism between generators following a severe disturbance
In this context, we propose in this work a new decentralized control for the stability of a class of non linear interconnected continuous systems based on polynomial modeling. The description of these systems using Kronecker product [19] and the use of a quadratic Lyapunov function have allowed the definition of sufficient conditions for the global asymptotic stability of the system equilibrium.

This paper is organized as follows: The next part exposes a brief summary of the main mathematical background that has supported this work. The third part will first present the studied systems, then expose the approach outcome of this work. The fourth and final part aims to show the applicability of the proposed design tool, on the basis of an illustrative example of a three-machine-based interconnected power system, followed by the concluding section.

2 Mathematical Notations and Properties

The dimensions of the matrices used in this section are the following:

$$A(p\times q),\quad B(r\times s),\quad C(q\times g),\quad D(s\times h),\quad E(n\times p),\quad P(n\times n),\quad X(n\times 1)\in\mathbb{R}^n,$$
$$Y(m\times 1)\in\mathbb{R}^m,\quad Z(q\times 1)\in\mathbb{R}^q.$$

Throughout the paper, the following notations are used: I_n is the identity matrix of order n, $\mathbb{O}_{n\times m}$ is the $(n\times m)$ null matrix and A^T is the transpose matrix of A.

2.1 Kronecker product

The Kronecker product of A and B denoted by $A\otimes B$ is the $(pr\times qs)$ matrix defined by:

$$A\otimes B=\begin{pmatrix} a_{11}B & \dots & a_{1q}B\\ \vdots & \ddots & \vdots\\ a_{p1}B & \dots & a_{pq}B\end{pmatrix}.$$

2.2 Kronecker power of vectors

The Kronecker power of order i, $X^{[i]}$, of the vector X is defined by

$$\begin{cases} X^{[0]} = 1, \\ X^{[i]} = X^{[i-1]} \otimes X = X \otimes X^{[i-1]}, X^{[i]} \in \mathbb{R}^{n^i}, \text{for } i \geq 1. \end{cases} \tag{1}$$

2.3 Permutation matrix

Let e_i^n denote the i^{th} vector of the canonic basis of $\mathbb{R}^n$, the permutation matrix denoted by $U_{n\times m}$ is defined by [19]:

$$U_{n\times m} = \sum_{i=1}^{n} \sum_{k=1}^{m} (e_i^n . e_k^{m^T}) \otimes (e_k^m . e_i^{n^T}). \tag{2}$$

This matrix is square $(nm \times nm)$ and has precisely a single "1" in each row and in each column. The main useful properties of this matrix are the following:

$$U_{n\times m}^{-1} = U_{n\times m}^{T} = U_{m\times n}, \tag{3}$$

$$U_{n\times 1} = U_{1\times n} = U_n. \tag{4}$$

This matrix ensures the following relations

$$B \otimes A = U_{r\times p}(A \otimes B)U_{q\times s}, \tag{5}$$

$$X \otimes Y = U_{n\times m}(Y \otimes X), \tag{6}$$

$$X^{[k]} = U_{n^i \times n^{k-i}} X^{[k]}, \quad \forall i \leq k. \tag{7}$$

2.4 Vec-function

The function Vec of a matrix was defined in [19] as follows:

$$A = [A_1 \quad A_2 \quad \dots \quad A_q], \quad vec(A) = \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_q \end{pmatrix}, \tag{8}$$

where $\forall i \in \{1, \dots, q\}$, A_i is a vector of $\mathbb{R}^p$. We recall the following useful rules of this function, given in [19]:

$$Vec(E.A.C) = (C^T \otimes E)Vec(A), \tag{9}$$

$$Vec(A^T) = U_{p\times q} Vec(A). \tag{10}$$

2.5 Mat-function

An important matrix-valued linear function of a vector, denoted by $Mat_{(n,m)}(.)$ was defined in [20] as follows. If V is a vector of dimension $p = n.m$ then $M = Mat_{(n,m)}(V)$ is the $(n \times m)$ matrix verifying: $V = Vec(M)$. We recall the following useful lemma for this function, given in [20].

Lemma 2.1 *Consider the matrix A with $p = n$ and $q = n^k$ ($k \in \mathbb{N}$), and let i and j be two integers verifying $i + j = k + 1$ and $i \geq 1$. Then*

$$Mat_{(n^i,n^j)}(Vec(PA)) = U_{n^{i-1}\times n}(P \otimes I_{n^{i-1}}).M_{i-1,j}(A) \tag{11}$$

with

$$M_{i-1,j}(A) = \begin{pmatrix} Mat_{(n^{i-1},n^j)}(A^{1^T}) \\ Mat_{(n^{i-1},n^j)}(A^{2^T}) \\ \vdots \\ Mat_{(n^{i-1},n^j)}(A^{n^T}) \end{pmatrix},$$

where A^i denotes the i^{th} row of the matrix A. i.e.,

$$A = \begin{pmatrix} A^1 \\ A^2 \\ \vdots \\ A^n \end{pmatrix}.$$

3 The Proposed Decentralized Stabilization Approach

3.1 Description of the studied systems

We consider the class of nonlinear systems, formed by the interconnection of n subsystems, and for which the r order polynomial development is composed only with the odd Kronecker power of vectors, i.e., $r = 2s - 1, \quad s \in \mathbb{N}$:

$$\begin{aligned} &\dot{X}_i = f_i(X_i) + B_iU_i + g_i(X_1, X_2, \dots, X_n), \\ &i = 1, 2, \dots, n \end{aligned} \tag{12}$$

with

$$f_i(X_i) = \sum_{k=0}^{s-1} A_{i,2k+1}X_i^{[2k+1]} \tag{13}$$

and

$$g_i(X_1,\dots,X_n) = \sum_{\substack{s_1,\dots,s_n \\ \sum_i s_i \leq r}}^{r} G_{s_1,\dots,s_n}X_1^{[s_1]}\otimes\dots\otimes X_i^{[s_i]}\otimes\dots\otimes X_n^{[s_n]}, \tag{14}$$

where $X_i \in \mathbb{R}^{n_i}$ is the state vector of the i^{th} subsystem, B_i is the control matrix of the i^{th} subsystem, U_i is the control of the i^{th} subsystem, $A_{i,2k+1} \in \mathbb{R}^{n_i\times n_i^{2k+1}}$, $G_{s_1,\dots,s_n}$ are matrices with appropriate dimensions.

The overall interconnected system is described by the following compact form:

$$\begin{aligned} \dot{\mathcal{X}} &= \mathcal{A}_1\mathcal{X} + \mathcal{A}_3\mathcal{X}^{[3]} + \mathcal{A}_5\mathcal{X}^{[5]} + \dots + \mathcal{A}_r\mathcal{X}^{[r]} + \mathcal{B}U \\ &= \sum_{j=0}^{s-1} \mathcal{A}_{2j+1}\mathcal{X}^{[2j+1]} + \mathcal{B}U, \quad r = 2s-1, \quad s \in \mathbb{N}, \end{aligned} \tag{15}$$

where $\mathcal{X} = [X_1^T, X_2^T, \dots, X_n^T]^T, \mathcal{X} \in \mathbb{R}^N$, $N = \sum_{i=1}^{n} n_i$, $\mathcal{A}_{2j+1} \in \mathbb{R}^{N\times N^{2j+1}}$,
$\mathcal{B} = diag(B_1, B_2, \dots, B_n)$, $U = (U_1^T, U_2^T, \dots, U_n^T)^T$.

3.2 Nonlinear decentralized control stabilization

We expose in this section our approach of a decentralized control synthesis of the interconnected global system (15). The decentralized control laws of the n subsystems are taken in the following form:

$$\begin{array}{l} U_i = -(K_{i1}X_i + K_{i3}X_i^{[3]} + K_{i5}X_i^{[5]} + \ldots + K_{ir}X_i^{[r]}), \\ i = 1, \ldots, n, \end{array} \tag{16}$$

which leads to the following global control law

$$\begin{array}{rcl} U & = & \begin{pmatrix} U_1 & \ldots & U_n \end{pmatrix}^T \\ & = & -(K_1\mathcal{X} + K_3\mathcal{X}^{[3]} + K_5\mathcal{X}^{[5]} + \ldots + K_r\mathcal{X}^{[r]}) \\ & = & -\sum_{j=0}^{s-1} K_{2j+1}\mathcal{X}^{[2j+1]}, \qquad r = 2s-1, \end{array} \tag{17}$$

where $K_1 = \mathrm{diag}(K_{i1})$, $i = 1, \ldots, n$ and matrices K_{2j+1}, $j = 1, \ldots, s-1$ are expressed from $K_{i,2j+1}$.

Let $Q_i(n_i \times n_i)$, $i = 1, \ldots, n$ be symmetric positive definite matrices, and α be a positive real. And let P_i $(i = 1, \ldots, n)$ be the symmetric positive definite matrices solution of the following Riccati equations

$$A_{i1}^T P_i + P_{i1}A_{i1} - P_i(B_i R_i^{-1} B_i^T)P_i + Q_i + 2\alpha P_i = 0, \tag{18}$$

where A_{i1} is the characteristic matrix of the i^{th} subsystem. And let the gains $K_{i,2j+1}$ $(i = 1, \ldots, n$ and $j = 1, \ldots, s-1)$ be given by

$$\left\{ \begin{array}{l} K_{i1} = R_i^{-1}B_i^T P_i, \\ M_{j,j+1}(K_{i,2j+1}) = (R_i^{-1}B_i^T P_i) \otimes I_{n_i^j}. \end{array} \right. \tag{19}$$

We have then the following theorem.

Theorem 3.1 *The decentralized control law (16) (or (17)) is globally and asymptotically stabilizable for system (15) if there exist $(n_i \times n_i)$ positive definite matrices Q_i, $i = 1, \ldots, n$ and $\alpha \in \mathbb{R}$ such that matrices F_1, F_3, F_{2s-1} defined by*

$$F_1 = Q + P\mathcal{B}R^{-1}\mathcal{B}^T P + 2\alpha P - (PH + H^T P) \tag{20}$$

with $Q = diag(Q_i)$, $P = diag(P_i)$, $R^{-1} = diag(R_i^{-1})$, H is the interconnection linear part, and for $j \geq 1$,

$$F_{2j+1} = (P\mathcal{B}R^{-1}\mathcal{B}^T P) \otimes I_{N^j} - (P \otimes I_{N^j})M_{j,j+1}(\mathcal{A}_{2j+1}) \tag{21}$$

are semi-positive definite.

Proof. The proof of the above theorem is based on Lyapunov direct method. Let V be the Lyapunov function defined by the following quadratic form:

$$V = \mathcal{X}^T P \mathcal{X}, \tag{22}$$

where $P = \mathrm{diag}(P_i)$ is an $(n \times n)$ definite symmetric matrix. The global asymptotic stability of the equilibrium state $\mathcal{X} = 0$ of system (15) is ensured when the time derivative $\dot{V}(\mathcal{X})$ of $V(\mathcal{X})$ is negative definite for all $\mathcal{X} \in \mathbb{R}^n$. One has

$$\dot{V} = \dot{\mathcal{X}}^T P\mathcal{X} + \mathcal{X}^T P\dot{\mathcal{X}}. \tag{23}$$

Using (15), expression (23) leads to

$$\begin{aligned} \dot{V} &= 2\sum_{j=0}^{s-1}(Vec(P\mathcal{A}_{2j+1} - P\mathcal{B}K_{2j+1}))^T \mathcal{X}^{[2j+2]} \\ &= 2\sum_{j=0}^{s-1}\mathcal{X}^{[j+1]^T} Mat_{(n^{j-1},n^j)}(Vec(P\mathcal{A}_{2j+1} - P\mathcal{B}K_{2j+1}))\mathcal{X}^{[j+1]}. \end{aligned} \tag{24}$$

Using Lemma 1, we get

$$Mat_{(n^{j+1},n^{j+1})}(Vec(P\mathcal{A}_{2j+1} - P\mathcal{B}K_{2j+1})) = U_{n^j,n}(P\otimes I_{n^j})M_{n^j,n^{j+1}}(\mathcal{A}_{2j+1} - \mathcal{B}K_{2j+1}). \tag{25}$$

The use of (25) and the following expression

$$\forall i, j \in \mathbb{N}; \quad U_{n^i \times n^j}\mathcal{X}^{[i+j]} = \mathcal{X}^{[i+j]} \tag{26}$$

yield

$$\begin{aligned} \dot{V} &= 2\sum_{j=0}^{s-1}\mathcal{X}^{[j+1]^T} Mat_{(n^{j-1},n^j)}(Vec(P\mathcal{A}_{2j+1} - P\mathcal{B}K_{2j+1}))\mathcal{X}^{[j+1]} \\ &= 2\sum_{j=0}^{s-1}\mathcal{X}^{[j+1]^T} U_{n^j,n}(P\otimes I_{n^j})M_{n^j,n^{j+1}}(\mathcal{A}_{2j+1} - \mathcal{B}K_{2j+1})\mathcal{X}^{[j+1]} \\ &= 2\sum_{j=0}^{s-1}\mathcal{X}^{[j+1]^T}(P\otimes I_{n^j})M_{n^j,n^{j+1}}(\mathcal{A}_{2j+1} - \mathcal{B}K_{2j+1})\mathcal{X}^{[j+1]} \\ &= 2\sum_{j=0}^{s-1}\mathcal{X}^{[j+1]^T}(P\otimes I_{n^j})(M_{n^j,n^{j+1}}(\mathcal{A}_{2j+1}) - M_{n^j,n^{j+1}}(\mathcal{B}K_{2j+1}))\mathcal{X}^{[j+1]} \\ &= 2\sum_{j=0}^{s-1}\mathcal{X}^{[j+1]^T}(P\otimes I_{n^j})(M_{n^j,n^{j+1}}(\mathcal{A}_{2j+1}) - \mathcal{B}R^{-1}B^T P\otimes I_{n^{j-1}})\mathcal{X}^{[j+1]}. \end{aligned} \tag{27}$$

Then we obtain the following expression:

$$\dot{V} = -\mathbb{X}^T\mathbb{M}\mathbb{X} \tag{28}$$

with

$$\mathbb{X} = \begin{pmatrix} \mathcal{X} \\ \vdots \\ \mathcal{X}^{[j+1]} \end{pmatrix}^T, \quad \mathbb{M} = \begin{pmatrix} F_1 & & \mathbb{O} \\ & \ddots & \\ \mathbb{O} & & 2(P\otimes I_{n^j})F_{2j+1}. \end{pmatrix}. \tag{29}$$

To ensure the asymptotic stability of system (15) with the control law (17), $\dot{V}$ should be negative definite, then the matrix $\mathbb{M}$ should be positive definite, which is equivalent to F_1 of expression (20) is positive definite and F_{2j+1}, for $j \geq 1$, of expression (21) are semi-positive definite.

3.2.1 Second version of decentralized stabilizability conditions

We consider system model (15) of the global interconnected system, and let

$$\mathcal{A}_{2j+1} = \mathcal{A}^1_{2j+1} + \mathcal{A}^2_{2j+1}, \tag{30}$$

where $\mathcal{A}^1_{2j+1}$ expressed from $A_{i,2k+1}$ (matrices of separated subsystems) and $\mathcal{A}^2_{2j+1}$ expressed from $G^{k,s}_{ij}$ (corresponding to interconnections).

If there exist symmetric positive definite matrices $Q_{i,j+1}(n_i^{j+1} \times n_i^{j+1})$, $j = 1, \dots, s-1$, such that the matrices P_i, solutions of Riccati equations (18), will be solutions of the following equations, for $i = 1, \dots, n$:

$$(P_i \otimes I_{n_i^j}) M_{j,j+1}(A_{i,2j+1}) + M^T_{j,j+1}(A_{i,2j+1})(P_i \otimes I_{n_i^j}) - (P_i B_i R_i^{-1} B_i^T P_i) \otimes I_{n_i^j} + Q_{i,j+1} = 0. \tag{31}$$

Each of isolated decoupled subsystems, in which all the interactions are assumed to be zero, can be stabilized with control vector U_i of (16), where the gains $K_{i,2j+1}$, $i = 1, \dots, n$ and $j = 1, \dots, s-1$ are given by

$$\begin{cases} K_{i1} = R_i^{-1} B_i^T P_i, \\ M_{j,j+1}(K_{i,2j+1}) = (R_i^{-1} B_i^T P_i) \otimes I_{n_i^j}. \end{cases} \tag{32}$$

Now the presence of interconnections will influence the stability, and it is necessary to obtain sufficient conditions to guarantee the stability of the overall system. This is given by the following theorem.

Theorem 3.2 *The decentralized control law (16) (or (17)) is globally and asymptotically stabilizable for system (15) if there exist $(n_i \times n_i)$ positive definite matrices Q_i, $i = 1, \dots, n$, $\alpha \in \mathbb{R}$, and $(n_i^{j+1} \times n_i^{j+1})$ positive definite matrices $Q_{i,j+1}$, $j \geq 1$, such that matrix F_1, defined by*

$$F_1 = Q_1 + P\mathcal{B}R^{-1}\mathcal{B}^T P + 2\alpha P - (PA_1^2 + A_1^{2^T} P), \tag{33}$$

where $Q_1 = diag(Q_i)$, $P = diag(P_i)$, $R^{-1} = diag(R_i^{-1})$, A_1^2 defined in (30) is positive definite, and for $j \geq 1$

$$F_{2j+1} = Q_{j+1} + (P\mathcal{B}R^{-1}\mathcal{B}^T P) \otimes I_{n^j} - [M^T_{j,j+1}(\mathcal{A}^2_{2j+1})(P \otimes I_{n^j}) + (P \otimes I_{n^j}) M_{j,j+1}(\mathcal{A}^2_{2j+1})], \tag{34}$$

where $Q_{j+1} = diag(Q_{i,j+1})$ are semi positive definite.

Proof. Let V be the Lyapunov function defined by the following quadratic form

$$V = \mathcal{X}^T P \mathcal{X}. \tag{35}$$

The development of $\dot{V}$ leads to

$$\begin{aligned} \dot{V} = {} & \mathcal{X}^T (P\mathcal{A}_1 + \mathcal{A}_1^T P - PBK_1 - K_1^T B^T P)\mathcal{X} + 2\sum_{j=1}^{s-1} \mathcal{X}^{[j+1]^T} (\otimes I_{n^j})(M_{j,j+1}(\mathcal{A}_{2j+1}) \\ & -(\mathcal{B}R^{-1}B^T P) \otimes I_{n^j})\mathcal{X}^{[j+1]}. \end{aligned} \tag{36}$$

Then using (18) and (31) in (36), we get

$$\begin{aligned} \dot{V} = {} & -\mathcal{X}^T (Q_1 + P\mathcal{B}R^{-1}\mathcal{B}^T P + 2\alpha P - (PA_1^2 + A_1^{2^T} P))\mathcal{X} - \sum_{j=1}^{s-1} \mathcal{X}^{[j+1]^T} \{Q_{j+1} \\ & + (P\mathcal{B}R^{-1}\mathcal{B}^T P) \otimes I_{n^j} - [M^T_{j,j+1}(\mathcal{A}^2_{2j+1})(P \otimes I_{n^j}) + (P \otimes I_{n^j}) M_{j,j+1}(\mathcal{A}^2_{2j+1})]\} \mathcal{X}^{[j+1]}. \end{aligned} \tag{37}$$

The expression (37) is then equivalent to

$$\dot{V} = -\mathbb{X}^T \mathbb{M} \mathbb{X} \tag{38}$$

with

$$\mathbb{X} = \begin{pmatrix} \mathcal{X} \\ \vdots \\ \mathcal{X}^{[j+1]} \end{pmatrix}^T, \quad \mathbb{M} = \begin{pmatrix} F_1 & & \mathbb{O} \\ & \ddots & \\ \mathbb{O} & & F_{2j+1} \end{pmatrix}. \tag{39}$$

To ensure the asymptotic stability of system (15) with the control law (17), $\dot{V}$ should be negative definite, then the matrix $\mathbb{M}$ should be positive definite, which is equivalent to F_1 is positive definite and F_{2j+1}, $j \geq 1$ is semi-positive definite.

4 Application of the Proposed Control to a Multimachine Power System

We propose in this part to show that it is possible to apply the proposed decentralized control method to an industrial process. It consists in studying the stability by decentralized control of a power system composed of three interconnected machines, (Figure 1), characterized by the parameters indicated in Table 1.

4.1 Multimachine power system modelisation

A three machine power system controlled by the steam valve opening, can be described with the interconnection of three subsystems as follows [21]:

$$\dot{X}_i(t) = A_i X_i(t) + B_i U_i(t) + \sum_{j=1, j\neq i}^{3} p_{ij} G_{ij} g_{ij}(X_i, X_j); \quad i = 1, \cdots, 3, \tag{40}$$

where $X_i(t)$ is the state vector defined by $X_i(t)^T = [\Delta\delta_i(t) \quad \omega_i(t) \quad \Delta P_{m_i}(t) \quad \Delta X_{e_i}(t)]$, $\Delta\delta_i(t) = \delta_i(t) - \delta_i 0$, $\Delta P_{m_i}(t) = P_{m_i}(t) - P_{m_i 0}$, $\Delta X_{e_i}(t) = X_{e_i}(t) - X_{e_i 0}$, $U_i(t)$ is the control, $U_i(t) = \Delta X_{ei}(t)$,

$$A_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\dfrac{D_i}{2H_i} & 0 & -\dfrac{\omega_0}{2H_i} & 0 \\ 0 & 0 & -\dfrac{1}{T_{m_i}} & \dfrac{K_{m_i}}{T_{m_i}} \\ -\dfrac{K_{e_i}}{T_{e_i} R_i \omega_0} & 0 & 0 & -\dfrac{1}{T_{e_i}} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{T_{e_i}} \end{bmatrix}, G_{ij} = \begin{bmatrix} 0 \\ -\dfrac{\omega_0 E'_{qi} E'_{qj} B_{ij}}{2H_i} \\ 0 \\ 0 \end{bmatrix},$$

$g_{ij}(x_i, x_j) = \sin(\delta_i(t) - \delta_j(t)) - \sin(\delta_{i0} - \delta_{j0})$, where:

p_{ij} a constant of either 1 or 0 (if 0, then j^{th} machine has no connection with i^{th} one);
δ_i the rotor angle for i^{th} machine, in *radian*;
ω_i the relative speed for i^{th} machine, in *radian/second*;
P_{m_i} the mechanical power for i^{th} machine, in *pu*;
X_{e_i} the steam valve opening for i^{th} machine, in *pu*;
H_i the inertia constant for i^{th} machine, in *second*;
D_i the damping coefficient for i^{th} machine, in *pu*;
T_{m_i} the time constant for i^{th} machine's turbine, in *second*;
K_{m_i} the gain of i^{th} machine turbine;
T_{e_i} the time constant of i^{th} machine's speed governor, in *second*;
T_{e_i} the gain of i^{th} machine's speed governor;
R_i the regulation constant of i^{th} machine, in *pu*;
B_{ij} the nodal susceptance between i^{th} and j^{th} machines, in *pu*;
ω_0 the synchronous machine speed, in *radian/second*;
E'_{qi} the internal transient voltage for i^{th} machine, in *pu*, which is a constant;
E'_{qj} the internal transient voltage for i^{th} machine, in *pu*, which is a constant;
x_{di} the direct axis reactance of the i^{th} generator, in *pu*;
x'_{di} the direct axis transient reactance of the i^{th} generator, in *pu*;
x_{Ti} the transformer reactance;
x_{adi} the mutual reactance between the excitation coil and the stator coil, in *p.u.*;
T'_{d0i} the direct axis transient short-circuit time constant, in *second*;
x_{ij} the transmission line reactance between the i^{th} and the j^{th} generators, in *pu*;

δ_{i0}, P_{m_i0} and X_{e_i0} are the initial values of $\delta_i(t)$, $P_{m_i}(t)$ and $X_{e_i}(t)$.

	Machine 1	Machine 2	Machine 3
$x_d(pu)$	1.863	2.36	2.36
$x'_d(pu)$	0.257	0.319	0.319
$x_T(pu)$	0.129	0.11	0.11
$x_{ad}(pu)$	1.712	0.712	0.712
$T'_{d0}(pu)$	6.9	7.96	7.96
$H(s)$	4	5.1	5.1
$D(pu)$	5	3	3
$T_m(s)$	0.35	0.35	0.35
$T_e(s)$	0.1	0.1	0.1
R	0.05	0.05	0.05
K_m	1	1	1
K_e	1	1	1

$x_{12}(pu)$	0.55
$x_{13}(pu)$	0.53
$x_{23}(pu)$	0.6
$\omega_0(rad/s)$	314.159

Table 1: Three-machine-based system parameters.

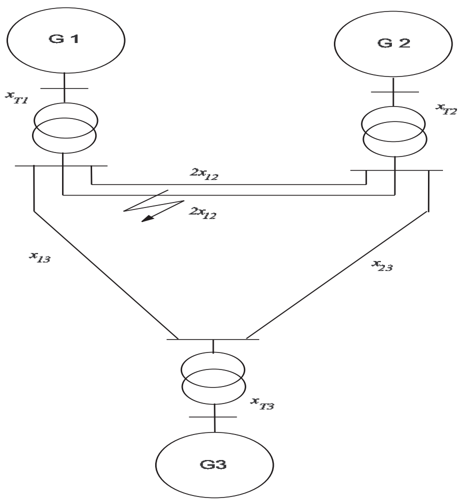

Figure 1: Three-machine example system.

4.1.1 Polynomial model

The nonlinear analytic model (40) can be represented with a third order truncated polynomial form, which is considered to be sufficient for the studied power system modeling:

$$\dot{X}_i = A_{i,1}X_i + A_{i,3}X_i^{[3]} + B_iU_i + \sum_{\substack{j=1\\ j\neq i}}^{3}\sum_{k=1}^{3}\sum_{s=1}^{k} p_{ij}G_{ij}^{k,s}X_i^{[k-s]}\otimes X_j^{[s]}, \quad i=1,2,3. \tag{41}$$

The global interconnected system is then modelled with the following polynomial form

$$\dot{\mathcal{X}} = \mathcal{A}_1\mathcal{X} + \mathcal{A}_3\mathcal{X}^{[3]} + \mathcal{B}U, \tag{42}$$

where $\mathcal{X} = [X_1^T, X_2^T, X_3^T]^T$,

$$\begin{cases} \mathcal{A}_1 = \mathrm{diag}(A_1, A_2, A_3), \mathcal{A}_1(2,1) = -54.98, \\ \mathcal{A}_1(2,5) = 27.49, \mathcal{A}_1(2,9) = 27.49, \mathcal{A}_1(6,5) = -46.2, \\ \mathcal{A}_1(6,1) = 23.1, \mathcal{A}_1(6,9) = 23.1, \mathcal{A}_1(10,9) = -50.59, \\ \mathcal{A}_1(10,1) = 23.1, \mathcal{A}_1(10,5) = 27.49, \end{cases}$$

$$\begin{cases} \mathcal{A}_3(2,1)=9.16, \mathcal{A}_3(6,769)=-13.745, \mathcal{A}_3(8,1537)=-13.745, \\ \mathcal{A}_3(2,65)=-13.745, \mathcal{A}_3(6,833)=7.7, \mathcal{A}_3(8,1601)=-13.745, \\ \mathcal{A}_3(2,129)=-13.745, \mathcal{A}_3(6,897)=-11.55, \mathcal{A}_3(8,1665)=8.43, \\ \mathcal{A}_3(2,257)=13.745, \mathcal{A}_3(6,577)=13.745, \mathcal{A}_3(8,1153)=13.745, \\ \mathcal{A}_3(2,513)=13.745, \mathcal{A}_3(6,1089)=11.55, \mathcal{A}_3(8,1409)=13.745, \\ \mathcal{A}_3(2,833)=-4.58, \mathcal{A}_3(6,1)=-3.85, \mathcal{A}_3(8,1)=-3.85, \\ \mathcal{A}_3(2,1664)=-4.58, \mathcal{A}_3(6,1665)=-13.745, \mathcal{A}_3(8,833)=-4.58, \\ \mathcal{A}_3(i,j) = 0 \text{ for the other values of i and j } 1 \leq i \leq 12, \\ \text{ et } 1 \leq j \leq 1728. \end{cases}$$

We want to compute the decentralized control laws given by (16) and (19) for $i = 1, 2, 3$. For α, R_i and Q_i, $i = 1, 2, 3$, given by $\alpha = 0$, $R_i = 2$, $Q_i = \mathrm{diag}\{0.001, 0.001, 0.01, 0.01\}$, we obtain:

$$K_{11} = [55.90 \quad 24.48 \quad 349.25 \quad 103.87], \; K_{21} = K_{31} = [55.90 \quad 33.06 \quad 359.28 \quad 106.62].$$

We can easily verify that matrices F_1 and F_3 given in Theorem 1 are positive defined, which guarantees the stability of system (42) by the decentralized control law.

Firstly, we want to know the behavior of the proposed power system in free operating conditions. The curves of Figure 2 show the strongly transient evolution of the power system state variables, when it is simulated under these conditions towards a perturbation on the first machine rotor angle.

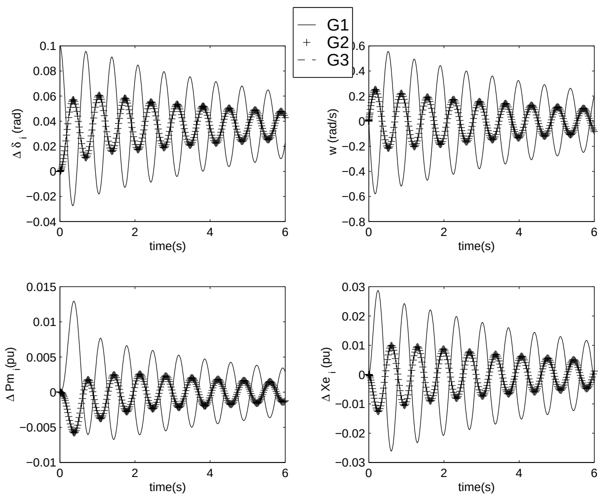

Figure 2: State variable evolution in free operating conditions, toward a perturbation on $\delta 1$.

Now, to test the performances of the established decentralized control law, we carry on the simulation of the controlled power system towards some perturbations occurred on state variables. Figure 3 shows the case when a perturbation is occurred on the rotor angle of the first machine. Figure 4 illustrates the corresponding control signal evolution. Regarding to Figure 5 and Figure 6 they show, respectively, the evolution of the three-machine state variables when a perturbation occurs on the relative speed of the second machine, and the corresponding control.

From the simulation results shown in these figures, it can be seen that the nonlinear decentralized control is able to damp the oscillations of the system and to enhance transient stability of the multimachine power system and this despite different fault locations that occur on state variables.

5 Conclusion

In this paper, we have developed and validated a new decentralized control approach of nonlinear interconnected polynomial systems. The studied systems are described by a polynomial model with odd Kronecker power of state vectors.

The nonlinear decentralized control law, which is also described by a polynomial form, can guarantee the asymptotic stability of the overall interconnected system when some sufficient conditions are verified.

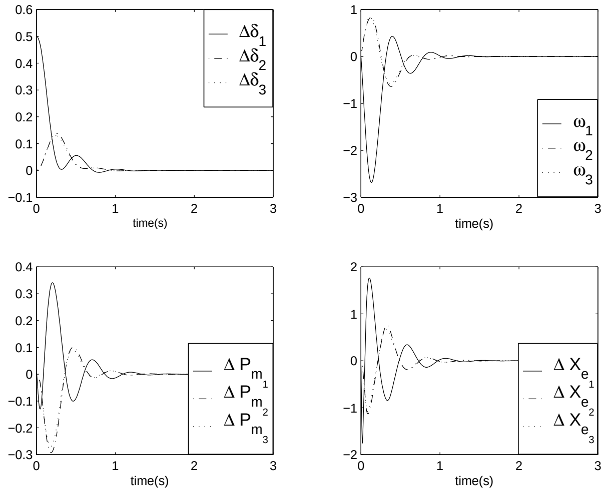

Figure 3: State variable evolution towards a perturbation on δ_1.

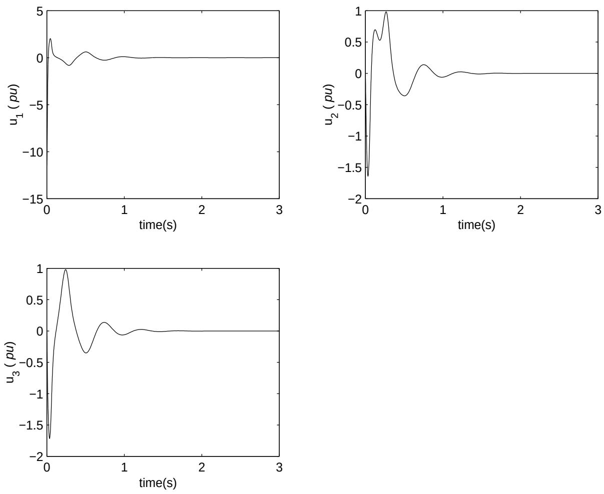

Figure 4: The corresponding control signal evolution.

This new approach is then validated by numerical simulation study on a three-interconnected-machine power system. The proposed study has shown the high performances of the considered control which is able to damp the system oscillations and to enhance the power system transient stability and this despite the high nonlinear interconnections between generators and different perturbations that can occur on the system state variables.

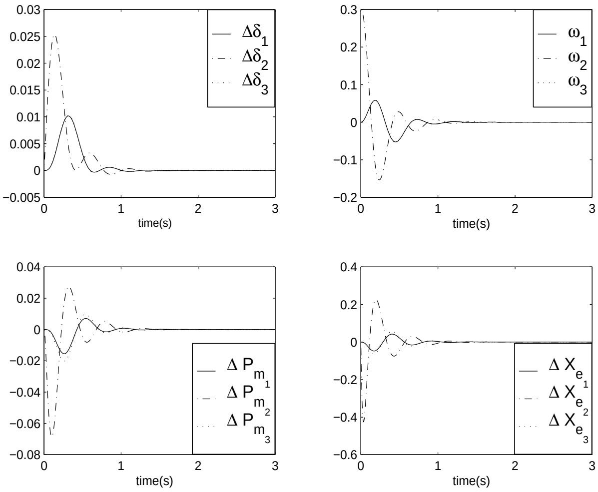

Figure 5: State variable evolution towards a perturbation on ω_2.

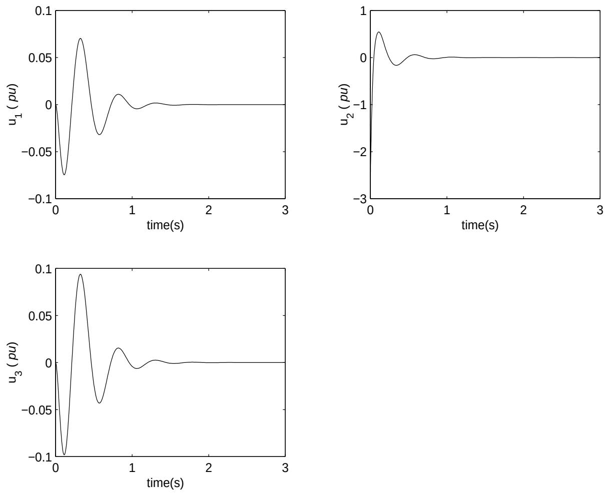

Figure 6: The corresponding control signal evolution.

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