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Boundary Stabilization of a Plate in Contact with a Fluid

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Abstract: This paper presents a solution to the boundary stabilization of a vibrating plate under fluid loading. The fluid is considered to be compressible, barotropic and inviscid. A linear control law is constructed to suppress the plate vibration. The control forces and moments consist of feedbacks of the velocity and normal derivative of the velocity at the boundaries of the plate. The novel features of the proposed method are that (1) it asymptotically stabilizes vibrations of a plate in contact with fluid (the fluid has a free surface) via boundary control and without truncation of the model; and (2) the stabilization of both plate vibrations and fluid motions are simultaneously achieved by using only a linear feedback from the plate boundaries.

Keywords: *semigroups of operators; LaSalle invariant set theorem; asymptotic stabilization; Kirchhoff plate; compressible Newtonain barotropic fluid.*

Mathematics Subject Classification (2010): 35M12, 35Q30.

1 Introduction

The vibration of a plate in contact with fluids has been thoroughly analyzed by many authors [1–3]. Such problems appear frequently in practice, for example when studying the veins, pulmonary passages and urinary systems which can be modeled as shells conveying fluid, aero-elastic instabilities around flexible aircraft, container conveying the fluids and dams [1–5].

One of the most challenging practical difficulties which is present in many of the fluid-structure applications is the vibration of the structures. This may be due to relatively low rigidity and small structural damping and a little excitation may lead long vibration decay time. Vibration is the most destructing source for the flexible structures.

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Therefore, vibration of flexible structures is capable for disturbance, discomfort, damage and destruction. In particular, many researchers have studied the problem of vibration suppression (stabilization) of plates (without and with being in contact with a fluid) since the plate is a necessary element in many applications such as aircraft's skin and flexible structures. In particular, it is widely used in fluid-structure systems [1, 2, 4, 5]. Therefore, an important question in the research of experimentalists and applied mathematicians in the field of flexible structures is the control and stability of vibrating plate under arbitrary loading (such as fluid loading) [6, 8–11]. That is, if the equilibrium state is slightly disturbed, do the perturbations grow or decay? Therefore, suppressing the vibration of such plates (under heavy fluid loading) takes attention of control researchers that investigate in this field.

Boundary stabilization methods are efficient methods to exclude the problems of both in-domain measurement and actuation. The boundary actuators designed for the nondiscretized PDE models are often simple compensators which ensure closed-loop stability for an infinite number of modes.

For some references in boundary stabilization methods, see [12]. Several researchers have proposed boundary actuators for a variety of flexible systems [9, 10, 12–15]. Some researches have been concerned with the fluid-structure stabilization problem, [3, 16]. In these studies, the fluid doesn't have free surface; however, in fact, in most of fluid-structure problems such as dams, large containers, the fluid has at least a free surface. Therefore, in this work we study the stabilization problem of vibrating plate in contact with a fluid having free surface; also we present the simulation results which verify our mathematical results. The fluid is considered to be barotropic compressible Newtonian fluid whereas the plate is taken to be Kirchhoff plate. We use the semigroup techniques to demonstrate the well-posedness of the system. Then benefitting from the Lyapunov stability method and the LaSalle's invariant set theorem, we prove the asymptotic stability of the closed loop system. The main objective of this paper is to use boundary control method for stabilizing the plate vibration in contact with a fluid having free surface via boundary actuators at the plate boundary. It should be noted that the Lyapunov methods are extended to various applications [17, 18]. The presented method uses control actuators at the boundaries of structure.

This article is arranged as follows. In Section 2, the dynamics of a plate and surrounding fluid are presented. Section 3 is devoted to well-posedness and boundary stabilization proof of the fluid-structure problem. Section 4 presents the simulation results. Section 5 is devoted to the conclusion.

2 Governing Equations of Motion

2.1 Fluid domain

The governing equations for the Newtonian barotropic fluid with low velocity can be simplified from the Navier–Stockes equation to the wave equation [19]. The related equations are listed below

$$\begin{cases} c^2\Delta\phi = \phi_{,tt} & \text{in} \quad \Theta, \\ \rho_0\phi_{,t} = -p(x,y,0,t) & \text{in} \quad \Omega, \\ \rho_0\phi_{,tt} + \rho_0 g\phi_{,n} + p_{e,t} = 0 & \text{in} \quad \Omega_2, \\ \phi_{,n} = 0 & \text{in} \quad \Omega_3, \end{cases} \tag{1}$$

where Ω, Ω_2 and Ω_3 are defined as follows:

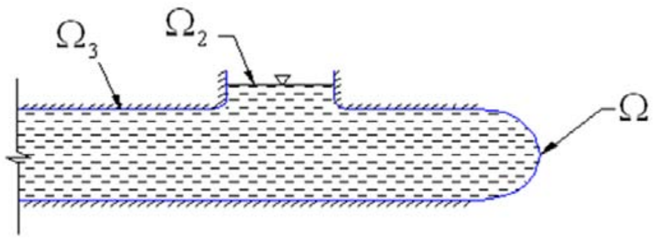

Figure 1: Different boundaries of the fluid-structure system.

1) The wet surface or the fluid structure interface (see Figure 1).

This is the most essential part of the fluid boundary. The motion of the structure and the normal component of the fluid motion coincide, that is [19]:

$$\mathbf{v_f}.\mathbf{n} = \mathbf{v}_s.\mathbf{n}, \tag{2}$$

where $\mathbf{v_f}$ is the fluid velocity and $\mathbf{v}_s$ is the structure velocity.

In this boundary the following equation can be attained [19]:

$$\rho_0\phi_{,t} = -p(x, y, 0, t). \tag{3}$$

2) A free surface with prescribed external pressure, where we allow the linearized (gravitational) waves Ω_2 (see Figure 2) [19]:

$$\rho_0\phi_{,tt} + \rho_0 g\phi_{,n} + p_{e,t} = 0. \tag{4}$$

3) Fixed surface with prescribed external pressure, Ω_3, see Figure1, [19]:

$$\phi_{,n} = 0, \tag{5}$$

where $\phi(x, y, z, t)$ is the velocity potential. This means that $\mathbf{v} = \nabla\phi$ and c is the sound speed in the fluid.

2.2 Structure Domain

The governing equation of a Kirchhoff's plate with external pressure $p(x, y, 0, t)$ can be written as follows [8]:

$$\left\{ \begin{array}{lll} D\nabla^4 w + \rho h w_{,tt} = p & \text{in} & \Omega, \\ w = \partial w/\partial n = 0 & \text{in} & \Gamma_0, \\ V^{(n)} + \partial M^{(ns)}/\partial s = U_1 \ , \ M^{(n)} = U_2 & \text{in} & \Gamma_1, \end{array} \right\} \tag{6}$$

$\forall(x, y, t) \in \Omega\times[0, \infty)$; where $w(x, y, t)$ represents the transverse displacement, $p(x, y, 0, t)$ is the external transverse force distribution (hydrodynamic pressure due to fluid loading) on the plate, h is the thickness of the plate, E is the Young's modulus of elasticity, ν is the Poisson's ratio and $D = Eh/(12(1-\nu^2))$ is the flexural rigidity. It should be noted that Ω is a bounded simple region and $\mathbf{n} = (n_1, n_2)$ is the unit outward normal vector to the boundaries of the plate. M_{11}, M_{12}, M_{22} and V_1, V_2 are defined in the Appendix.

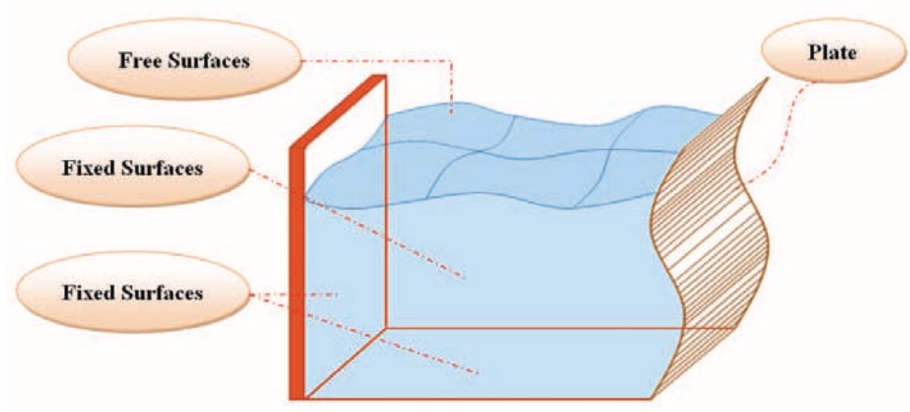

Figure 2: Schematic view of the fluid-structure problem.

3 Stabilization of Plate Under Heavy Fluid Loading

In this section, we consider the stabilization problem of the vibration of a plate without any boundary attachment. For this purpose, first, the following definitions will be used. The inner product on the space $\mathbf{H} = H^1_{\Omega_3}(\Theta) \times \mathrm{L}^2(\Theta) \times H^2_{\Gamma_0}(\Omega) \times L^2(\Omega)$ will be presented as

$$< X, Y >_H = \int_\Theta [\frac{\rho_0}{2c^2}\tau_1\tau_2 + \frac{\rho_0}{2}\Pi(\kappa_1,\kappa_2)]d\theta + \int_\Omega [\frac{\rho_0}{2g}\tau_1\tau_2 + \frac{\rho h}{2}\zeta_1\zeta_2 + \Lambda(\eta_1,\eta_2)]d\Omega, \quad (7)$$

where $X, Y \in \mathbf{H}, X = (\kappa_1, \tau_1, \eta_1, \zeta_1), Y = (\kappa_2, \tau_2, \eta_2, \zeta_2), H^2_{\Omega_3}(\Theta) = \{\kappa_1 : \kappa_1 \in H^2(\Theta) : \partial\kappa_1/\partial n = 0|_{\Omega_1}\}$ and $H^2_{\Gamma_0}(\Omega) = \{\xi_1 : \xi_1 \in H^2(\Omega) : \xi_1 = 0|_{\Gamma_0}, \partial\xi_1/\partial n|_{\Gamma_0} = 0\}$; also the following relations hold

$$\begin{cases} \Pi(\kappa_1,\kappa_2) &= \kappa_{1,x}\kappa_{2,x} + \kappa_{1,y}\kappa_{2,y} + \kappa_{1,z}\kappa_{2,z}, \\ \Lambda(\eta_1,\eta_2) &= (1/2)\Delta\eta_1\Delta\eta_2. \end{cases} \quad (8)$$

It should be noticed that $\Pi(\kappa,\kappa)$ and $\Lambda(\eta,\eta)$ take the roles of the strain energy of the plate and fluid respectively and therefore must be nonnegative.

The plate governing equations and related boundary conditions are as follows (see [8]):

$$\left\{ \begin{array}{llll} D\Delta^2 w + \rho h w_{,tt} & = & p, & \\ w = \partial w/\partial n & = & 0 & \text{in} \quad \Gamma_0, \\ V^{(n)} + \partial M^{(ns)}/\partial s & = & U_1 & \text{in} \quad \Gamma_1, \\ M^{(n)} & = & U_2 & \text{in} \quad \Gamma_1. \end{array} \right\} \quad (9)$$

For this problem, our main intention is to show that the system (9) under boundary feedbacks $U_1 = -w_{,t}$ and $U_2 = \partial(w_{,t})/\partial n$ is well-posed and asymptotically stable. Note that $\partial\Omega = \Gamma = \Gamma_1 \cup \Gamma_2$ and

$$\begin{array}{lll} M^{(n)} & = & M_{11}n_1^2 + M_{22}n_2^2 - 2M_{11}n_1n_2, \\ M^{(ns)} & = & (M_{11} - M_{22})n_1n_2 + M_{12}(n_1^2 - n_2^2), \\ V_1 & = & M_{11,x} + M_{12,y}, \\ V_2 & = & M_{12,x} + M_{22,y}, \\ V^{(n)} & = & V_1n_1 + V_2n_2, \end{array} \quad (10)$$

where $\overrightarrow{n} = (n_1, n_2)$ is the unit outward vector normal to the boundary. V_1, V_2 stand for transversal forces which lay in the planes being perpendicular to unit vectors in x and y directions. $V^{(n)}, M^{(n)}$, are, respectively, transverse force and bending moment which lay perpendicular to the normal direction. For definitions of the remaining parameters see Appendix. To analyze the system using the notion of the linear operators, we utilize the following notation

$$AX = \begin{bmatrix} \tau_1 \\ c^2 \Delta \kappa_1 \\ \zeta_1 \\ \frac{-D}{\rho h} \Delta^2 \eta_1 + p \end{bmatrix}. \tag{11}$$

The state space representation of the system (9) is

$$\left\{ \begin{array}{lll} \dot{\Xi} = A\Xi, & & \\ w = 0, \partial w / \partial n = 0 & \text{in} & \Gamma_0, \\ V^{(n)} + M_{,s}^{(ns)} = -w_{,t}, \ , \ M^{(n)} = \partial(w_{,t})/\partial n & \text{in} & \Gamma_1, \\ \rho_0 \phi_{,t} = -p & \text{in} & \Omega, \\ \rho_0 \phi_{,tt} + \rho_0 g \phi_{,n} = 0 & \text{in} & \Omega_2, \\ \phi_{,n} = 0 & \text{in} & \Omega_3, \\ \Xi(0) = \Xi_0, & & \end{array} \right\} \tag{12}$$

where $\Xi = (\xi_1, \xi_2, \xi_3, \xi_4), \phi = \xi_1, \phi_{,t} = \xi_2, w = \xi_3$ and $w_{,t} = \xi_4$. At first, it will be shown that the operator A with the following domain is a dissipative operator.

$$\begin{array}{rcl} D(A) & = & \{(\xi_1, \xi_2, \xi_3, \xi_4) | \xi_1 \in H^2(\Theta) \cap H^1_{\Omega_3}(\Theta), \xi_2 \in H^1_{\Omega_3}(\Theta), \\ & & \xi_3 \in H^2_{\Gamma_0}(\Omega) \cap H^4(\Omega), \xi_4 \in H^2_{\Gamma_0} \text{ such that } \rho_0 \xi_2|_{\Omega} = -p, \} \end{array} \tag{13}$$

where $H^4_{\Gamma_0}(\Omega) = \{\xi_3 : \xi_3 \in H^4(\Omega) : \xi_3 = 0|_{\Gamma_0}, \partial \xi_3/\partial n|_{\Gamma_0} = 0\}$ and $H^2_{\Omega_3}(\Theta) = \{\xi_1 : \xi_1 \in H^2(\Theta), \xi_1 = 0|_{\Omega_3}\}$.

Lemma 3.1 *A is a dissipative operator.*

Proof. We start from the fact that the total mechanical energy of the systems is equal to the following inner product $E(t) =< \Xi, \Xi >$, therefore

$$\dot{E}(t) = 2 < \Xi, \dot{\Xi} >= 2 < \Xi, A\Xi > . \tag{14}$$

With the above premise and referring to the Lemma 5.1 of Appendix, the proof will be complete.

Lemma 3.2 *The resolvent* $(\alpha I - A)^{-1}$ *exists and is compact* $(\forall \alpha > 0)$.

Proof. For this purpose, we utilize the following relation

$$(\alpha I - A)X = X_0, X_0 \in \mathbf{H} \tag{15}$$

it can be seen that

$$< (\alpha I - A)X, X >_{\mathbf{H}} = \alpha \|X\|^2_{\mathbf{H}} + \|\xi_4\|^2_{L^2(\Gamma_1)} + \|\partial \xi_4 / \partial n\|^2_{L^2(\Gamma_1)} \geq \alpha \|X\|^2_{\mathbf{H}}, \tag{16}$$

where $\|X\|^2_{\mathbf{H}} =< X, X >$.

Using Lax-Milgram lemma, one can easily prove that the above equation has a unique weak solution (see [20–22]). In particular one can infer that:

$R(\alpha I - A) = H^2(\Theta) \times H^1(\Theta) \times H^4(\Omega) \times H^2(\Omega)$, where $\alpha > 0$.

On the other hand, it is clear that $D(A)$ is dense in $H^2(\Theta) \times L^2(\Theta) \times H^4(\Omega) \times \mathrm{Ł}^2(\Omega)$, hence, according to Lumer-Phillips theorem; it is proved that A generates a C_0-semigroup of contractions (see [24]). Finally one can obtain the following result

$$\|X_0\|_{\mathbf{H}} \geq \alpha \|X\|_{\mathbf{H}}. \tag{17}$$

Using Sobolev embedding theorem (Rellich-Kondrachov compact embedding theorem), since $(\alpha I - A)^{-1} V$ is compactly embedded in $\mathrm{Ł}^2(\Theta) \times \mathrm{Ł}^2(\Theta) \times \mathrm{Ł}^2(\Omega) \times \mathrm{Ł}^2(\Omega)$, therefore the compactness of the above-mentioned resolvent is evident.

Theorem 3.1 *Let in the system (22), the initial condition Ξ_0 belong to $D(A)$. Then the system (22) is well-posed.*

Proof. Based on Lemma 3.1, it is evident that the system (22) is well-posed [24]. Also its strong solution has the following regularity (see [23, 24]).

$$\begin{array}{rcl} \phi(t) & \in & C^0([0,t], H^2(\Theta) \cap H^1_{\Omega_3}(\Theta)) \cap C^1([0,t], H^1_{\Omega_3}(\Theta)) \cap C^2([0,t], L^2(\Theta)), \\ w(t) & \in & C^0([0,t], H^4(\Omega) \cap H^2_{\Gamma_0}(\Omega)) \cap C^1([0,t], H^2_{\Gamma_0}(\Omega)) \cap C^2([0,t], L^2(\Theta)). \end{array} \tag{18}$$

Now, we turn our attention to the proof of the asymptotic stability of the closed loop system.

Theorem 3.2 *Using the boundary feedback control laws (19), the states of the system Ξ will eventually tend toward zero,*

$$U_1 = -\xi_4 \text{ and } U_2 = \partial \xi_4 / \partial n. \tag{19}$$

Proof. The mechanical energy of the system as discussed previously, is

$$E(t) =< \Xi, \Xi > . \tag{20}$$

By performing some algebraic operations and using Green's Lemma, the following can be obtained (see Appendix):

$$\dot{E}(t) = -\|\xi_4\|^2_{L^2(\Gamma_1)} - \|\partial \xi_4 / \partial n\|^2_{L^2(\Gamma_1)} \leq 0. \tag{21}$$

At this step, because of the compactness of the resolvent $(\alpha I - A)^{-1}$, one can use LaSalle's invariant set theorem and therefore, it is sufficient to show that the following system has the trivial solution as its unique solution:

$$\left\{ \begin{array}{ll} \dot{\Xi} = A\Xi & \text{in } \ \Omega, \\ \xi_4 = \partial \xi_4/\partial n = 0 \ \textit{and} \ M^{(n)} = V^{(n)} = 0 & \text{in } \ \Gamma_1, \\ \xi_3 = \partial \xi_3 / \partial n = 0 & \text{in } \ \Gamma_0, \\ \rho_0 \phi_{,t} = -p & \text{in } \ \Omega, \\ \rho_0 \phi_{,tt} + \rho_0 g \phi_{,n} = 0 & \text{in } \ \Omega_2, \\ \phi = 0 & \text{in } \ \Omega_3, \\ \Xi(0) = \Xi_0. & \end{array} \right\} \tag{22}$$

Using the Holmgren uniqueness theorem [25], one can easily show that the above system of equations admits only trivial solution. Then, by regarding the LaSalle's invariant set theorem,

$$\lim_{t \to \infty} E(t) = 0, \tag{23}$$

which yields the desired stability.

4 Simulation Results

In this section, we compare the controlled vibration of the plate in contact to a fluid with the uncontrolled one. We plot displacements of some points of the plate in the controlled and uncontrolled cases. We will see the effect of the boundary actuators.

4.1 Geometric Properties of the Plate and the Acoustic Fluid Models

Acoustic fluid region is a $0.5m \times 0.5m \times 0.5m$ cubic space. All sides of the fluid except one which is in contact with the plate are fixed and; therefore, the normal velocities of the fluid at those faces are zero. One face is in contact with the plate and the other face is a free surface (see Figure 2).

4.2 Mechanical Properties of Plate and Acoustic Fluid

The mechanical properties of the fluid and plate are shown in Table 1 and Table 2, respectively.

Bulk Modulus	Density
225e7	1000 Kg/m^3

Table 1: Material properties of the fluid.

Young Modulus	Poisson's Ratio	Density
200e9 Pa	0.3	1920 Kg/m^3

Table 2: Material properties of the plate.

4.3 Results

We present two sets of results. First, the results of the vibration of middle point of the plate without boundary actuators at the plate boundaries are presented and then the other set is for the vibrations of the same point of the plate in the presence of the boundary actuators. We attach a set of boundary actuators with controller gain $k_f = 3N.s/m$ at two controlled sides of the plate. First, the results for the free vibrations of the plate are presented and subsequently the simulation results for the controlled vibrations of the plate are demonstrated. The displacements of the mentioned points of the plate are illustrated by Figures 3–8.

5 Conclusion

Asymptotic stability of the vibration of plates in contact with a fluid was proved. It is shown that the mechanical energy of the systems would converge asymptotically toward zero. Since the control laws consisted only of the feedback from the shear force and bending moment at the boundary of plate, measurement cost was minimized. Also, the proposed method avoids installation of distributed actuators / sensors which meant observation of vibration data along the plate or in the interior of the fluid is not required and the asymptotical stability of the fluid is accomplished without using any actuation in the fluid domain or its boundary.

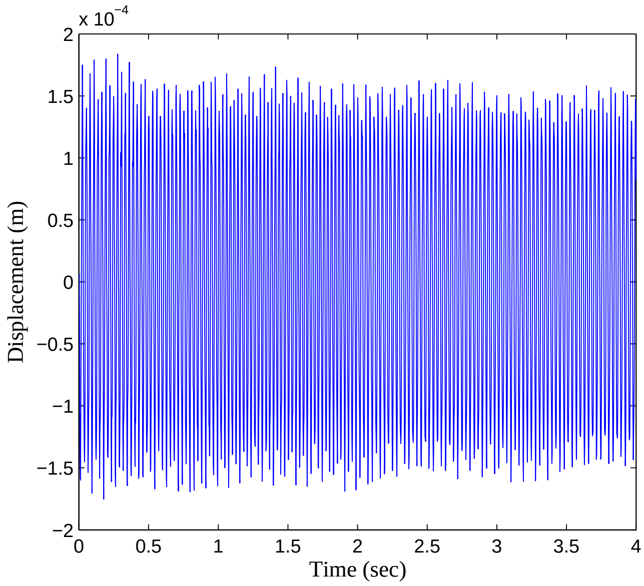

Figure 3: Displacement of the point (0.25, 0) of the plate in contact with the fluid in its free vibration.

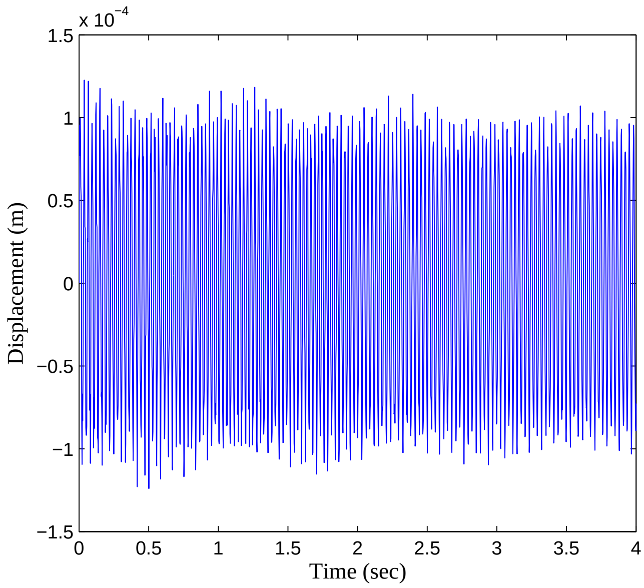

Figure 4: Displacement of point (0.25, 0.25) of the plate in contact with the fluid in its free vibration.

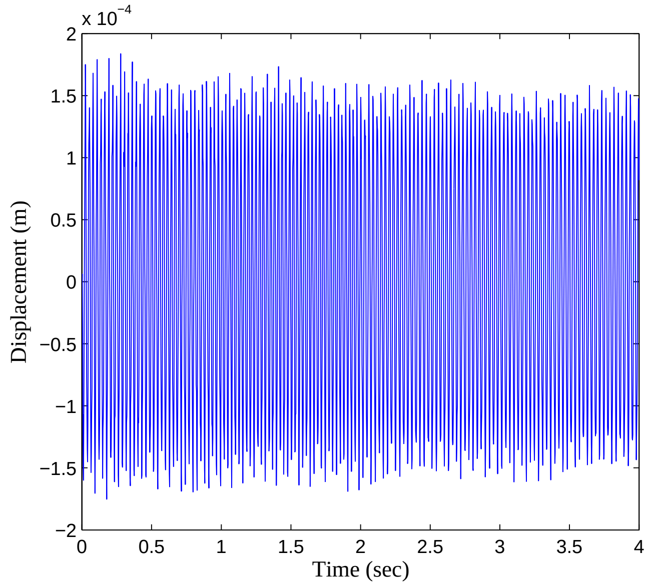

Figure 5: Displacement of point (0.25, 0.5) of the plate in contact with the fluid in its free vibration.

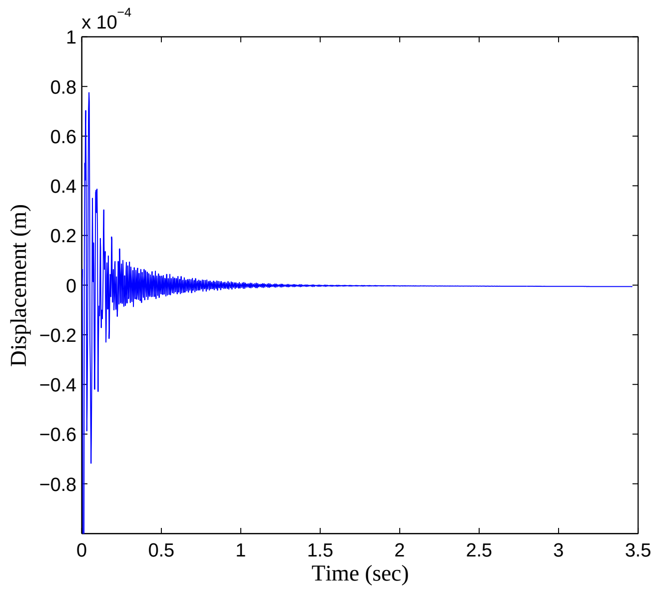

Figure 6: Displacement of point (0.25, 0) of the plate in contact with the fluid in the presence of the boundary actuators.

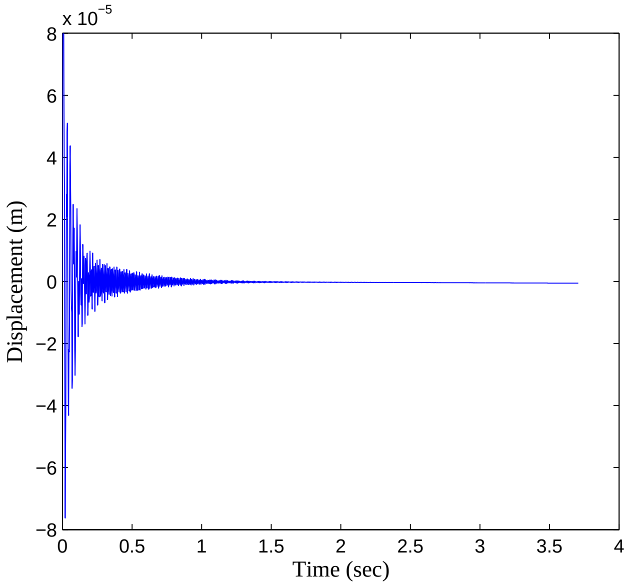

Figure 7: Displacement of point (0.25, 0.25) of the plate in contact with the fluid in the presence of the boundary actuators.

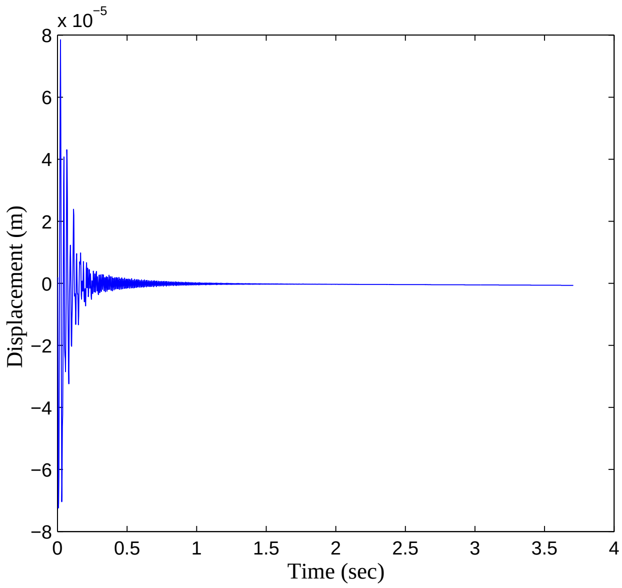

Figure 8: Displacement of point (0.25, 0.5) of the plate in contact with the fluid in the presence of the boundary actuators.

Appendix

In this section it will be shown that the time derivative of the mechanical energy of the system is negative semi-definite and in the sequel we show that the operator A is dissipative.

Lemma 5.1 *For the operator A, with definition (11), one can have*

$$\dot{E}(t) = 2 < \Xi, A\Xi >= -\|\xi_4\|^2_{L^2(\Gamma_1)} - \|\partial\xi_4/\partial n\|^2_{L^2(\Gamma_1)}. \tag{24}$$

Proof. It is clear that $\dot{E}(t) = 2 < \Xi, A\Xi >$. For the rest of the proof, we define some parameters

$$M_{11} = -D(w_{,xx} + \nu w_{,yy}), \tag{25}$$
$$M_{22} = -D(w_{,yy} + \nu w_{,xx}), \tag{26}$$
$$M_{12} = -D(1-\nu)w_{,xy}, \tag{27}$$
$$\kappa_{11} = -w_{,xx}\ , \quad \kappa_{22} = -w_{,yy}\ , \quad \kappa_{12} = -2w_{,xy}, \tag{28}$$
$$V_1 = M_{11,x} + M_{12,y}, \tag{29}$$
$$V_2 = M_{12,x} + M_{22,y}. \tag{30}$$

We notice that the governing equation of motion can be rewritten in the following form [8]

$$M_{11,xx} + 2M_{12,xy} + M_{22,yy} = \rho h w_{,tt}. \tag{31}$$

The energy functional takes the following form

$$\begin{aligned} E(t) = &\tfrac{1}{2}\int_\Omega [M_{11}\kappa_{11} + M_{22}\kappa_{22} + M_{12}\kappa_{12} + \rho h w_{,t}^2]d\Omega \\ &+ \int_\Theta [\tfrac{\rho_0}{2c^2}\phi_{,t}^2 + \tfrac{\rho_0}{2}|\nabla\phi|^2]d\Theta + \int_{\Omega_2}[\tfrac{\rho_0}{2g}\phi_{,t}^2]d\Omega. \end{aligned} \tag{32}$$

Therefore, time derivative of $E(t)$ will be

$$\begin{aligned} \dot{E}(t) = &\frac{1}{2}\int_\Omega [\dot{M}_{11}\kappa_{11} + \dot{M}_{22}\kappa_{22} + \dot{M}_{12}\kappa_{12} + M_{11}\dot{\kappa}_{11} + M_{22}\dot{\kappa}_{22} + M_{12}\dot{\kappa}_{12} + 2\rho h w_{,t}w_{,tt}]d\Omega \\ &+ \int_{\Omega_2}[\frac{\rho_0}{g}\phi_{,t}\phi_{,tt}]d\Omega + \int_\Theta [\frac{\rho_0}{c^2}\phi_{,t}\phi_{,tt} + \rho_0(\phi_{'tx}\phi_{,x} + \phi_{,ty}\phi_{'y})]d\Theta \end{aligned} \tag{33}$$

and therefore

$$\begin{aligned} 2\dot{E}(t) = &\int_\Omega [M_{11}\dot{\kappa}_{11} + M_{22}\dot{\kappa}_{22} + M_{12}\dot{\kappa}_{12} + (M_{11,xx} + M_{22,yy} + 2M_{12,xy})w_t]d\Omega + \\ &\int_\Omega [\dot{M}_{11}\kappa_{11} + \dot{M}_{22}\kappa_{22} + \dot{M}_{12}\kappa_{12} + (M_{11,xx} + M_{22,yy} + 2M_{12,xy})w_{,t}]d\Omega + \\ &\int_{\Omega_2}[\tfrac{\rho_0}{g}\phi_{,t}\phi_{,tt}]d\Omega + \rho_0\int_\Theta [\tfrac{\partial}{\partial x}((\phi_{,t}\phi_{,tt}) + \tfrac{\partial}{\partial y}(\phi_{,t}\phi_{,tt})]d\Theta. \end{aligned} \tag{34}$$

Employing the relations for the resultant moments in directions x and y (see (24)– (28)), we get

$$\begin{aligned} 2\dot{E}(t) = &\int_\Omega [(M_{11,xx}w_{,t} - M_{11}w_{,xxt}) + (M_{22,yy}w_{,t} - M_{22}w_{,yyt}) + 2(M_{12,xy}w_{,xyt})]d\Omega + \\ &\int_\Omega D[w_{,xx}w_{,xxt} + \nu w_{,yyt}w_{,xx}]d\Omega + \int_\Omega D[\nu w_{,yy}w_{,xxt} + w_{,yyt}w_{,yy}]d\Omega + \\ &\int_\Omega 2D(1-\nu)w_{,xyt}w_{,xy}d\Omega - \int_\Omega D[w_{,xxxx}w_{,t} + \nu w_{,xxyy}w_{,t}]d\Omega - \\ &\int_\Omega 2D(1-\nu)w_{,xxyy}w_{,t}d\Omega - \int_\Omega D[w_{,yyyy}w_{,t} + \nu w_{,xxyy}w_{,t}]d\Omega + \\ &\int_\Omega p w_{,t}d\Omega + \int_{\Omega_2}\tfrac{\rho_0}{g}\phi_{,t}\phi_{,tt}d\Omega + \rho_0\int_\Omega \phi_{,t}\phi_{,n}d\Omega + \\ &\rho_0\int_{\Omega_2}\phi_{,t}\phi_{,n}d\Omega + \rho_0\int_{\Omega_3}\phi_{,t}\phi_{,n}d\Omega. \end{aligned} \tag{35}$$

Rearranging the terms and using the boundary conditions for the fluid yield

$$\begin{aligned} 2\dot{E}(t) &= 2\int_\Omega [(M_{11,x}w_{,t} + M_{12,y}w_{,t} - M_{11}w_{,tx} - M_{12}w_{,yt})_{,x}d\Omega + \\ &\int_\Omega [(M_{22,y}w_{,t} - M_{12,x}w_{,t} - M_{22}w_{,ty} - M_{12}w_{,xt})_{,y}d\Omega + \\ &\int_\Omega pw_{,t}d\Omega + \int_{\Omega_2} \frac{\rho_0}{g}\phi_{,t}\phi_{,tt}d\Omega + \rho_0\int_\Omega \phi_{,t}\phi_{,n}d\Omega + \rho_0\int_{\Omega_2}\phi_{,t}\phi_{,n}d\Omega. \end{aligned} \tag{36}$$

Applying Green's Lemma and also boundary conditions of the fluid yield

$$\begin{aligned} 2\dot{E}(t) &= \oint_\Gamma (M_{11,x}w_{,t} + M_{12,y}w_{,t} - M_{11}w_{,tx} - M_{12}w_{,yt})n_1 d\Gamma + \\ &\oint_\Gamma [(M_{22,y}w_{,t} - M_{12,x}w_{,t} - M_{22}w_{,ty} - M_{12}w_{,xt})n_2 d\Gamma + \\ &\int_\Omega pw_{,t}d\Omega - \rho_0\int_{\Omega_2}\phi_{,t}\phi_{,n}d\Omega - \int_\Omega pw_{,t}d\Omega + \rho_0\int_{\Omega_2}\phi_{,t}\phi_{,n}d\Omega. \end{aligned} \tag{37}$$

Grouping the terms and noting that

$$\frac{\partial\Delta}{\partial x} = n_1\frac{\partial\Delta}{\partial n} - n_2\frac{\partial\Delta}{\partial s}, \tag{38}$$

$$\frac{\partial\Delta}{\partial y} = n_1\frac{\partial\Delta}{\partial s} - n_2\frac{\partial\Delta}{\partial n}, \tag{39}$$

yield the following result

$$\dot{E}(t) = \oint_\Gamma [(V^{(n)} + \frac{\partial M_{ns}}{\partial s})w_{,t} - M^{(n)}(w_{,t})_{,n}]d\Gamma. \tag{40}$$

By applying the assumptions of Theorem 2, and using the related boundary conditions, the following result is attained:

$$\dot{E}(t) = -\|\xi_4\|^2_{L^2(\Gamma_1)} - \|\frac{\partial\xi_4}{\partial n}\|^2_{L^2(\Gamma_1)}. \tag{41}$$

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