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Huang-Hilbert Transform Based Wavelet Adaptive Tracking Control for a Class of Uncertain Nonlinear Systems Subject to Actuator Saturation

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Abstract: In this paper a novel Huang-Hilbert Transform (HHT) based adaptive tracking control strategy is proposed for a class of uncertain systems subjected to actuator saturation. HHT is used in this work for the online feature extraction of the uncertainties in the systems which are approximated by Wavelet Neural Networks (WNNs). Adaptation laws are developed iteratively using the Intrinsic Modal Functions (IMF) for the online tuning of wavelets parameters. The uniformly ultimate boundedness of the closed-loop tracking error is verified even in the presence of WNN approximation errors and bounded unknown disturbances, using the Lyapunov approach and with novel weight updating rules. Finally some simulations are performed to verify the effectiveness and performance of the theoretical development.

Keywords: *Hilbert-Huang transform; empirical mode decomposition; intrinsic mode function; wavelet neural networks; adaptive control; Lyapunov functional.*

1 Introduction

In many practical systems, the system model always contains some uncertain elements; these uncertainties may be due to additive unknown internal or external noise, environmental influence, nonlinearities such as hysteresis or friction, poor plant knowledge, reduced-order models, and uncertain or slowly varying parameters. The analytical study of adaptive nonlinear control systems involving online approximation structures has evolved considerably during the last decade [1–3] The design of online approximation based controllers can be broken up into two stages: first, the unknown nonlinearity is represented by some online approximators. Hence, the designer needs to choose a specific

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adaptive network configuration, including the general structure of the online approximator, the number of layers (in case of multi layer neural networks), the number of adjustable weights, etc. In the second stage, the designer needs to develop an appropriate feedback control law for updating the adjustable weights.

Characteristics of practical actuators are in general nonlinear, usually described by the nonlinearities such as saturation, hysteresis, backlash etc. Nonlinear behavior of the actuator causes the detuning of plant as well as controller parameters which may lead to the poor performance or even may cause the destabilization of the system. Out of these nonlinearities saturation is the frequently encountered nonlinearity and is addressed by several researchers [4, 5].

In recent years, learning-based control methodology using Neural networks (NNs) has become an alternative to adaptive control since NNs are considered as general tools for modeling nonlinear systems. Work on adaptive NN control using the universal NN approximation property is now pursued by several groups of researchers [6, 7]. By using neural network (NN) as an approximation tool, the assumptions on linear parameterized nonlinearities in adaptive controller designing aspects have greatly been relaxed. It also broadens the class of the uncertain nonlinear systems which can be effectively dealt by adaptive controllers. However there are some difficulties associated with NN based controller. The basis functions are generally not orthogonal or redundant; i.e., the network representation is not unique and is probably not the most efficient one and the convergence of neural networks may not be guaranteed. Also the training procedure for NN may be trapped in some local minima depending on the initial settings. Wavelet neural networks are the modified form of the NN having the properties of space and frequency localization properties leading to a superior learning capabilities and fast convergence. Thus WNN based control systems can achieve better control performance than NN based control systems [6–9].

Recently, a new signal analysis approach, Hilbert-Huang transform (HHT), is proposes by Huang et. al. [10, 11]which is a combination of empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA). By EMD, a signal is decomposed into a series of mono-component modes defined as intrinsic mode functions (IMFs), and Hilbert transform can thus be applied to each IMF to obtain the instantaneous frequency and the instantaneous magnitude. Unlike Fourier series representation in which base functions are always sinusoidal functions, HHT adopts different IMFs to describe various signals, resulting in adaptive base functions. Also HHT is valid for nonlinear and nonstationary signals. Because of the distinct characteristics of HHT, it has attracted considerable research interest in exploring its potential as a frequency identification tool.

A straightforward method could be that, after application of HHT to a signal, comparisons are made between Fourier spectra of the obtained IMFs and that of the original signal to find out the relationships between IMFs and vibration modes. Then by computing the amplitude weighted average frequencies based on the Hilbert spectra, modal frequencies can be identified. Besides, Yang et al. [12] proposed a method in which, before they are analyzed by HHT, the signals are processed by some pre-selected bandpass filters, the thresholds of which are determined by referring to the Fourier spectra of the signals. Efficacious as they are, these two HHT-based frequency identification methods however have to rely on some a priori information about the natural frequencies to be identified, whether by comparing Fourier spectra of original signals and those of the IMFs or by selecting the thresholds of the bandpass filters. From a practical point of view, it is difficult to obtain some a priori information about the frequencies of random signals.

So the theoretical Eigen analysis techniques are not appropriate to provide a sufficiently accurate estimation of natural frequencies. That is, frequency information is usually unavailable before identification procedures are carried out. Koh et al. [13] and Chhoa et al. [14] introduced a criterion that the IMF component with the highest energy compared to other IMF components most probably represents the fundamental frequency of the system. The criterion was applied to experimental signals collected from the real time systems and successfully identified the IMFs related to the fundamental frequencies. Due to noise contamination, however, the fundamental frequency of a system may relegate from one IMF to the next IMF during the time range of the signal [15], and the identification of the relationships between IMFs and multiple physical vibration modes might be more involved as a modal frequency may be contained along specific segments of the whole time duration of one or more IMFs.

The major limitation of HHT and EMD is that the signal under analysis must be known so that its maxima and minima can be calculated. But in this work, the nonlinear function present in the dynamics of the system is uncertain in nature. To overcome this problem we have proposed a technique to estimate the uncertain function by WNN first and then through iterative EMD algorithm, the uncertain function is approximated very accurately. Multiple WNNs are cascaded to solve this problem. Every layer has different number of nodes and different tuning laws derived by gradient descent rule. The output of each WNN is used for the derivation of the adaptive tuning laws of the next cascaded WNN. This process is repeated until the residue becomes zero, which means the approximation is best possible. This novel technique of using HHT and EMD to approximate the features of an uncertainties present in the nonlinear systems has never been cited in the literature to the best of the knowledge of authors and hence reflects the contribution of this work.

This paper deals with the designing of HHT based wavelet adaptive tracking controller for a class of uncertain nonlinear systems. WNN are used for approximating the system uncertainty as well as to optimize the performance of the control strategy. HHT algorithm generates the features of these uncertainties to be fed to the consecutive WNN.

The paper is organized as follows: Section 2 deals with the system preliminaries, system description is given in Section 3. WNN based controller designing aspects are discussed in Section 4. Section 5 describes the proposed HHT based wavelet adaptive controller design. The stability analysis of the proposed control scheme is given in Section 6. Effectiveness of the proposed strategy is illustrated through an example in Section 7 while Section 8 concludes the paper.

2 System Preliminaries

2.0.1 Actuator Saturation

The output of an actuator $u(t)$with input $v(t)$ subjected to the condition of saturation is defined as

$$u = \begin{cases} u_{\max}, & v \geq u_{\max}, \\ v, & u_{\min} < v < u_{\max}, \\ u_{\min}, & v \leq u_{\min}, \end{cases} \tag{1}$$

where $u_{\max}$ and $u_{\min}$ are upper and lower saturation limits as shown in Figure 1.

For symmetric actuator saturation $u_{\min} = -u_{\max}$ part of the control effort which can

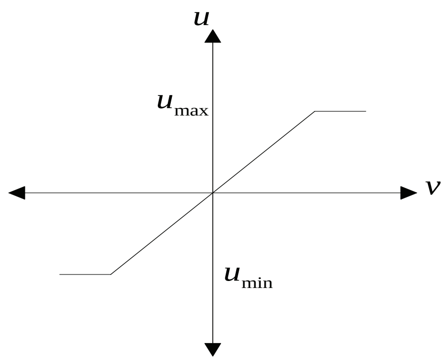

Figure 1: Saturation function.

not be implemented under this condition is defined as

$$\Delta u = \begin{cases} u_{\max} - v, & v \geq u_{\max}, \\ 0, & u_{\min} < v < u_{\max}, \\ u_{\min} - v, & v \leq u_{\min}, \end{cases} \tag{2}$$

where Δu describes the effect of actuator saturation and can be effectively approximated by using a wavelet neural network.

2.0.2 Wavelet neural network

Wavelet network is a type of building block for function approximation. The building block is obtained by translating and dilating the mother wavelet function. In contrast to conventional wavelets, a biased wavelet has a nonzero mean and can better reproduce signal components that are in the low-frequency region on the time-frequency plane since the nonzero mean enlarges low-frequency gain. Output of a biased n dimensional wavelet network with m nodes is

$$f = \alpha^T \varphi(x, w, c) + \beta^T \phi(x, w, c), \tag{3}$$

where $x = [x_1, x_2, ..., x_n]^T \in R^n$ is the input vector, $\varphi = [\varphi_1, \varphi_2, ..., \varphi_m]^T \in \Re^m$ and $\phi = [\phi_1, \phi_2, ..., \phi_m]^T \in \Re^m$are wavelet and bias functions respectively; $w = [w_1, w_2, ..., w_m]^T \in R^{mxn}$ and $c = [c_1, c_2, ..., c_m]^T \in R^{mxn}$ are dilation and translation parameters respectively ; $\alpha = [\alpha_1, .., \alpha_m]^T \in R^m$ and $\beta = [\beta_1, .., \beta_m]^T \in R^m$ are weights of wavelet and bias function respectively.

Let f^* be the optimal function approximation using an ideal wavelet approximator then

$$f = f^* + \Delta = \alpha^{*T} \varphi^* + \beta^{*T} \phi^* + \Delta, \tag{4}$$

where $\varphi^* = \varphi(x, w^*, c^*)$ and $\phi^* = \phi(x, w^*, c^*)$, α^*, β^*, w^*, c^* are the optimal parameter vectors of α, β, w, c respectively and Δ denotes the approximation error and is assumed to be bounded by $|\Delta| \leq \Delta^*$, in which Δ^* is a positive constant.

Optimal parameter vectors needed for best approximation of the function are difficult to determine so define an estimate function as

$$\hat{f} = \hat{\alpha}^T \hat{\varphi} + \hat{\beta}^T \hat{\phi}, \tag{5}$$

where $\hat{\varphi} = \varphi(x, \hat{w}, \hat{c}), \hat{\phi} = \phi(x, \hat{w}, \hat{c})$ and $\hat{\alpha}, \hat{\beta}, \hat{w}, \hat{c}$ are the estimates of $\alpha^*, \beta^*, w^*, c^*$ respectively. Define the estimation error as

$$\tilde{f} = f - \hat{f} = f^* - \hat{f} + \Delta = \alpha^T \tilde{\varphi} + \hat{\alpha}^T \tilde{\varphi} + \tilde{\alpha}^T \hat{\varphi} + \tilde{\beta}^T \tilde{\phi} + \hat{\beta}^T \tilde{\phi} + \tilde{\beta}^T \hat{\phi} + \Delta, \tag{6}$$

where $\tilde{\alpha} = \alpha^* - \hat{\alpha}$, $\tilde{\beta} = \beta^* - \hat{\beta}$, $\tilde{\varphi} = \varphi^* - \hat{\varphi}$, $\tilde{\phi} = \phi^* - \hat{\phi}$.

By properly selecting the number of nodes, the estimation error $\tilde{f}$ can be made arbitrarily small on the compact set so that the bound $\left\|\tilde{f}\right\| = \tilde{f}_m$ holds for all $x \in \Re$.

Use Taylor expansion linearization technique to transform the nonlinear function into a partially linear form as a step towards the derivation of online tuning laws for the wavelet parameters to achieve the favorable estimation of system dynamics

$$\tilde{\varphi} = A_1^T \tilde{w} + B_1^T \tilde{c} + h_1 \tilde{\phi} = A_2^T \tilde{w} + B_2^T \tilde{c} + h_2, \tag{7}$$

where $\tilde{w} = w^* - \hat{w}, \tilde{c} = c^* - \hat{c}$ and h_1, h_2 are the vectors of higher order terms and

$$A_1 = \left[\frac{d\varphi_1}{dw}, \frac{d\varphi_2}{dw}, ..., \frac{d\varphi_m}{dw}\right]\Bigg|_{w=\hat{w}}, \quad A_2 = \left[\frac{d\phi_1}{dw}, \frac{d\phi_2}{dw}, ..., \frac{d\phi_m}{dw}\right]\Bigg|_{w=\hat{w}},$$
$$B_1 = \left[\frac{d\varphi_1}{dc}, \frac{d\varphi_2}{dc}, ..., \frac{d\varphi_m}{dc}\right]\Bigg|_{c=\hat{c}}, \quad B_2 = \left[\frac{d\phi_1}{dc}, \frac{d\phi_2}{dc}, ..., \frac{d\phi_m}{dc}\right]\Bigg|_{c=\hat{c}},$$

with

$$\begin{aligned}
\frac{d\hat{\varphi}_i}{dw} &= \left[0...0\frac{d\hat{\varphi}_i}{dw_{1i}}, \frac{d\hat{\varphi}_i}{dw_{2i}}, ..., \frac{d\hat{\varphi}_i}{dw_{ni}}, 0...0\right]^T, \\
\frac{d\hat{\varphi}_i}{dc} &= \left[0...0\frac{d\hat{\varphi}_i}{dc_{1i}}, \frac{d\hat{\varphi}_i}{dc_{2i}}, ..., \frac{d\hat{\varphi}_i}{dc_{ni}}, 0...0\right]^T, \\
\frac{d\hat{\phi}_i}{dw} &= \left[0...0, \frac{d\hat{\phi}_i}{dw_{1i}}, \frac{d\hat{\phi}_i}{dw_{2i}}, ..., \frac{d\hat{\phi}_i}{dw_{ni}}, 0...0\right]^T, \\
\frac{d\hat{\phi}_i}{dc} &= \left[0...0, \frac{d\hat{\phi}_i}{dc_{1i}}, \frac{d\hat{\phi}_i}{dc_{2i}}, ..., \frac{d\hat{\phi}_i}{dc_{ni}}, 0...0\right]^T.
\end{aligned}$$

Substituting (7) into (6)

$$\begin{aligned}
\tilde{f} = \tilde{\alpha}^T \left(\hat{\varphi} - \mathrm{A}_1^T \hat{w} - B_1^T \hat{c}\right) + \tilde{w}^T \left(A_1 \hat{\alpha} + A_2 \hat{\beta}\right) + \tilde{c}^T \left(B_1 \hat{\alpha} + B_2 \hat{\beta}\right) \\
+ \tilde{\beta}^T \left(\hat{\phi} - \mathrm{A}_2^T \hat{w} - B_2^T \hat{c}\right) + \varepsilon,
\end{aligned} \tag{8}$$

where the uncertain term is given by the following expression

$$\varepsilon = \ \alpha^{*T} h_1 + \tilde{\alpha}^T A_1^T w^* + \tilde{\alpha}^T B_1^T c^* + \beta^{*T} h_2 + \tilde{\beta}^T A_2^T w^* + \tilde{\beta}^T B_2^T c^*.$$

2.0.3 Hilbert-Huang Transform

This section briefly summarizes the principles and procedures of HHT. HHT is an adaptive data analysis method designed for analyzing non-stationary signals. In HHT, the signal is decomposed into a finite small number of components, called Intrinsic Mode Functions (IMF). This process of decomposition is called Empirical Mode Decomposition (EMD). Presented by Huang et al. [8], HHT essentially consists of two steps:

empirical mode decomposition and Hilbert spectral analysis. By EMD, a complicated signal is decomposed into a series of simple oscillatory modes, designated as intrinsic mode functions, and a residue. Hilbert spectral analysis is then invoked for each IMF to obtain the instantaneous frequencies and the instantaneous magnitudes, which comprise the Hilbert-Huang spectrum of the signal.

i. Empirical Mode Decomposition (EMD) The EMD decomposes the signal in terms of IMFs, each of which is a mono-component function. Given an arbitrary signal $x(t)$ following the EMD method, sifting processes are used to extract the IMFs. In a typical single sifting process, the local maxima are first identified and connected by cubic spline functions, resulting in an upper envelope $u_1^{(1)}(t)$ of the signal. A lower envelope $l_1^{(1)}(t)$ is similarly obtained based on local minima. Then a function $m_1^{(1)}(t)$ is defined as the mean of the upper and lower envelopes. Finally, subtracting the mean function $m_1^{(1)}(t)$ from signal $x(t)$, the first iterate $h_1^{(1)}(t)$, or the first proto-IMF is obtained. The above procedures are iterated until the proto-IMF $h_1^{(k+1)}(t)$ converges to the first IMF q_1 if the following conditions are satisfied:

- For $h_1^{(k+1)}(t)$, the number of extrema and the zeros differ at most by 1.
- The difference between the mean $m_1^{(k)}(t)$ and zero is within the pre-selected tolerance.

The above sifting process is shown in (9)

$$m_1^{(k+1)} = \frac{u_1^{(k+1)} + l_1^{(k+1)}}{2} h_1^{(k+1)} = h_1^k - m_1^{(k+1)}, \tag{9}$$

where $k = 0, 1, 2, \ldots$and $h_1^0 = x$. One kind of iteration stopping criterion is that the value of standard deviation SD is less than a preselected value, where SD is defined as

$$SD_k = \sum_i \frac{\left(h^{(k+1)}(t_i) - h^{(k)}(t_i)\right)^2}{\left(h^{(k)}(t_i)\right)^2} \tag{10}$$

or

$$SD_k = \frac{\sum\limits_i \left(h^{(k+1)}(t_i) - h^{(k)}(t_i)\right)^2}{\sum\limits_i \left(h^{(k)}(t_i)\right)^2}. \tag{11}$$

The shifting process is stopped, when SD_k becomes smaller than a pre-determined value. Once the shifting process is stopped, the first IMF q_1 can be obtained, which contains the finest scale or the shortest period component of the signal. After separating q_1 from the original signal $x(t)$, the residue of the signal is obtained

$$x(t) - q_1 = r_1. \tag{12}$$

A new sifting process is applied to r_1, which leads to the second IMF q_2 and the residue r_2:

$$r_1 - q_2 = r_2 \tag{13}$$

Similarly, for n^{th} IMF,

$$r_{n-1} - q_n = r_n. \tag{14}$$

The sifting processes are iterated until r_n becomes a constant, a monotonic function, or a function with only one extremum. Therefore, by EMD, the original signal $x(t)$ is denoted as

$$x(t) = \sum_{i=1}^{n} q_i + r_n. \tag{15}$$

Thus the decomposition of a signal in n-empirical modes is achieved. The components of the EMD are physically meaningful, as the characteristic scales are defined by the physical data. The instantaneous frequency can be computed by finding the Hilbert Transform of the IMF components.

ii. Feature Extraction using Hilbert-Huang Transform.

The features of the disturbance signals are extracted by finding the energy of the IMFs which are derived from each of the disturbance waveforms. Let q_1, q_2, q_3 be the first three IMF components and E_1, E_2 and E_3 be their corresponding energies. Energy of the IMF is calculated using the following equations

$$E_1 = \|q_1\|^2, \tag{16}$$

$$E_2 = \|q_2\|^2, \tag{17}$$

$$E_3 = \|q_3\|^2. \tag{18}$$

3 System Description

Consider a nonlinear system of the form

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = x_3, \\ &\vdots \\ &\dot{x}_n = f(x) + gu, \\ &y = x_1, \end{aligned} \tag{19}$$

where $x = [x_1, x_2, ..., x_n]^T, u, y$ are state variable, control input and output respectively. $f(x)$ is a smooth unknown, nonlinear function of state variables.

Rewriting the system (19) as

$$\begin{aligned} &\dot{x} = Ax + B(f(x) + u(t)), \\ &y = Cx, \end{aligned} \tag{20}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix}.$$

Using the actuator saturation defined in Section 2 system (20) can be transformed to

$$\begin{aligned} &\dot{x} = Ax + B(\delta(x) + (v + \Delta u)), \\ &y = Cx, \end{aligned} \tag{21}$$

where $\delta(x, \bar{y}_d) = f(x) + \Delta u$. Let $\bar{y}_d = [y_d, \dot{y}_d, \ldots, \overset{n-1}{y_d}]^T$ be the vector of desired tracking trajectory. The objective is to formulate a state feedback control law to achieve the desired tracking performance simultaneously nullifying the effect of actuator saturation. The control law is formulated using the transformed system (21). The following assumptions are taken for the systems under consideration.

Assumption 3.1 Desired trajectory $y_d\,(t)$ is assumed to be smooth, continuous C^n and available for measurement.

4 Basic Controller Design Using Filtered Tracking Error

Define the state tracking error vector $e(t)$ as

$$e(t) = x(t) - \bar{y}_d(t). \tag{22}$$

The filter tracking error is defined as

$$r = Ke, \tag{23}$$

where $K = [k_1, k_2, \ldots k_{n-1}, 1]$ is an appropriately chosen coefficient vector such that $e \to 0$exponentially as $\Re \to 0$.

Applying the feedback linearization method, the control laws for every iteration level are defined in the subsequent section.

5 Proposed Adaptive WNN Controller Design

A novel adaptive control strategy is proposed in this section which uses WNN to approximate the nonlinear uncertainties $\delta(x)$ present in the systems through HHT algorithm. A separate WNN network with different number of nodes and different adaptation laws is implemented for every iteration level of HHT algorithm. The tuning laws for the WNN at various iterations are derived as follows.

The cost function derived for the tuning of WNN parameters using (23) is given by

$$S = \frac{1}{2}\dot{r}^T\dot{r}. \tag{24}$$

Using the gradient descent algorithm, the online tuning laws for the WNN parameters are

$$\begin{aligned} \dot{\alpha} &= -\eta\tfrac{\partial S}{\partial \alpha} = -\eta\dot{r}\tfrac{\partial \dot{r}}{\partial \alpha}, \\ \dot{w} &= -\eta\tfrac{\partial S}{\partial w} = -\eta\dot{r}\tfrac{\partial \dot{r}}{\partial w}, \\ \dot{c} &= -\eta\tfrac{\partial S}{\partial c} = -\eta\dot{r}\tfrac{\partial \dot{r}}{\partial c}. \end{aligned} \tag{25}$$

i. First iteration.

Assuming $q_1$be the WNN approximation for the first EMD, the control law can be derived as

$$u = (\overset{n}{y_d} - \frac{K_e e}{k_n} - r - q_1), \tag{26}$$

where $K_e = [0, k_1, k_2, \ldots, k_{n-1}]$.

From (23), we get

$$\dot{r} = K_e e + k_n(\delta + u + \overset{n}{y}_d). \tag{27}$$

And the online tuning laws for the first WNN are given by

$$\begin{aligned} \dot{\alpha}_1 &= \eta_1 k_n \dot{r} \tfrac{\partial q_1}{\partial \alpha_1}, \\ \dot{w}_1 &= \eta_1 k_n \dot{r} \tfrac{\partial q_1}{\partial w_1}, \\ \dot{c}_1 &= \eta_1 k_n \dot{r} \tfrac{\partial q_1}{\partial c_1}. \end{aligned} \tag{28}$$

ii. Second iteration.

Assuming q_2 be the WNN approximation for the second EMD, the control law can be derived as

$$u = (\overset{n}{y}_d - \frac{K_e e}{k_n} - r - q_1 - q_2). \tag{29}$$

Also the corresponding online tuning laws for WNN are derived as

$$\begin{aligned} \dot{\alpha}_2 &= \eta_2 k_n \dot{r} \tfrac{\partial q_2}{\partial \alpha_2} \\ \dot{w}_2 &= \eta_2 k_n \dot{r} \tfrac{\partial q_2}{\partial w_2}, \\ \dot{c}_2 &= \eta_2 k_n \dot{r} \tfrac{\partial q_2}{\partial c_2}. \end{aligned} \tag{30}$$

iii. n^{th} iteration.

Similarly assuming $q_n$be the WNN approximation for the n^{th} EMD, the final control law can be derived as

$$u = (\overset{n}{y}_d - \frac{K_e e}{k_n} - r - (\sum_{i=1}^{n} q_i)). \tag{31}$$

Also the corresponding online tuning laws for WNN are derived as

$$\begin{aligned} \dot{\alpha}_n &= \eta_n k_n \dot{r} \tfrac{\partial q_n}{\partial \alpha_n}, \\ \dot{w}_n &= \eta_n k_n \dot{r} \tfrac{\partial q_n}{\partial w_n}, \\ \dot{c}_n &= \eta_n k_n \dot{r} \tfrac{\partial q_n}{\partial c_n}. \end{aligned} \tag{32}$$

Stability of the system (21) with the proposed control strategy will be analyzed in the next section.

5.0.4 Stability Analysis

Consider a Lyapunov functional of the form [16]

$$V = \frac{1}{2} r^2. \tag{33}$$

Differentiate it along the trajectories of the system,

$$\dot{V} = r(K_e e + K(\delta(x) + u(t) - v_r - \overset{n}{y}_d).$$

By the substitution of control law $u(t)$ in the above equation, we get

$$\dot{V} = r(-Kr + \tilde{\delta}(x) - v_r),$$

where $\tilde{\delta}(x)$ is the error between the actual value and the approximated value of

$$\dot{V} \leq -Kr^2 + |r| \left|\tilde{\delta}(x)\right| - rv_r).$$

Substitute the robust control term $v_r = -\frac{(\rho^2+1)r}{2\rho^2}$ in the above equation,

$$\dot{V} \leq -s_1 r^2 + s_2(|r| \left|\tilde{\delta}(x)\right|)^2,$$

where $s_1 = (K + \frac{K}{2})$ and $s_2 = \frac{K\rho^2}{2}$. The system is stable as long as

$$s_1 r^2 \geq s_2(|r| \left|\tilde{\delta}(x)\right|)^2. \tag{34}$$

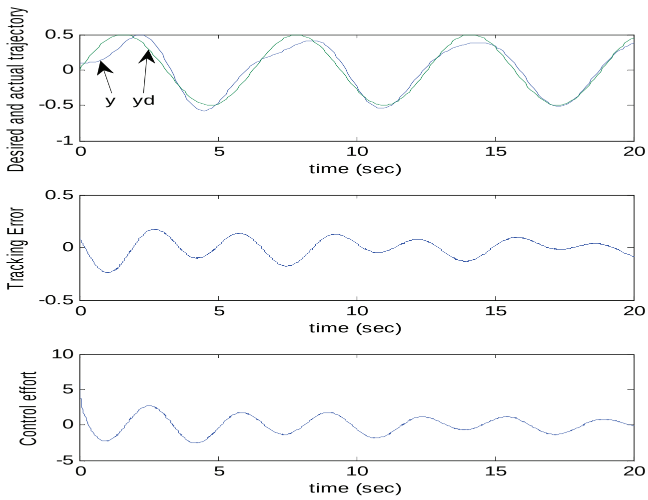

Figure 2: Desired trajectory, actual trajectory, tracking error and control effort after first iteration level.

5.0.5 Simulation results

Simulation is performed to verify the effectiveness of proposed HHT-WNN based control strategy. Consider a system of the form

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = x_3, \\ &\dot{x}_3 = x_4, \\ &\dot{x}_4 = 0.01 x_1 \sin x_2 + u, \\ &y = x_1. \end{aligned} \tag{35}$$

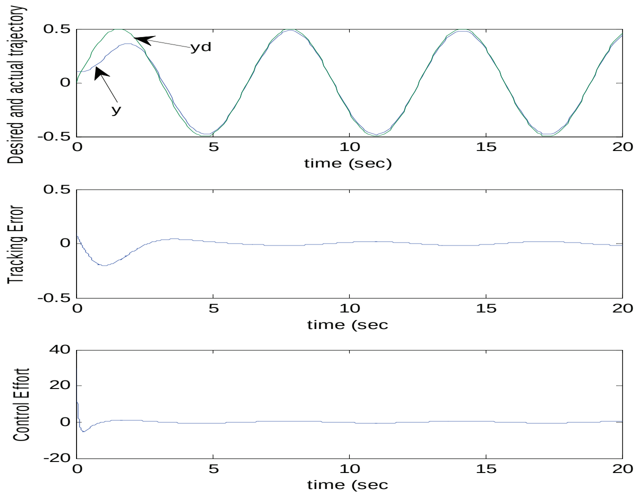

Figure 3: Desired trajectory, actual trajectory, tracking error and control effort after second iteration level.

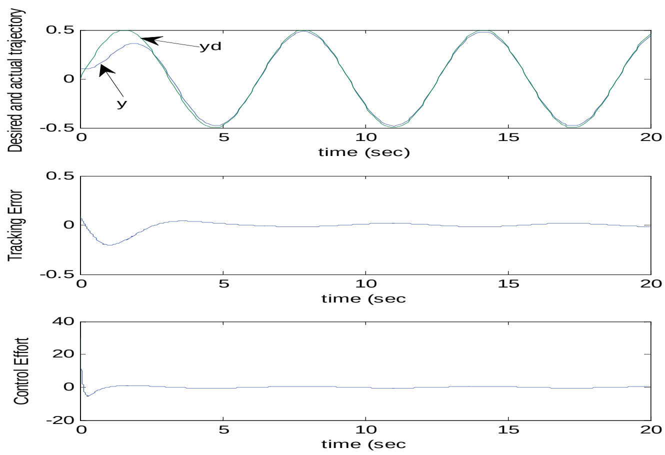

Figure 4: Desired trajectory, actual trajectory, tracking error and control effort after third iteration level.

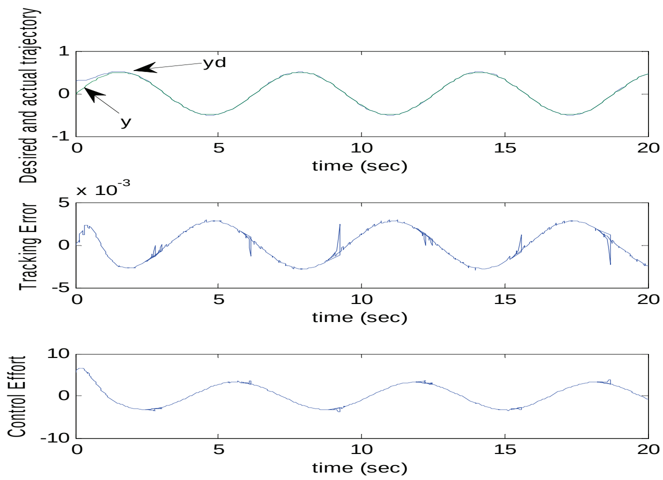

Figure 5: Desired trajectory, actual trajectory, tracking error and control effort after fourth iteration level.

The system belongs to the class of uncertain nonlinear systems defined by (21) with $n = 4$. The proposed controller strategy is applied to this system with an objective to solve the tracking problem of system. Four iteration levels are used for the simulation. The desired trajectory is taken as $y_d = 0.5 \sin t$. Initial conditions are taken as $x(0) = [0.3, 0, 0, 0]^T$. Attenuation levels for robust controller are taken as 0.01. Controller gain vector is taken as (31). Wavelet networks with Mexican Hat wavelet as the mother wavelet is used for approximating the unknown system dynamics. Wavelet parameters for these wavelet networks are tuned online using the proposed adaptation laws. Initial conditions for all the wavelet parameters are set to zero. Simulation results are shown in Figures 2–5. As observed from the figures, system response tracks the desired trajectory rapidly in consecutive iterations and after the fourth iteration the trajectory is perfectly tracked. This reflects the efficiency of the proposed control strategy.

6 Conclusion

A novel HHT based Wavelet adaptive tracking control strategy is proposed for a class of systems with unknown system dynamics and actuator saturation. Adaptive wavelet networks are used for approximating the unknown system dynamics. HHT algorithm is used for the better online feature extraction of uncertainties present in the dynamics of the system. Adaptation laws are developed for online tuning of the wavelet parameters. The stability of the overall system is guaranteed by using the Lyapunov functional. The theoretical analysis is validated by the simulation results.

References

[1] Chang, K. M. Reference adaptive control for uncertain systems with sector-like bounded nonlinear inputs. In: *Proc. of IEEE International conference on Control and Automation (ICCA05)* (2005) 611–616.

[2] Stefanovic, M. and Safonov, M. G. Safe Adaptive Switching Control: Stability and Convergence. *IEEE Transactions on Automatic Control* **53**(9) (2008) 2012–2021.

[3] Chen, M., Ge, S. S. and How, B. Robust adaptive neural network control for a class of uncertain MIMO nonlinear systems with input nonlinearities. *IEEE Transaction on neural networks* **21**(5) (2010) 796–812.

[4] He, P. and Jagannathan, S. Reinforcement learning neural-network-based controller for nonlinear discrete-time systems with input constraints. *IEEE Transactions on Systems, Man and Cybernetics – Part B: Cybernetics* **37**(2) (2007) 425–436.

[5] Zhou, J., Joo, M. and Zhou, Y. Adaptive neural network control of uncertain nonlinear system in presence of input saturation. In: *Proceedings of the ICARCV* (2006) 895–899.

[6] Zhang, Q. and Benveniste, A. Wavelet networks. *IEEE Transactions on Neural Networks* **3**(6) (1992) 889–898.

[7] Zhang, J., Walter, G. G., Miao, Y. and Lee, W. Wavelet neural networks for function learning. *IEEE Transactions on Signal Processing* **43**(6) (1995) 1485–1497.

[8] Jaddu, H. and Hiraishi, K. A Wavelet Approach for Solving linear quadratic optimal control problems. In: *SICE-ICASE International Joint Conference* (2006) 694–698.

[9] Minu, K. K., Lineesh, M. C. and John, C. J. Wavelet Neural Networks for Nonlinear Time Series Analysis. *Applied Mathematical Sciences* **4** (1992) 2485–2495.

[10] Huang, N. E., Shen, Z., Long, S. R., Wu, M. C., Shih, H. H., Zheng, Q., Yen, N.C., Tung, C. C. and Liu, H. H. The Empirical Mode Decomposition Method and the Hilbert Spectrum for Non-linear and Non-stationary Time Series Analysis. In: *Proceedings of Royal Society, London* (1998) 903–915.

[11] Huang, N. E., Wu, M. C., Long, S. R. , Shen, S. P., Qu, W. and Gloersen, P. A confidence limit for the empirical mode decomposition and the Hilbert spectral analysis. In: *Proceedings of Royal Society, London* (2003) 18–21.

[12] Yang, J. N., Lei, Y., Lin, S. and Huang, N. Hilbert-Huang based approach for structural damage detection. *Journal of Engineering Mechanics, ASCE* (2004) 85-95.

[13] Koh, S. J. A., Maalej, M. and Quek, S. T. Damage quantification of flexurally loaded RC slab using frequency response data. *Journal of Structural Health Monitoring* (2004) 293-311.

[14] Chhoa, C. Y., Maalej, M. and Quek, S. T. Damage detection in concrete beam using Hilbert-Huang transform. In: *Proc. of 17th KKCNN symposium on civil engineering* (2004) 85-90.

[15] Flandrin, P., Gonalvs, P. and Rilling, G. EMD equivalent filter banks, from interpretation to applications in Hibert-Huang Transform: Introduction and Application *World Scientific, Singapore* **8**(5) (2005) 67–87.

[16] Sharma, M. and Verma, A. Wavelet Neural Network Based Adaptive Tracking Control for a Class of Uncertain Nonlinear Systems Using Reinforcement Learning. *Nonlinear Dynamics and System Theory* **12**(3) (2012) 272–279.