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# Designing a Compensator Based on Extended Kalman Filter for Elimination of Noise and Delay Effect in Tracking Loop

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**Abstract:** This paper introduces a new control structure in tracking loop. In this new structure a position sensor has been used to eliminate noise and delay effect of tracking sensor. The EKF based Compensator estimates the desired error and provides the controller with the appropriate control signal with respect to gimbal position that is reported by the position sensor and the tracking loop error (output of tracking sensor). We have shown that in this new structure, notwithstanding the existence of noise and delay, designing of the tracking loop controller can be done and EKF based Compensator is practical for compensating noise and delay effect of the tracking sensor. Indeed a considerable feature of the presented control structure is that designing method of the controller is simple and utilizing of the delay and noise compensator has significantly reduced complication of the design.

**Keywords:** *tracking system; stochastic error; two degree of freedom gimbal; thermal noise; extended Kalman filter (EKF); constant delay; variable time delay.*

**Mathematics Subject Classification (2010):** 93C15, 93C55, 93B52.

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## 1 Introduction

Tracking loop is designed in order to locate the position of mobile objects in space. Indeed the main issue is designing and developing a system that is able to report position of the target object in relation to a certain reference.

Sensors with radar or optic nature are applicable for reporting the position of the (target) object. Such sensors usually have a limited field of view with regard to the technology that is used in designing tracking sensors [1-2].

Hence for reporting position of an object that is moving in space, a servomechanism system is needed to align field of view of the tracking sensor with the target position so that the target is always inside of the field of view. In other words, if the tracking sensor (with limited field of view) is set over a two degree of freedom gimbal that is tracking the target, we can be aware of precise position of the target.

The loop that is closed for target tracking and extraction of its position data is known as tracking loop. Usually it is desirable that tracking error (measured amounts by the tracking sensor) tends toward zero [3]. A simple diagram of tracking loop can be supposed as below figure [4-5].

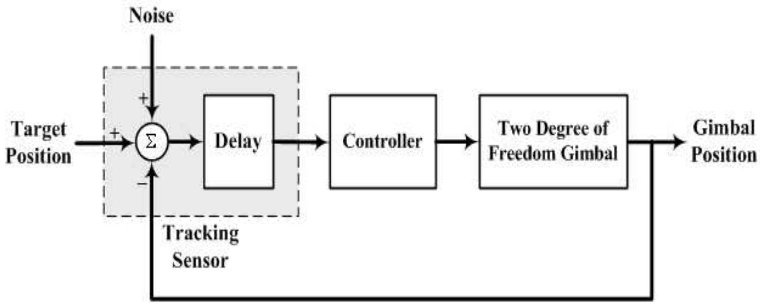


**Figure 1**: A simple diagram of tracking loop.

As it is shown in the above figure, it has been tried to simply model the issues of tracking precision and delay due to the process of the tracking sensor that are fundamental characteristics influencing on the tracking loop performance.

Tracking error will be different dependent on the nature of the sensor quantitatively or qualitatively. Radar sensors often have errors of stochastic nature that arise from thermal noise, glint noise and so on and impact on tracking precision [3-11]. Also Angle measurement delay causes stability margin of the tracking loop to be reduced and so the performance band width of the tracking system. Radar sensors usually have a time varying process delay.

Many texts have been written on designing an appropriate controller for reducing time delay. Most of the authors have dealt with the control problem of the time-delay system via designing robust-based controllers and predictor-based controllers. In systems that use the robust controller the desired performance with respect to time-delay is guaranteed in specified changes [7-8]. In predictor-based controllers it has been tried to remove the time delay effect from the closed-loop system [9-11].

In this paper, a structure like Figure 2 is offered for compensating the noise and delay effect of the tracking sensor. In this new structure we have used the output of position

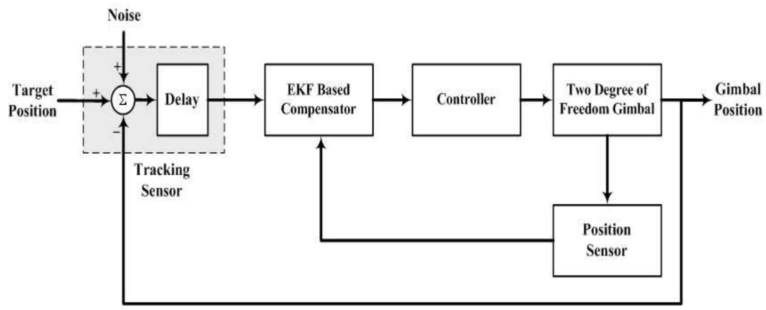


**Figure 2**: Proposed tracking loop structure.

sensor that ordinarily is used in gimbal frame, for eliminating the delay effect. In the structure, an EKF based compensator not only eliminates effect of the input noise, but also it is useful in eliminating the effect of delay by processing the reported data of the position sensor.

It should be noted that in designing of the mentioned compensator two main assumptions related to the position sensor have been regarded. First, the precision of the position sensor is equal to that of the tracking sensor, and the second is that the mentioned sensor is free delay (that is not far away from the reality).

The tracking loop controller is also a usual controller that can be designed despite the quantity of the delay and noise of the tracking sensor. Indeed first we can design the controller based on the desirable conditions regardless of the delay and noise that enter into the tracking loop and then with the aid of the EKF based compensator, compensate bad effects of the noise and delay inside of the tracking loop. Thus designing of the controller accomplishes with less complication that is regarded as an excellent advantage.

In the following, we are going to provide the necessary arrangements for utilizing the position sensor beside the tracking sensor in the tracking loop by deriving the kinematic relationship between them. Then we will describe the structure of the EKF based compensator by reviewing the mechanism of EKF and rewriting the related formulas. After that we will analyze the obtained results in order to evaluate the performance of this new control structure, in two cases of constant delay and variable time delay.

## 2 Deriving Kinematic Relationship Between Tracking Sensor and Position Sensor

Tracking sensors usually report angular position of the target in relation to their bore sight. Likewise, position sensor is able to measure angular position of the bore sight of the tracking sensor in relation to a definite reference. The purpose of this section is to determine the relationship between the reported angles by these sensors.

For this purpose, first consider the kinematic related to the tracking of a substance in space as shown in Figure 3.

In this figure, three coordinate systems are defined that are target frame (T), tracking sensor frame (S) and a frame that is attached to the ground (I). Indeed if tracking gimbal has been fixed on the ground, this ground attached frame can be considered as a reference for tracking gimbal position.

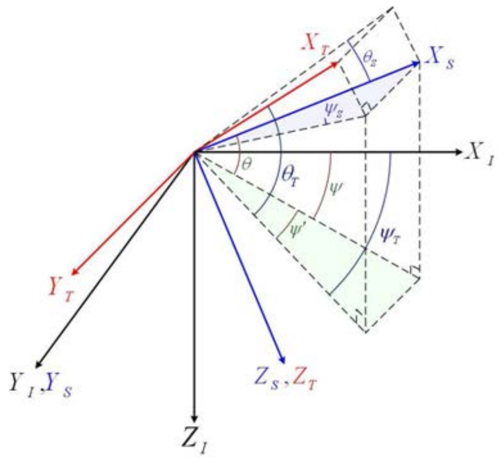


**Figure 3**: Target kinematic related to definite coordinate systems.

According to the figure, angles $\theta_T$ and $\psi_T$ are elevation and azimuth position of target in relation to position sensor. Angles $\theta_S$ and $\psi_S$ also are measured quantities of the tracking sensor. Now if we show elevation and azimuth angles of the bore sight of the tracking sensor in relation to gimbal position sensor with $\theta$ and $\psi$, the purpose is determining $\psi^{'}$ (image of $\psi_R$ in the $X_I Y_I$ plane) as an explanation of $\psi_S$in relation to gimbal position sensor.

Without reducing anything from the issue, we will assume $\psi$ and $\theta_S$ is zero. In this condition the transform matrix of I frame to T frame is obtained with rotation about $Y_I$ by an amount of $\theta$ and then rotation about $Z_S$ by an amount of $\psi_S$. This process can be shown mathematically according to formula (1)

$$\{I\} \overset{C_{Y_I}(\theta)}{\longrightarrow} \{S\} \overset{C_{Z_S}(\psi_S)}{\longrightarrow} \{T\}. \tag{1}$$

Now with the calculation of the rotation matrix we can determine target position relative to $I$ frame.

$$\begin{aligned} {}^I_T C &= \begin{bmatrix} C\psi_S\, C\theta & -S\psi_S\, C\theta & S\theta \\ S\psi_S & C\psi_S & 0 \\ -C\psi_S\, S\theta & S\psi_S\, S\theta & 0 \end{bmatrix}, \quad {}^T R_T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \\ \Rightarrow {}^I R_T &= \begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix} = {}^I_T C\, {}^T R_T = \begin{bmatrix} C\psi_S\, C\theta \\ S\psi_S \\ -C\psi_S\, S\theta \end{bmatrix}. \end{aligned} \tag{2}$$

On the other hand according to Figure 3 and above formula we have:

$$tg\psi' = \frac{y_I}{x_I} = \frac{1}{C\theta}\, tg\psi_S. \tag{3}$$

Now whereas $\psi_S$ is a small value, $\psi^{'}$ and thereupon the relationship between the measured values of the tracking and position sensors can be approximated as follows

$$\psi' \approx \frac{1}{C\theta}\,\psi_S \quad \Rightarrow \quad \begin{cases} \psi_T = \psi + \frac{1}{C\theta}\psi_S, \\ \theta_T = \theta + \theta_S. \end{cases} \tag{4}$$

According to above formula, the values measured by the position sensor and tracking sensor relate to each other via $C\theta$ factor.

## 3 Explanation of Compensator Structure

This section allocated to explanation of structure of EKF based compensator. Before anything else first, we will introduce EKF and then we will describe performance of the compensator with mathematical formulas with regard to equations governing the issue.

### 3.1 A review of EKF equations

EKF is used as a recursive estimate algorithm for nonlinear systems. Indeed because of the ability of this algorithm in Gaussian-nonlinear filtering, EKF is used extensively. This filter is based on linearization of measurements and alteration of models by using Taylor series expansion. In other words, EKF is an estimation algorithm in which the nonlinear system is linearized firstly, and then recursive Kalman filter equations are used for time update [6].

In order to illustrate the issue, consider related equations in a nonlinear system as follows

$$\begin{aligned} &x(k) = f_{k-1}\left(x(k-1), u(k-1), w(k-1)\right), \\ &y(k) = h_k\left(x(k), v(k)\right), \\ &w(k) \sim (0, Q(k)), \\ &v(k) \sim (0, R(k)), \end{aligned} \tag{5}$$

where $x(k)$ is state variable, $u(k\ )$ is process input, $y(k)$ is process output and $v(k)$ is output noise. Meanwhile process and output noise are assumed Gaussian with zero mean and $Q(k)$ and $R(k)$ covariance.

Since Kalman filter is going to minimize the expected value of the estimated square error, the initial quantification of the state estimation $\hat{x}^+(0)$ and error covariance $P^+(0)$ is as follows [6]

$$\begin{aligned} &\hat{x}^+(0) = E\left(x(0)\right), \\ &P^+(0) = E\left[\left(x(0) - \hat{x}^+(0)\right)\left(x(0) - \hat{x}^+(0)\right)^T\right]. \end{aligned} \tag{6}$$

With this initial quantification, the first step that is estimation based on premeasured values is done. The equations related to this step are as follows [6]

$$\begin{aligned} &F(k-1) = \left.\frac{\partial f_{k-1}}{\partial x}\right|_{\hat{x}^+(k-1)}, \\ &L(k-1) = \left.\frac{\partial f_{k-1}}{\partial w}\right|_{\hat{x}^+(k-1)}, \\ &P^-(k) = F_{k-1}P^+(k-1)F_{k-1}^T + L_{k-1}Q(k-1)L_{k-1}^T, \\ &\hat{x}^-(k) = f_{k-1}(\hat{x}^+(k-1), u(k-1), 0). \end{aligned} \tag{7}$$

In this formula $F(k\text{-}1)$ and $L(k\text{-}1)$ are Jacobians of the process model and $P(k)$ is estimation error covariance. Then in the second step, update is done based on the present output of measured samples according to the following equations [6]

$$\begin{array}{l} H(k) = \frac{\partial h_k}{\partial x}\big|_{\hat{x}^-(k)}, \\ M(k) = \frac{\partial h_k}{\partial v}\big|_{\hat{x}^-(k)}, \\ K(k) = P^-(k)H^T(k)\left(H(k)P^-(k)H^T(k) + M(k)R(k)M^T(k)\right)^{-1}, \\ \hat{x}^+(k) = \hat{x}^-(k) + K(k)\left[y(k) - h_k\left(\hat{x}^-(k), 0\right)\right], \\ P^+(k) = \left(I - K(k)H(k)\right)P^-(k). \end{array} \tag{8}$$

In this equation *H(k)* and *M(k)* are Jacobians of the output model and *K(k)* is filter gain.

### 3.2 Description of compensator performance with the aid of mathematical equations

Due to the fact that the considered issue is tracking a substance in three dimension space, related equations and state variables are as follows

$$\left\{\begin{array}{l} \ddot{x} = a_x, \\ \ddot{y} = a_y, \\ \ddot{z} = a_z, \end{array}\right. \quad \Rightarrow \quad \left\{\begin{array}{l} x_1 = x, \\ x_2 = y, \\ x_3 = z, \\ x_4 = \dot{x}, \\ x_5 = \dot{y}, \\ x_6 = \dot{z}. \end{array}\right. \tag{9}$$

That $a_x$, $a_y$ and $a_z$ are components of target acceleration in direction of Cartesian coordination axes.

Thus state space model is as formula (10)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} & & & 1 & 0 & 0 \\ & 0 & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \\ 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & 0 & \\ 0 & 0 & 0 & & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_x \\ a_y \\ a_z \end{bmatrix}. \tag{10}$$

For digitalizing based on sampling time $T$, we have:

$$\begin{array}{ll} \dot{x}_i = \frac{x_i(k+1) - x_i(k)}{T} & (i = 1, ..., 6) \\ \quad = x_{i+3}(k) & (i = 1, 2, 3). \end{array} \tag{11}$$

Thus with regard to the above equation, discrete state space equations are obtained as formula (12).

$$\underbrace{\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ x_5(k+1) \\ x_6(k+1) \end{bmatrix}}_{x_{k+1}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ & & & 1 & 0 & 0 \\ & 0 & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \end{bmatrix}}_{x_k} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ T\,a_x \\ T\,a_y \\ T\,a_z \end{bmatrix}}_{B\,u_k}. \tag{12}$$

The output of the system in Cartesian coordination based on the state variables is rewritable as follows

$$y_k = \begin{bmatrix} R \\ \psi \\ \theta \end{bmatrix} = \begin{bmatrix} \sqrt{x_1^2(k) + x_2^2(k) + x_3^2(k)} \\ tg^{-1}\left(\frac{x_2(k)}{x_1(k)}\right) \\ tg^{-1}\left(\frac{x_3(k)}{\sqrt{x_1^2(k)+x_2^2(k)}}\right) \end{bmatrix} = h(x_k). \tag{13}$$

Due to the fact that control structure of the tracking loop in both channels of elevation and azimuth is similar and for simplifying the equations, from now on we just regard azimuth channel equations. Azimuth channel related to the control structure has been shown in Figure 4. In this figure *C(z)* and *G(z)* are discrete model of controller and process sequently. Meanwhile $\Delta$ is function of delay. Also error of the tracking sensor modeled with *N(0,σ)* that is Gaussian noise with zero mean and variance $\sigma$. *Cθ* is the relationship factor between position sensor and tracking sensor that is obvious in formula (4). Also *1/Cθ* is considered to compensate *Cθ* effect. It is very clear that the only difference in elevation channel is absence of these factors.

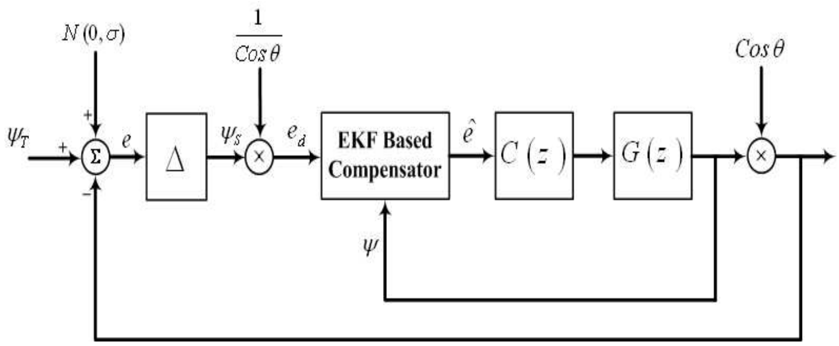


**Figure 4**: Mathematical model of tracking loop structure.

Thus consider output of the tracking loop in azimuth channel as formula (14).

$$\psi(k) = tg^{-1}\left(\frac{x_2(k)}{x_1(k)}\right) = h_\psi\left(x(k)\right). \tag{14}$$

According to this equation, the output Jacobian matrix follows as

$$H_\psi(k) = \left.\frac{\partial h_\psi}{\partial x}\right|_{x(k)} = \begin{bmatrix} -\frac{x_2(k)}{x_1^2(k)+x_2^2(k)} & \frac{x_1(k)}{x_1^2(k)+x_2^2(k)} & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{15}$$

Due to the fact that designing of the tracking loop often is done to reset to zero the steady state error for ramp input [3], in the following we will assume that the tracking target flies with constant speed (acceleration equals to zero).

Now for implementing EKF based compensator, *F(k-1)* and *H(k)* matrices in equation sets of (7) and (8) are placed as follows.

$$H(k) = H_\psi(k), \quad F(k-1) = A. \tag{16}$$

Also relative to the nature and the value of the loop input noise, appropriate values for *Q(k),R(k),M(k)* and *L(k-1)* can be selected.

But with regard to the mathematical model of the control loop of the azimuth channel and according to Figure 4 and formula (8), *y(k)* is assumed as below in which $n$ states number of the delay samples that is considered in compensator structure

$$y(k) = e_d(k) + \psi(k-n). \tag{17}$$

Indeed $n$ is a designing parameter of compensator that depends on the nature of the delay of the tracking sensor, appropriate value should be chosen for it. In the next section we will see how the selection of this parameter is done.

On the other hand, according to discrete state space model we have:

$$\begin{aligned} & x(k+1) = A\,x(k) \\ & \Rightarrow\ x(k+n) = A^n\,x(k). \end{aligned} \tag{18}$$

Thus in update stage, we will define $\tilde{\psi}$ as follows.

$$\begin{aligned} & \tilde{X} = A^n\,\hat{x}^+(k) \\ & \Rightarrow\ \tilde{\psi} = tg^{-1}\left(\frac{\tilde{X}_2}{\tilde{X}_1}\right). \end{aligned} \tag{19}$$

Therefore estimated error follows formula (20)

$$\hat{e} = \tilde{\psi} - \psi. \tag{20}$$

With applying this estimation error to controller we expect that tracking loop delay and noise effect decrease prominently.

## 4 Performance Evaluation of the New Control Structure in Tracking Loop

In this section we are going to evaluate the performance of the control structure that was introduced in the last section. In first stage we assume that the value of tracking sensor delay is constant and definite and we will evaluate the ability of the EKF based compensator in compensating the given delay and noise according to this assumption. In the following for creating the condition closer to the reality, we will assume that sensor delay is a time variable and then we will evaluate the performance of the control structure with this assumption.

The evaluations are done with the aid of the simulation in which certain values are allocated to parameters and components of the tracking loop. In the case of discrete model *C(z)* and *G(z)* with *T=0.025 sec* we have the following function

$$C(z)\,G(z) = \frac{0.055z^2 + 0.006z - 0.05}{z^3 - 2.58z^2 + 2.17z - 0.58}. \tag{21}$$

Also regarding the tracking loop input noise we assume that

$$\begin{aligned} & Q(k) = (60T_c)^2 \begin{bmatrix} \frac{1}{3}T_c^2\,I_{3\times3} & \frac{1}{2}T_c\,I_{3\times3} \\ \frac{1}{2}T_c\,I_{3\times3} & I_{3\times3} \end{bmatrix}, \\ & R(k) = 2\ ,\ L(k-1) = 1\ ,\ M(k) = 0. \end{aligned} \tag{22}$$

In this formula $T_c$ is regarded equal to *5T* ( $T_c = 5T$).

### 4.1 Study for constant delay

If constant delay value is supposed $d$ sample, according to Figure 4, delay model $\Delta$ can be considered as formula (23)

$$\Delta = T\,d. \tag{23}$$

The above equation shows that in formulas (17) and (19) parameter $n$ must be set equal to $d$ and EKF based compensator equation is updated each $T$ second. For *d=4* and *T=0.025 sec*, tracking error is estimated according to Figure 5 by means of compensator. As you can see in this figure, EKF based compensator is carefully accomplished compensating for noise and delay effect.

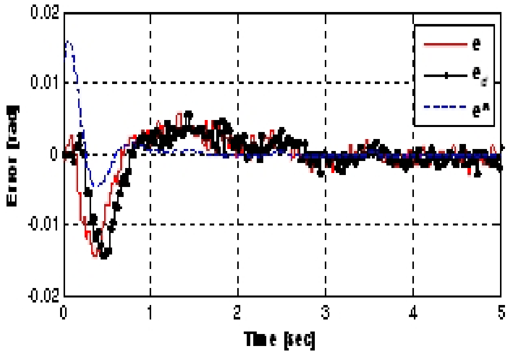


**Figure 5**: Comparing error signal of tracking loop with compensator estimation error.

Therefore system response to ramp input (target with constant speed) will be according to Figure 6. In this figure, three cases are supposed to show the value of compensator power. First, to the to tracking loop, that a controller is designed for *0.1sec.* delay is applied, that according to Figure 6-(a) means the loop is unstable. In Figure 6-(b) and Figure 6-(c) ideal loop (delay free) behavior is compared with delay and compensator applied loop. Figure 6-(b) shows response to ramp input in two cases and in comparison with another.

Also in Figure 6-(c) output error (difference between input and output of loop) is shown. According to the presented results you see that the designed compensator, besides catering proper stability margin, could eliminate noise effect truly.

### 4.2 Study for time variable delay

Now assume the considered delay is variable and the following function

$$\Delta(r) = r(k - \delta(k)) \;\;,\;\; \delta(k) \in \{D_{\min}, ..., D_{\max}\}. \tag{24}$$

According to this formula the upper and lower bound of variable delay is assumed definite and is shown with $D_{max}$ and $D_{min}$ .

In this situation a conservative selection for parameter $n$ is the upper bound of delay. Thus, in compensator considered equation (25)

$$n = D_{\max}. \tag{25}$$

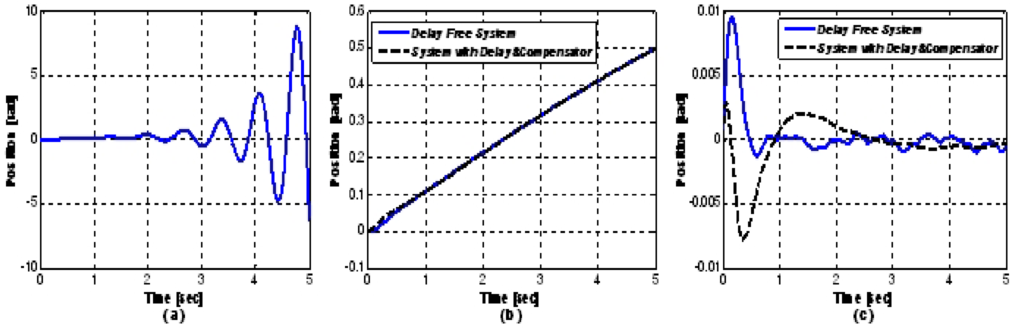


**Figure 6**: Simulation results for constant delay.

Another offer for selection of $n$ in compensator related equations is as following.

$$n = \left[ \frac{D_{\max} + D_{\min}}{2} \right], \tag{26}$$

where *[]* is integer fraction. With this choice, it is adequate to consider integer fraction of mean of delay bounds in designing of compensator. Then, the estimated error will be going to zero and tracking loop output is going to desired value in steady state.

Studies have shown that the second method is better for selection of $n$ and obtains better response. In Figure 7 the estimated error signal is compared in both of the above methods. According to this figure in the second method the estimated error has a lower oscillation magnitude.

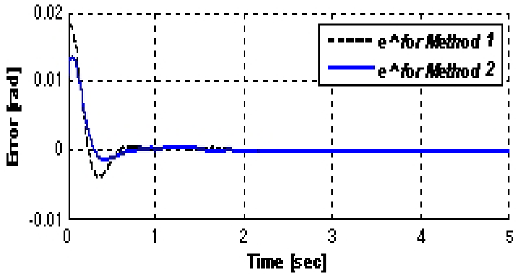


**Figure 7**: Comparison of the estimated error signal for $n$ selection in both methods.

In Figure 8 tracking loop simulation results in variable delay case and for $D_{min}=2$, $D_{max}=5$ and thus $n=3$ according to formula (26) have been shown. In Figure 8-(a) you see that for applied delay, output is unstable whereas Figures 8-(b) and 8-(c) show that with the aid of EKF based compensator a near to ideal response is obtained. However the difference between input and output of tracking loop in compensator applying state depreciates in longer time, but in less than *3 sec* with well precision goes to zero. This subject can be seen clearly in Figure 8-(c).

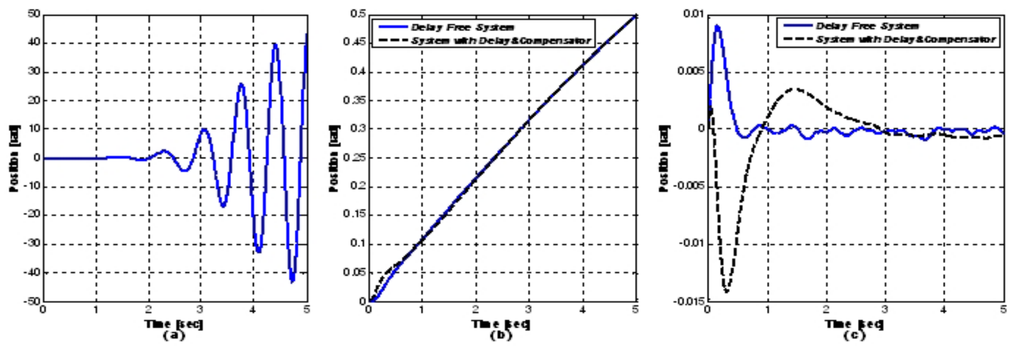


**Figure 8**: Simulation results for variable time delay.

Another remarkable note is that designed control structure is responsible for acceleration input tracking. In fact, however usually tracking loop is designed for constant speed input, but ability of acceleration input tracking is a significant and notable subject.

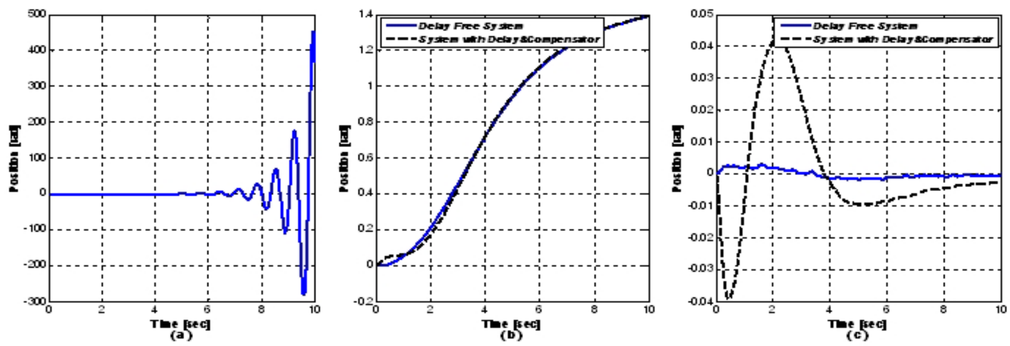


**Figure 9**: Simulation results for acceleration input.

Figure 9 depicts results for applying an acceleration input to tracking loop. Likewise Figure 9-(a) shows that with existence of tracking sensor delay, loop is unstable. But Figure 9-(b) shows that existence of EKF based compensator does not reduce ability of controller in acceleration input tracking. In Figure 9-(c), with comparison of error signal in ideal state (delay free) and the state when new control structure is applied in addition to ordinary controller, this fact is illustrated clearly. According to this figure, we can say that existence of compensator due to longer transient state response is practical for eliminating noise and delay effect of tracking sensor.

## 5 Conclusion

In this paper a new control structure is presented to compensate destructive effect of delay and noise on performance of tracking loop. We show that in the introduced structure an EKF based compensator with access to position of two degree of freedom gimbal via a position sensor, is able to estimate tracking loop error as effect of tracking sensor delay and noise reduce on output.

A significant feature of the presented structure is that notwithstanding the existence of noise and delay, and with assumption of ideal condition, designing of the tracking loop controller can be done. Then we utilize EKF based compensator to eliminate noise and delay effect without reduction in performance of tracking loop design.

In two sections the performance of the mentioned control structure for constant and time varying delay is evaluated. The obtained results confirmed that EKF based compensator, beside providing appropriate stability margin, has a good performance in reduction of delay and noise effect.

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