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Robust Neural Output Feedback Tracking Control for a Class of Uncertain Nonlinear Systems Without Time-delay

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Abstract: This paper investigates the problem of adaptive tracking control by output feedback for a class of uncertain nonlinear systems. These nonlinear systems are subjected to various structured and unstructured uncertainty due essentially to modelling errors, parameter variations and unmodelled dynamics. With the help of error signals generated by the simple linear observer, a radial basis function neural network (RBF NN) is established to approximately compensate on line for these uncertainties. In this note, the neural network operates over system input/output signals without time delay. The stability analysis and tracking performance of the closed-loop system are confirmed through Lyapunov stability theory. The potential of the theoretical results is demonstrated through computer simulations of both nonlinear systems, Van der Pol and tunnel diode circuit.

Keywords: *nonlinear systems; feedback control; perturbations; adaptive or robust stabilization; neural nets and related approaches; stability; simulation.*

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1 Introduction

In practical engineering, a large range of physical systems and devices, such as electromagnetism, mechanical actuators, electronic relay circuits and chaotic systems possess nonlinear and uncertain characteristics [8, 18]. On the other hand, the magnitude of control signal is always limited due to the poorly modelled dynamics of these systems, i.e., for most practical processes, obtaining an exact model is a difficult task or is not possible at all [6]. Therefore, modelling errors, unmodelled dynamics and uncertain parameter variations should be explicitly considered in the control design to enhance robust control performance. If these uncertainties (referred to as inversion errors) are ignored in the control design, the closed-loop control performance will be strongly damaged, and instability may occur. Thus, it is very important to develop powerful robust control techniques for nonlinear systems subjected to high uncertainty.

In recent years, there has been growing attention paid to the control problems of uncertain systems [5,8,26]. As is well known, various adaptive state feedback and output feedback controls have been known as efficient algorithms for designing feedback controllers for a large class of nonlinear systems in the presence of uncertainties [1–3,6,16,20]. These algorithms are expected to exhibit more excellent performance in order to have its outputs track given reference signals. In the same area, [20] discusses backstepping-based approaches to adaptive output feedback control of uncertain systems that are linear with respect to unknown parameters. For systems in which nonlinearities depend only upon the available measurement, [23] and [16] give a solution to the output feedback stabilization problem. In brief, the controller designs and stability analysis of highly uncertain nonlinear dynamic systems have been an important research topic. Unfortunately, the majority of the existing references are deterministic since the exact models are not available and/or their parameters are not precisely known, which prevent the error signals from tending to zero [6].

Recent years have witnessed advances in approximation of high nonlinearity by incorporating neural networks (NNs) and fuzzy logic systems (FLSs) in the control design to achieve excellent tracking performances. Taking advantage of this fact, these intelligent techniques have been widely employed for nonlinear control and identification since they can approximate any nonlinear functions without a priori knowledge of system dynamics [6]. With the help of FLSs and NNs, many approximator based adaptive control approaches were proposed for uncertain nonlinear systems; see, for example, [10, 19, 21, 22, 25, 26] and references therein. In [21, 22, 25], adaptive fuzzy or NN state feedback control schemes for a class of single-input single-output (SISO) nonlinear systems without or with time delays are developed; in [10, 19], adaptive output feedback controllers for SISO nonlinear systems are developed without unmeasured states, while the adaptive fuzzy or NN decentralized output feedback stabilization problem for a class of nonlinear systems is discussed in [26]. [20] proposes a robust adaptive output-feedback controller based on the small-gain theorem in order to overcome the effect of the unmodelled dynamics involved in the considered uncertain systems, whereas a RBF NN augmented backstepping controller for the nonlinear system dynamics is applied in [4] to gain from the approximation ability of NNs and ensure the stability of the closed loop system by an augmented Lyapunov function. Thus, authors in [1, 2, 5] augment adaptive output feedback linearization control using single hidden layer NNs in order to overcome the effect of uncertain parameter and unmodelled dynamics for highly uncertain nonlinear systems, and excellent tracking performances were achieved. With the aid of NN techniques, [27] presents a novel robust adaptive trajectory linearization control (RATLC) method for a class of uncertain nonlinear systems, in which RBF NNs are introduced to approximate the uncertainties online from available measurements. In [3], first, an adaptive neural network (NN) state-feedback controller for a class of nonlinear systems with mismatched uncertainties is proposed. Then, a bound of unknown nonlinear functions is approximated using RBFNNs so that no information about the upper bound of mismatched uncertainties is required.

Moreover, in most real cases, the state variables are unavailable for direct online measurements, and merely input and output of the system are measurable. Therefore, estimating the state variables by observers plays an important role in the control of processes to achieve better performances. During the past several decades, many nonlinear observers have been developed to obtain the estimated states. Thus, [24] and [17] present an output feedback control using a high-gain observer that is applied to estimate the unmeasurable states of the nonlinear systems. A sliding mode observer is proposed in [9] for a class of nonlinear systems to achieve finite time convergence for all error states. Notice that this previous observer makes use of fractional powers to reduce other non-output errors to zero in finite time. For a special class of single-output nonlinear systems, [15] has developed a sliding mode high-gain observer for state and unknown input estimations, so that the disturbance can be estimated from the sliding surface by ensuring the observability of the unknown input with respect to the output. However, these conventional nonlinear observers, such as high-gain observers [17, 24], and sliding mode observers [9, 15] are only applicable to systems with specific model structures.

Recently, observer-based adaptive fuzzy-neural control schemes are proposed for a large class of uncertain nonlinear dynamical systems. [11] proposes an indirect adaptive fuzzy neural network controller with state observer and supervisory controller for a class of uncertain nonlinear dynamic time-delay systems, in which the free parameters of the indirect adaptive fuzzy controller can be tuned on-line by observer based output feedback control law and adaptive laws by means of Lyapunov stability criterion. A novel state and output feedback control law that are developed for the tracking control of a class of multi-input-multi-output (MIMO) continuous time nonlinear systems with unknown dynamics and disturbance input can be found in [23], in which a high-gain observer is utilized to estimate the unmeasurable system states and an output feedback based controller is designed.

In the present paper, we contribute to design only one robust adaptive output feedback controller augmented using a RBF NN to handle uncertainties that exist in two switched SISO nonlinear systems. In the simple strategy followed in this work, first, we involve feedback linearization. Then, we design the adaptive control signal coupled with the robustifying term to compensate adaptively for inversion errors. A vector, that contains a linear combination of the tracking error generated by the linear observer and the compensator states, is exploited in the adaptation laws for the NN parameters. Furthermore, input/output data of the considered systems (without time-delay) is employed as a teaching signal for the NN. Consequently, the obtained robust control scheme not only guarantees the stability of the closed-loop system, but also has strong robustness to uncertainties existing in both nonlinear systems. Computer simulations of switched nonlinear systems, Van der Pol example having fourth-order nonlinear system of relative degree two and tunnel diode circuit model having full relative degree, are used to demonstrate the effectiveness of the proposed approach.

The rest of this paper is organized as follows. First, the system description and control problem are introduced in the next section. Then, the control structure is well detailed in Section 3. Section 4 develops a robust adaptive controller, in which NN augmentation is discussed. In Section 5, faithful stability analysis is elaborated to guarantee the boundedness of the tracking error signals. The efficiency of the proposed control approach is revealed throughout computer simulation in Section 6.

2 Problem Formulation

Let the dynamics of an observable uncertain SISO system be given as follows

$$\begin{aligned} \dot{x} &= f(x, u), \\ y &= h(x), \end{aligned} \tag{1}$$

where $x \in \mathfrak{R}^{\mathbf{n}}$ is the state of the plant, $u \in \mathfrak{R}$, and $y \in \mathfrak{R}$ are the control and measurement, respectively.

Assumption 1. The functions $f : \mathfrak{R}^{n+1} \longrightarrow \mathfrak{R}^n$ and $h : \mathfrak{R}^n \longrightarrow \mathfrak{R}$ are partially known, and the dynamical system of (1) satisfies the output feedback linearization conditions [14] with relative degree r for all $(x, u) \in \Omega \times \mathfrak{R}$ where $\Omega \subset \mathfrak{R}^n$. Moreover, n need not to be known. Therefore, there exists a mapping that transforms the system in (1) into the so-called normal form [12]:

$$\begin{aligned} \dot{\xi}_i &= \xi_{i+1}, \quad i = 1, ..., r-1, \\ \dot{\xi}_r &= h(\xi, u), \\ \xi_1 &= y, \end{aligned} \tag{2}$$

where $h(\xi, u) = L_f^{(r)} h$ are the Lie derivatives, and $\xi = [\xi_1 \quad ... \quad \xi_r]^T$.

The key objective is to design a robust neural output feedback tracking control that utilizes the available measurement y, so that $y(t)$ tracks a reference trajectory $y_{ref}(t)$ with bounded error.

3 Controller Design

3.1 Feedback linearization

Approximate feedback linearization is performed by defining the following control input signal:

$$v = \widehat{h}^{-1}(y, u), \tag{3}$$

where v is a pseudo-control. The function $\widehat{h}(y, u)$ represents the best available approximation of $h(y, u)$. Then, the system dynamics can be formulated as

$$y^{(r)} = v + \vartheta, \tag{4}$$

where

$$\vartheta(\xi, v) = h(\xi_1, \widehat{h}^{-1}(\xi_1, v)) - \widehat{h}(\xi_1, \widehat{h}^{-1}(\xi_1, v)) \tag{5}$$

is the inversion error. Note that the pseudo-control mentioned in (4) is chosen to have the form

$$v = y_{ref}^{(r)} + L_d^c - V_c^s + R_t, \tag{6}$$

where $y_{ref}^{(r)}$ is the r^{th} derivative of the input signal y_{ref} generated by a stable command filter, L_d^c is the output of a linear dynamic compensator, V_c^s and R_t, namely adaptive control signal and robustifying term, are designed to overcome ϑ.

With (6), the dynamics in (4) will be expressed as follows

$$y^{(r)} = y_{ref}^{(r)} + L_d^c - V_c^s + R_t + \vartheta. \tag{7}$$

From (5), notice that ϑ depends on V_c^s and R_t through v, whereas $V_c^s - R_t$ has been designed to approximately cancel ϑ.

3.2 Linear Dynamic Compensator Design and Tracking Error Dynamics

The output tracking error is defined as $e = y_{ref} - y$. Then the dynamics in (7) can be rewritten as

$$e^{(r)} = -L_d^c + V_c^s - R_t - \vartheta. \tag{8}$$

Note that the adaptive term coupled with the robustifying term $V_c^s - R_t$ are not required when $\vartheta = 0$. Consequently, the error dynamics in (8) reduces to

$$e^{(r)} = -L_d^c. \tag{9}$$

The following linear compensator is introduced to stabilize the dynamics in (9):

$$\begin{cases} \dot{\lambda} = A_q\lambda + b_q e, \\ L_d^c = c_q\lambda + d_q e, \quad \lambda \in \Re^{r-1}. \end{cases} \tag{10}$$

Note that λ needs to be at least of dimension $(r-1)$ [7]. This follows from the fact that (9) corresponds to error dynamics that has r poles at the origin. One could elect to design a compensator of dimension $\geq$ r as well. In the future, we will assume that the minimum dimension is chosen.

Returning to (8), notice that the vector $e_r = [e \quad \dot{e} \quad ... \quad e^{(r-1)}]^T$ mutually with the compensator state λ will obey the following dynamics, referred to as tracking error dynamics:

$$\begin{cases} \dot{E} = A_k E + b_k[V_c^s - R_t - \vartheta], \\ z = C_k E, \end{cases} \tag{11}$$

where z is the vector of available measurements.

Remind that

$$A_k = \begin{bmatrix} A - d_q bc & -bc_q \\ b_q c & A_q \end{bmatrix}, \; b_k = \begin{bmatrix} b \\ 0 \end{bmatrix}, \; c_k = \begin{bmatrix} c & 0 \\ 0 & I \end{bmatrix} \tag{12}$$

and a new vector

$$E_d = \begin{bmatrix} e_r^T & \lambda^T \end{bmatrix}^T, \tag{13}$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T.$$

Note that A_q, b_q, c_q and d_q in (10) should be designed such that A_k is Hurwitz.

3.3 Observer Design for the Error Dynamics

Lyapunov-like stability analysis of the error dynamics results in update laws for the adaptive control parameters in terms of (E) for the full-state feedback application [2, 5]. In [12] and [13], adaptive state observers are used to provide the necessary estimates in the adaptation terms. In the present paper, we propose a simple linear observer for the tracking error dynamics in (11), and confirm through Lyapunov's direct method that the adaptive part of the control signal coupled with the robustifying term $(V_c^s - R_t)$ cancels successfully the inversion error (ϑ), if the output of this observer is introduced as an error signal for the adaptive laws. Moreover, a minimal-order observer of dimension $(r-1)$ may be designed for the dynamics in (11).

In what follows, we consider the case of a full-order observer of dimension $(2r-1)$ [12]. To this end, consider the following simple linear observer for the tracking error dynamics in (11):

$$\begin{cases} \dot{\widehat{E}} = A_k \widehat{E} + K(z - \widehat{z}), \\ \widehat{z} = C_k \widehat{E}, \end{cases} \tag{14}$$

where K is a gain matrix, and z that is defined in (11) should be chosen such that $(A_k - KC_k)$ is asymptotically stable.

Let

$$\widetilde{A} = A_k - KC_k, \quad \widetilde{E} = \widehat{E} - E, \quad \widetilde{z} = \widehat{z} - z. \tag{15}$$

Then, the observer error dynamics can be written as

$$\begin{cases} \dot{\widetilde{E}} = \widetilde{A}\widetilde{E} - b_k[V_c^s - R_t - \vartheta], \\ \widetilde{z} = c_k \widetilde{E}. \end{cases} \tag{16}$$

4 RBF NN Augmented Controller

4.1 NN approximation

Following [12], given a compact set $\mathcal{D} \subset R^{n+1}$ and $\epsilon^* > 0$, the model inversion error $\vartheta(\xi, v)$ can be approximated over $\mathcal{D}$ by a radial basis function neural network (RBF NN)

$$\vartheta(\xi, v) = M^T \phi(\varrho) + \epsilon(d, \varrho), \quad |\epsilon| < \epsilon^*, \tag{17}$$

using the input vector

$$\varrho(t) = [v \quad y]^T \in \mathcal{D}, \quad \|\varrho\| \le \varrho^*, \quad \varrho^* > 0. \tag{18}$$

The adaptive signal is designed as follows

$$V_c^s = \widehat{M}^T \phi(\widehat{\varrho}), \tag{19}$$

where $\widehat{M}$ is the estimate of M that is updated according to the following adaptation law:

$$\dot{\widehat{M}} = -\beta_M[2\phi(\widehat{\varrho})\widehat{E}^T P b_k + \alpha_M(\widehat{M} - M_0)] \tag{20}$$

in which M_0 is the initial value of M, P is the solution of the Lyapunov equation

$$A_k^T P + P A_k = -Q \tag{21}$$

for some $Q > 0$, $k > 0$, β_M is the adaptation gain matrix, and $\widehat{\varrho}$ is an implementable input vector to the NN on the compact set $\Omega_{\widehat{\varrho}}$, defined as $\widehat{\varrho} = [v^T(t) \quad \widehat{y}^T(t)]^T \in \Omega_{\widehat{\varrho}}$, $\widehat{y}_i = \widehat{E}_i + y_{ref}^{(i-1)}, i = 1, ..., r-1$.

Notice that in (19), there is an algebraic loop, since $\widehat{\varrho}$, by definition, depends upon V_c^s through v, see (18). However, with bounded squashing functions, this algebraic loop has at least one fixed-point solution as long as $\phi(.)$ is made up of bounded basis functions.

The robustifying term is designed as follows

$$R_t = \widehat{\Psi} sgn(2\widehat{E}^T P b_k), \tag{22}$$

where the adaptive gain $\widehat{\Psi}$ is updated according to the following adaptation law

$$\dot{\widehat{\Psi}} = -\beta_\Psi[2\widehat{E}^T P b_k sgn(2\widehat{E}^T P b_k) + \alpha_\Psi(\widehat{\Psi} - \Psi_0)]. \tag{23}$$

in which Ψ_0 is an initial value of $\widehat{\Psi}, \beta_\Psi > 0, \alpha_\Psi > 0$.

Using (17) and (19), we can write the mismatch between the adaptive signal and the real NN as:

$$V_c^s - \vartheta = \widehat{M}^T \phi(\widehat{\varrho}) - M^T \phi(\varrho) - \epsilon = \widetilde{M}^T \widehat{\phi} + M^T \widetilde{\phi} - \epsilon, \tag{24}$$

where $\widetilde{M} = \widehat{M} - M, \widehat{\phi} = \phi(\widehat{\varrho}), \widetilde{\phi} = \phi(\widehat{\varrho}) - \phi(\varrho)$.

Using (24), the error dynamics in (11) and the observer error dynamics in (16) can be reformulated as

$$\dot{E} = A_k E + b_k[\widetilde{M}^T \widehat{\phi} + M^T \widetilde{\phi} - \epsilon - \widehat{\Psi} sgn(2\widehat{E}^T P b_k)], \tag{25}$$

$$\dot{\widetilde{E}} = \widetilde{A}\widetilde{E} + b_k[\widetilde{M}^T \widehat{\phi} + M^T \widetilde{\phi} - \epsilon - \widehat{\Psi} sgn(2\widehat{E}^T P b_k)]. \tag{26}$$

Notice that for radial basis function and many other activation functions that satisfy $|\phi_i| \leq 1, i = 1, ..., N$, there exists an upper bound over the set $\mathcal{D}$

$$\|\phi(\varrho)\| \leq \varpi, \quad \varpi = \max_{\varrho \in \mathcal{D}} \|\phi(\varrho)\|, \tag{27}$$

where ϖ remains of the order one, even if N is large. With this, we have the following upper bound:

$$|M^T \widetilde{\phi}| \leq 2\|M\|\varpi. \tag{28}$$

5 Stability Analysis

We confirm through Lyapunov's direct method that if the initial errors of the variables $E^T, \widetilde{E}^T, \widetilde{E}, \widehat{M}^T$ and $\widetilde{\Psi}$ belong to a presented compact set, then the composite error vector $\zeta = \begin{bmatrix} E^T & \widetilde{E}^T & \widehat{M}^T & \widetilde{\Psi} \end{bmatrix}^T$ is ultimately bounded, where $\widetilde{\Psi} = \widehat{\Psi} - \Psi$ and $\Psi = \epsilon^* + 2\varpi\|M\|$. Notice that ζ can be viewed as a function of the state variables $y, \lambda, \widehat{E}, \widehat{Z}$, the command vector y_{ref}, and a constant vector Z

$$\zeta = F\big(y, \lambda, \widehat{E}, \widehat{Z}, y_{ref}, Z\big), \tag{29}$$

where $\widehat{Z} = [\widehat{M}^T \quad \widehat{\Psi}]^T$, $Z = [M^T \quad \Psi]^T$. The relation in(29) represents a mapping from the original domains of the arguments to the space of the error variables

$$F : \Omega_y \times \Omega_{y_{ref}} \times \Omega_\lambda \times \Omega_{\widehat{E}} \times \Omega_{\widehat{Z}} \times \Omega_Z \longrightarrow \Omega_\zeta. \tag{30}$$

Recall that (18) introduces the compact set $\mathcal{D}$ over which the NN approximation is valid. From (18), it follows that

$$\varrho \in \mathcal{D} \Longleftrightarrow y \in \Omega_y, \quad v \in \Omega_v. \tag{31}$$

Also, notice that, since the observer in(14) is driven by the output tracking error $e = y_{ref} - y$ and compensator state λ, having $y \in \Omega_y$, $y_{ref} \in \Omega_{y_{ref}}$, $\lambda \in \Omega_\lambda$, implies that $\widehat{E} \in \Omega_{\widehat{E}}$, the latter being a compact set. According to (6)

$$v = F_v\big(\lambda, \widehat{E}, \widehat{Z}, y_{ref}\big), \tag{32}$$

where $F_v : \Omega_\lambda \times \Omega_{\widehat{E}} \times \Omega_{\widehat{Z}} \times \Omega_{y_{ref}} \longrightarrow \Omega_v$.

Thus, (29), (31) and (32) ensure that Ω_ζ is a bound set. Introduce the largest ball, which is included in Ω_ζ in the error space

$$L_B = \big\{|\zeta| \|\zeta\| \leq R\big\}, \quad R > 0. \tag{33}$$

For every $\zeta \in L_B$, we have $\varrho \in \mathcal{D}, Z \in \Omega_Z$, where both $\mathcal{D}$ and Ω_Z are bounded sets.

Assumption 2. Assume

$$R > \gamma\sqrt{\frac{T_M}{T_m}} \geq \gamma. \tag{34}$$

where T_M and T_m are the maximum and minimum eigenvalues of the following matrix

$$T = \tfrac{1}{2}\begin{bmatrix} 2P & 0 & 0 & 0 \\ 0 & 2P & 0 & 0 \\ 0 & 0 & \beta_M^{-1} I & 0 \\ 0 & 0 & 0 & \beta_\Psi^{-1} \end{bmatrix} \tag{35}$$

and

$\gamma = \max\Big(\sqrt{\frac{4(\Theta\Psi)^2 + \overline{Z}}{\alpha_{\min}(Q) - 2}}, \sqrt{\frac{4(\Theta\Psi)^2 + \overline{Z}}{\alpha_{\min}(\widetilde{Q}) - 2}}, \sqrt{\frac{4(\Theta\Psi)^2 + \overline{Z}}{\rho}}$, where $\overline{Z} = \frac{\alpha_M}{2}\|M - M_0\|^2 + \frac{\alpha_\Psi}{2}|\Psi - \Psi_0|^2, \Theta = \|Pb_k\| + \|\widetilde{P}b_k\|$, $\rho = \alpha - \Theta^2(\varpi + 1)^2 > 0$, $\alpha = \frac{1}{2}\min(\alpha_M, \alpha_\Psi)$ and $\widetilde{P}$ satisfies $\widetilde{A}^T\widetilde{P} + \widetilde{P}\widetilde{A} = -\widetilde{Q}$ for some $\widetilde{Q} > 0$ with minimum eigenvalues $\alpha_{min}(\widetilde{Q}) > 2$.

Theorem 1. *Let the assumption (1) hold, and let $\alpha_{min}(Q) > 2$ for Q introduced in (21). Then, if the initial errors belong to the set Ω_α defined in (37), the feedback control laws given by (3) and (6), along with adaptation laws (20) and (23) ensure that the error signals $E, \widetilde{E}$, $\widetilde{M}$ and $\widetilde{\Psi}$ in the closed-loop system are ultimately bounded.*

Proof. Take into account the following Lyapunov function:

$$V = E^T PE + \widetilde{E}^T \widetilde{P}\widetilde{E} + \frac{1}{2}\widetilde{M}^T \beta_M^{-1}\widetilde{M} + \frac{1}{2}\widetilde{\Psi}^T \beta_\Psi^{-1}\widetilde{\Psi}. \tag{36}$$

The derivative of V along the tracking error dynamics(25), the observer error dynamics (26), NN weight and adaptive gain adaptation laws (20) and (23) can be formulated as

$$\begin{aligned}\dot{V} = &-E^T PE - \widetilde{E}^T \widetilde{Q}\widetilde{E} - 2\widetilde{E}^T(\widetilde{P}+P)b_k[\widetilde{M}^T\widehat{\phi} + M^T\widetilde{\phi} - \epsilon - \widehat{\Psi}sgn(2\widehat{E}^T Pb_k)] \\ &- 2\widetilde{E}^T Pb_k[\epsilon - M^T\widetilde{\phi} + \Psi sgn(2\widehat{E}^T Pb_k)] - [\alpha_M \widetilde{M}^T(\widehat{M} - M_0)] - \widetilde{\Psi}\alpha_\Psi(\widehat{\Psi} - \Psi_0),\end{aligned}$$

where $\widetilde{E} = \widehat{E} - E, \widehat{\Psi} = \Psi + \widetilde{\Psi}$. Using the following property for vectors $[\widetilde{M}^T(\widehat{M} - M_0)] = \frac{1}{2}\|\widetilde{M}\|^2 + \frac{1}{2}\|\widehat{M} - M_0\|^2 - \frac{1}{2}\|M - M_0\|^2$, and with (28), the upper bound becomes [13]

$$\dot{V} \le -[\alpha_{min}(Q) - 2]\|\widetilde{E}\|^2 - [\alpha_{min}(\widetilde{Q}) - 2]\|E\|^2 - [\alpha - \Theta^2(\varpi + 1)^2]\|\widetilde{Z}\|^2 + \overline{Z} + 4(\Theta\Psi)^2.$$

Either of the following conditions:
$\|\widetilde{E}\| > \sqrt{\frac{4(\Theta\Psi)^2+\overline{Z}}{\alpha_{\min}(Q)-2}}, \|E\| > \sqrt{\frac{4(\Theta\Psi)^2+\overline{Z}}{\alpha_{\min}(\widetilde{Q})-2}}, \|\widetilde{Z}\| > \sqrt{\frac{4(\Theta\Psi)^2+\overline{Z}}{\rho}}$ will render $\dot{V} < 0$ outside a compact set: $B_\gamma = \{\zeta \in L_B, \|\zeta\| \le \gamma\}$.

Note from (34) that $B_\gamma \subset L_B$. Then, consider the Lyapunov function candidate in (36) and write it as: $V = \zeta^T T\zeta$. Let Υ be the maximum value of the Lyapunov function V on the edge of B_γ: $\Upsilon = \max_{\|\zeta\|=\gamma} V = \gamma^2 T_M$. Introduce the level set $\Omega_\gamma = \{\zeta V \le \Upsilon\}$. Let α_v be the minimum value of the Lyapunov function V on the edge of L_B: $\alpha_v = \min_{\|\zeta\|=R} V = R^2 T_m$. Define the level set

$$\Omega_\alpha = \{\zeta \in L_B, V = \alpha_v\}. \tag{37}$$

Consequently, the condition in (34) guarantees that $\Omega_\gamma \subset \Omega_\alpha$, and thus ultimate boundedness of ζ.

6 Application

This paper addresses the design of a robust adaptive controller augmented using a NN to handle the uncertainty of two switched nonlinear systems: Van der Pol model having a fourth-order nonlinear system of relative degree two and the tunnel diode circuit example with full relative degree. This part is devoted to illustrating the performance of the proposed approach. First, we present the dynamics of the considered uncertain systems:

6.1 Tunnel diode circuit model

$$\begin{cases} \dot{x}_1 = \dfrac{1}{C}x_2 - \dfrac{1}{C}h(x_1), \\ \dot{x}_2 = -\dfrac{R}{L}x_2 - \dfrac{1}{L}x_1 + \dfrac{u}{L}, \end{cases} \tag{38}$$

where x_1 the voltage across the capacitor C and x_2 is the current through the inductor L. The initial conditions were set as $x_1(0) = 0.1, x_2(0) = 0.0005$, and the element values of the circuit are $R = 1.5k\Omega, L = 1nH$, and $C = 2pF$. Notice that the function $h : \mathfrak{R} \longrightarrow \mathfrak{R}$ represents the characteristic curve of the tunnel diode, $h(x_1) = x_1 + 2x_1^2 + x_1^3 - x_1^4 - 2x_1^5$. We assume that the output y has a full relative degree of $n = r = 2$.

6.2 Van der Pol model

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -0.2(x_1^2 - 1)x_2 - 0.2x_3 + \dfrac{u}{\sqrt{|u| + 0.1}}, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = -0.2x_4 - x_2 + x_1, \end{cases} \tag{39}$$

with initial conditions $x_1(0) = 0.5, x_2(0) = 1.5, x_3(0) = 0$ and $x_4(0) = 0$. The output y has a relative degree of $r = 2$.

The command signals y_{ref} and $y_{ref}^{(2)}$ are generated through a second -order command filter with natural frequency of $1rad/s$ and damping of 0.7. The following dynamic compensator:

$$\begin{cases} \dot{\lambda} = -6.4\lambda + 4e, \\ L_d^c = -18.2\lambda + 13.04e, \end{cases} \tag{40}$$

places the poles of the closed-loop error dynamics in (9) of both nonlinear systems at $-3.6, -1.4 \pm j$. The observer dynamics in (16) was designed so that its poles are four times faster than those of the error dynamics. A radial basis function NN with five neurons was used in the adaptive control. The functional form for each RBF neuron was defined by

$$\phi_i(\varrho) = e^{-(\varrho - \kappa_{c_i})^T(\varrho - \kappa_{c_i})/\sigma^2}, \quad \sigma = 1, \quad i = \overline{1, 6}. \tag{41}$$

The centers κ_{c_i}, $i = \overline{1, 6}$, were arbitrarily selected over a grid of possible values for the vector ϱ. The adaptation gains were set to $\beta_M = 1.2$, with sigma modification gain $\alpha_M = 0.001$. The other parameters are : $\alpha_\Psi = 0.012, \beta_\Psi = 0.0015$.

In this paper, we contribute to design one robust adaptive control scheme augmented using a RBF NN in order to make up adaptively for the nonlinearities that exist in both uncertain systems (Van der Pol and tunnel diode circuit model). Therefore, the designed controller forces the system response to track a given reference trajectory with bounded errors. First, set the output $y = x_1$ for each system. Then, we employ feedback linearization, coupled with an on-line NN to handle the inversion errors, according to the equation (7). The dynamic compensator, described in (10) and (40), is designed to stabilize the linearized systems [1,2]. A signal, constituted of a linear combination of the measured tracking error and the compensator states is used to adapt the control laws, such as presented in (20), (22) and (23).

Figure 1 compares the system measurement y without NN augmentation (dashed line) with the reference model output y_{ref} (solid line), clearly demonstrating the almost unstable oscillatory behavior caused by the nonlinear elements (ϑ) in the Van der Pol model in the first half time (0 to 50 seconds) and the nonlinearities of the tunnel diode equation in the last half time (50 to 100 seconds). Meanwhile, with the aid of NN augmentation, Figure 2 shows that the effect of these nonlinearities is successfully eliminated. This is

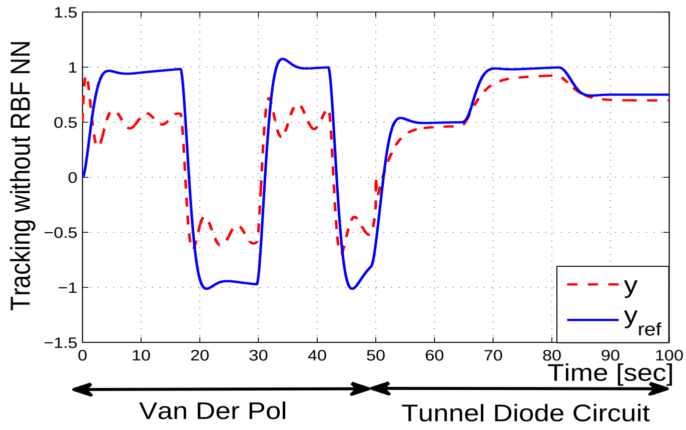

Figure 1: Tracking without RBF NN.

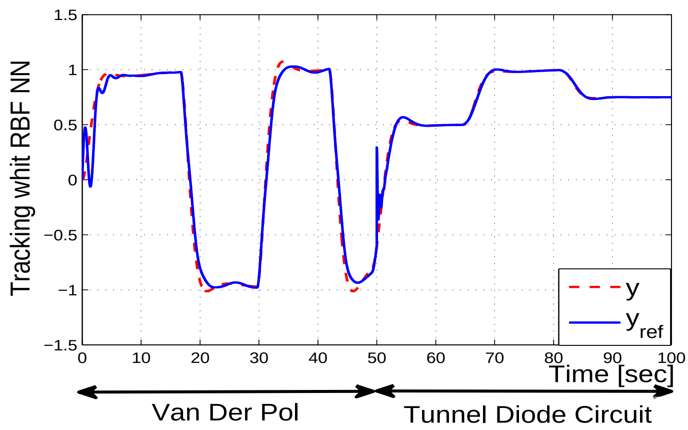

Figure 2: Tracking with the aid of RBF NN.

due essentially to the excellent identification of the model inversion error (ϑ) (dashed line) by adaptive control signal and robustifying term ($V_c^s - R_t$) (solid line), which is illustrated in Figure 3.

Figure 4 compares the control efforts ($y_{ref} - y$) without and with adaptation, in which the NN based robust adaptive controller exhibits a steady state tracking error.

As expected, the RBF NN improves the tracking performance due to its ability to "model" nonlinearities. Consequently, simulation results show that the NNs augmented robust adaptive output feedback controller compensates successfully for the uncertainties existing in two different nonlinear systems.

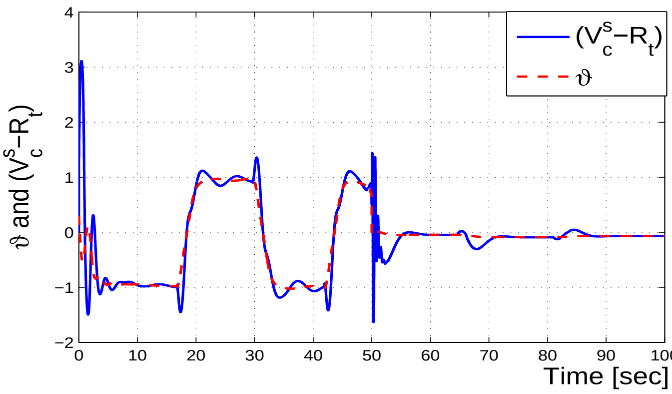

Figure 3: Identification of uncertainties (ϑ) by NN ($V_c^s - R_t$).

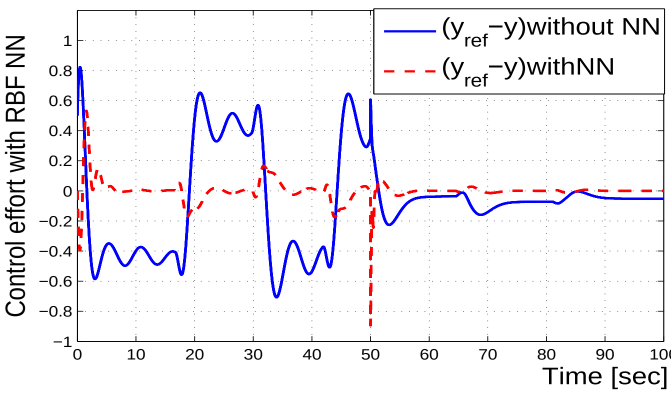

Figure 4: Control effort without and with RBF NN.

7 Conclusion

In this paper, one robust adaptive output feedback control augmented via RBF NN has been designed to overcome the effect of nonlinearities for both highly uncertain nonlinear systems: Van der Pol and Tunnel Diode Circuit. The derivatives of the tracking error are estimated by the simple linear observer. These estimates are used in the adaptation laws for the NN parameters. Ultimate boundedness of the tracking and observation errors are proven using Lyapunov's direct method. The methodology is applicable for observable and stabilizable systems of unknown but bounded dimension when the relative degree is known. Through Lyapunov-based theoretical analysis and computer simulation, we were able to demonstrate that the proposed RBF NN-based robust adaptive output feedback controller was robust to modeling inaccuracies, and excellent tracking performance was succeeded.

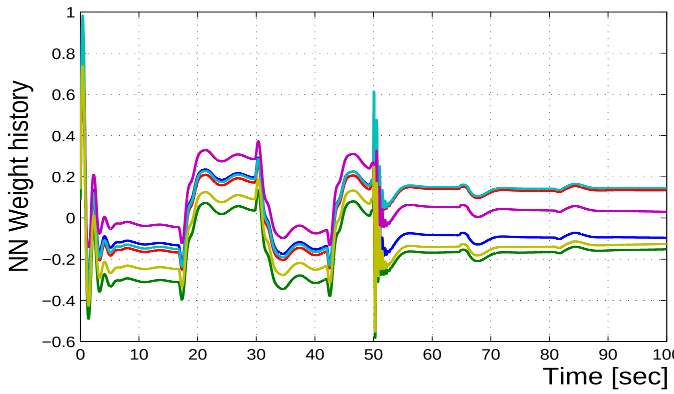

Figure 5: NN weights history.

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