Nonlinear Dynamics and Systems Theory, **19** (3) (2019) 427–443

Comparison of Quadrotor Performance Using Forwarding and PID-Backstepping Control

F. Ouendi [1,3]*, S. Houti [2] and M. Tadjine [1]

[1] *Process Control Laboratory, Department of Electrical Engineering, National Polytechnic School of Algiers, 10 avenue Pasteur, Hassan Badi, BP 182, El Harrach, Algiers, Algeria.*
[2] *University of Versailles Saint Quentin, France.*
[3] *University of Mouloud Mammeri Tizi-Ouzou, Algeria.*

Received: November 25, 2018; Revised: June 29, 2019

Abstract: The purpose of this paper is to discuss a comparative evaluation of the performance of two different controllers, namely a controller based on the forwarding control and a hybrid controller based on the PID-backstepping control in the quadrotor dynamic system which is a sub-system actuated with a high non-linearity. As only four states can be controlled at the same time in the quadrotor, the trajectories are designed on the basis of the four states while the position and the three-dimensional rotation along the axis, called yaw movement, are taken into account.

This paper deals with the forwarding controller and the hybrid controller composed of PID controllers for attitude control and backstepping for controlling the position. The forwarding approach is applied for the nonlinear model of the quadrotor to track the trajectories. Meanwhile the hybrid controller approach for nonlinear model is designed on the basis of a linear model for the PID controller and a nonlinear model for the controller backstepping quadrotor because the performance of the linear model and the nonlinear model around some nominal points is almost similar. Simulink and MATLAB software are used to design the controllers and evaluate the performance of both controllers.

Keywords: *nonlinear systems; feedback control; perturbations; stability; boundedness; simulation.*

Mathematics Subject Classification (2010): 93C10; 93B52; 93C73; 70K20; 34C11.

* Corresponding author: mailto:fatima_ouendi@yahoo.fr

1 Introduction

Quadrotors are flying robots that have been investigated in recent years, this is due to their low manufacturing cost as well as their maneuverability, their ability to execute vertical takeoffs and landings and their large fields of application, both military and civil, and, in particular, when human intervention becomes difficult or dangerous [5]. The quadrotors consist of four rotors; two of these rotors rotate in one direction and the two others in the opposite direction. By varying the rotational speeds of these rotors, the quadrotor can make different movements both in translation and in rotation [9].

The quadrotor is classified in the category of the most complex flying systems given the number of physical effects that affect its dynamics, namely aerodynamic effects, gravity, gyroscopic effects, friction and moment of inertia [7]. This complexity results essentially from the fact that the expression of these effects differs for each flight mode. The operation of the quadrotor is so particular. While varying astutely rotors throttles, it is possible to make it go up/go down, to incline it on the left/right (rolling motion: rotation around the x-axis) or forward /back (pitching motion: rotation around the y-axis) or to make it swivel on itself (yaw motion: rotation around the z-axis) The desired roll and pitch angles are deduced from nonholonomic constraints [6].

Finally, all synthesized control laws are validated by simulations for the complete model [6]. A quadrotor is a dynamic vehicle with four input forces, six output coordinators, highly coupled and unstable dynamics [5]. Hence the design of a control law is an interesting challenge [15]. Based on the dynamical system of UAV, many researchers focus on the stability of the attitude control. This is because the moving of the UAV depends on the torque, and the torque has a strong relationship with the UAV attitude [15]. [4] presented the optimized PID method to control the UAV attitude. [13] proposed the fuzzy-PD controller structure with the purpose of combining the behaviors of several PD controller configurations. [3] developed a hybrid optimal backstepping and adaptive fuzzy control for the quadrotor with time-varying disturbance. [18] proposed the synthesis control method to perform the position and attitude tracking control of the dynamical model of the small quadrotor. [7] has combined the backstepping control with the sliding mode control.

Several linear methods, such as the PID and LQR control methods have been applied to control a quadrotor [5, 9, 10]. Since the quadrotor is a nonlinear system, and for a good performance, the nonlinear control methods have been attempted such as the feedback linearization, sliding mode [19], and backstepping control [2, 9, 15]. In [17] the authors used the backstepping strategy and BP nerual network. [9] proposed a PID cascade control of a quadrotor path tracking problem when velocity and acceleration are small. [20] has developped a forwarding algorithm for designing a tracking controller for a high-order nonlinear system in the presence of bounded unknown disturbances. [8] has developped an adaptive backstepping controller design for a quadrotor with unknown disturbances. [15] proposed a trajectory tracking control method for a quadrotor, by using the backstepping control and the dual-loop cascade control.

This paper presents two control techniques applied to a quadrotor for developing a reliable control system for stabilization and trajectory tracking.

A nonlinear forwarding control technique forces the whole system to be able to drive the quadrotor to the desired trajectory of the Cartesian position and yaw angle [20]. A forwarding control algorithm was proposed to stabilize the desired trajectory of position and attitude. Then, we present a control technique based on the development and the synthesis of a control algorithm based upon a hybrid approach, which combines the backstepping and the PID classic regulation controls to navigate a quadrotor. The backstepping controller was developed to ensure the Lyapunov stability and to follow the desired trajectories [15].

2 Dynamics Models

The quadrotor essentially contains a reticulated rigid structure with four independent rotors propellers launched. Three basic movements are needed to describe all the movements of a quadrotor. The rotational movement about the x-axis is described as a roll motion (φ) and about the y-axis is noted as a pitching motion (θ). The roll motion and the pitching motion can be achieved by balancing the motor speed 2 and 4 and motor 1 and 3, respectively. Lateral and longitudinal acceleration are possible, respectively, by changing the angle φ and θ.

Let $E = \{\overrightarrow{E}_x, \overrightarrow{E}_y, \overrightarrow{E}_z\}$ be the fixed inertial reference, and $B = \{\overrightarrow{B}_x, \overrightarrow{B}_y, \overrightarrow{B}_z\}$ be the reference associated with the center of mass of the quadrotor (see Figure 1).

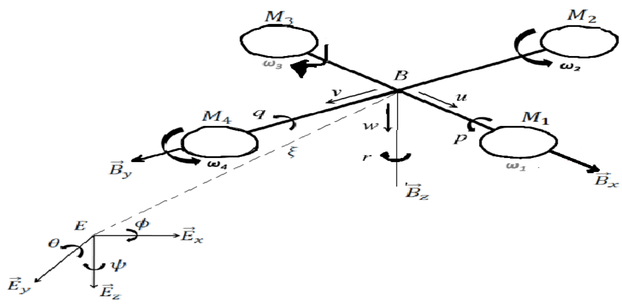

Figure 1: Definition of the frame.

3 The Transformation Matrix

Since it is necessary to deal with two different coordinate systems, namely an inertial marker and a reference linked to the quadrotor's center of mass to explain the position and motion of a quadrotor, a matrix transformation must be used. Here, R is the required transformation matrix that can describe the position and movement of the earth-inertial frame to the fixed frame of the body [9].

The rotation matrix connecting the two marks is defined by the orthogonal matrix: $R(h) : B \to E; R(h) = R_\Psi R_\theta R_\Phi$ in which : C_x and S_x denote $cos(x)$ and $sin(x)$, respectively, and :

$$R_\Psi = \begin{bmatrix} C_\Psi & -S_\Psi & 0 \\ S_\Psi & C_\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ R_\theta = \begin{bmatrix} C_\theta & -S_\Psi & S_\theta \\ 0 & 1 & 0 \\ -S_\theta & 0 & C_\theta \end{bmatrix}, \ R_\Phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\Phi & -S_\Phi \\ 0 & S_\Phi & C_\Phi \end{bmatrix}. \tag{1}$$

So

$$R(h) = \begin{bmatrix} C_\theta C_\Psi & S_\Phi S_\theta C_\Psi - C_\Phi S_\Psi & C_\Phi S_\theta C_\Psi + S_\Phi S_\Psi \\ C_\theta S_\Psi & S_\Phi S_\theta S_\Psi + S_\Phi C_\Psi & C_\Phi S_\theta S_\Psi - S_\Phi C_\Psi \\ -S_\theta & S_\Phi C_\theta & C_\Phi C_\theta \end{bmatrix}. \tag{2}$$

4 Translational Movement

The gravitational force (F_g) and the aerodynamic drag force (F_a) must be introduced to describe the translational motion of a quadrotor while these forces must be overcome by the thrust of the engines (F_{tb}) to achieve any horizontal motion or movement lace. Here, the translational motion of the quadrotor is described by Newton's second law as follows:

$$m \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} + R \begin{pmatrix} 0 \\ 0 \\ -U_1 \end{pmatrix} - k_t \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}. \tag{3}$$

5 Rotary Mouvement

The Newton-Euler method is used to obtain rotational motion equations for the quadrotor [10].

$$I_w = - \begin{pmatrix} p \\ q \\ r \end{pmatrix} \times I \begin{pmatrix} p \\ q \\ r \end{pmatrix} - \begin{pmatrix} p \\ q \\ r \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ I_r w_r \end{pmatrix} + \begin{pmatrix} IU_2 \\ IU_3 \\ IU_4 \end{pmatrix} - k_r \begin{pmatrix} p \\ q \\ r \end{pmatrix}. \tag{4}$$

The state model of the quadrotor is given as follows: $[x \;\; y \;\; z \;\; \dot{x} \;\; \dot{y} \;\; \dot{z} \;\; \Phi \;\; \theta \;\; \Psi \;\; \dot{\Phi} \;\; \dot{\theta} \;\; \dot{\Psi}]^T$. A representation of the state space can be obtained as follows by considering equations (1) for the dynamic model :

$$\begin{cases} \dot{x}_1 = x_4, \\ \dot{x}_2 = x_5, \\ \dot{x}_3 = x_6, \\ \dot{x}_4 = \dfrac{U_1}{m} U_x, \\ \dot{x}_5 = \dfrac{U_1}{m} U_y, \\ \dot{x}_6 = \dfrac{U_1}{m}(Cx_7 Cx_8) - g, \\ \dot{x}_7 = x_{10}, \\ \dot{x}_8 = x_{11}, \\ \dot{x}_9 = x_{12}, \\ \dot{x}_{10} = a_1 x_{11} x_{12} + b_1 U_2 \quad with : a_1 = \dfrac{j_y - j_x}{j_x}, a_2 = \dfrac{j_{rz}}{j_x}, b_1 = \dfrac{d}{j_x}, \\ \dot{x}_{11} = a_3 x_{10} x_{12} + b_2 U_3 \quad with : a_3 = \dfrac{j_z - j_x}{j_y}, a_4 = -\dfrac{j_{rz}}{j_y}, b_2 = \dfrac{d}{j_y}, \\ \dot{x}_{12} = a_5 x_{10} x_{11} + b_3 U_4 \quad with : a_5 = \dfrac{j_x - j_y}{j_z}, b_3 = \dfrac{1}{j_z}, \end{cases} \tag{5}$$

where w_i represents the speed of the engine, $U_x = Cx_7Sx_8Cx_9 + Sx_7Sx_9$ and $U_y = Cx_7Sx_8sx_9 - Sx_7Cx_9$. So, we end up with the following dynamic model:

$$\begin{cases} \dot{x}_1 = x_4, \\ \dot{x}_2 = x_5, \\ \dot{x}_3 = x_6, \\ \dot{x}_4 = \dfrac{U_1}{m}(Cx_7Sx_8Cx_9 + Sx_7Sx_9), \\ \dot{x}_5 = \dfrac{U_1}{m}(Cx_7sx_8sx_9 - sx_7Cx_9), \\ \dot{x}_6 = \dfrac{U_1}{m}(Cx_7Cx_8) - g, \\ \dot{x}_7 = x_{10}, \\ \dot{x}_8 = x_{11}, \\ \dot{x}_9 = x_{12}, \\ \dot{x}_{10} = a_1x_{11}x_{12} + b_1U_2, \\ \dot{x}_{11} = a_3x_{10}x_{12} + b_2U_3, \\ \dot{x}_{12} = a_5x_{10}x_{11} + b_3U_4. \end{cases} \tag{6}$$

The writing of the control inputs according to the rotational speeds of the rotors is as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -Ib & 0 & Ib \\ -Ib & 0 & Ib & 0 \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \\ w_4^2 \end{bmatrix}, \tag{7}$$

where w_i represents the speed of the rotor i. Notice that from the equation of u_x and u_y we find $\Phi_d = arcsin(U_x sin(\Psi_d) - U_y cos(\Psi_d))$ and $\theta_d = arcsin(U_x cos(\Psi_d) + U_y sin(\Psi_d))/(cos(\Phi_d))$.

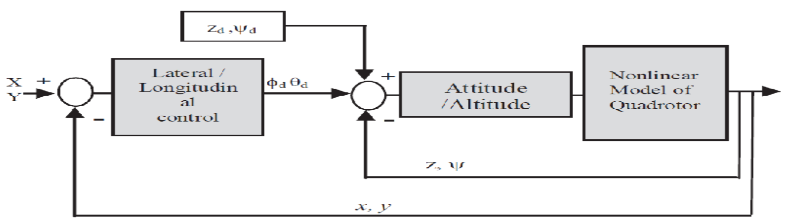

Figure 2: General structure of the quadrotor control system.

In what follows, we develop two strategies to stabilize the orientation and position of the quadrotor. Their performance is then compared in the next section, based on the numerical simulation results.

6 Strategy of Control

State and output feedback controllers design for dynamic systems with the prescribed and desired properties is a key problem of the strategy of control. At the same time, the properties of control systems such as asymptotic stability, robustness and optimality of the performance indexes are in the foreground [1].

7 The Forwarding Control

It is a non-linear control technique that is considered robust; the synthesis of such a control is done in a systematic way and based on the Lyapunov approach as well as the control in a recursive way for a system that can be written in the following form [14] :

$$\begin{cases} \dot{x}_1 = f_1(x_2, \cdots, x_n) + g_1(x_2, \cdots, x_n)u, \\ \dot{x}_2 = f_1(x_2, \cdots, x_n) + g_1(x_2, \cdots, x_n)u, \\ \vdots \\ \dot{x}_{n-1} = f_{n-1}(x_n) + g_{n-1}(x_n)u, \\ \dot{x}_n = u, \end{cases} \tag{8}$$

where $x \in R^n, u \in R$ and $g = [g_1, \cdots, g_{n-1}, 1]^T$.

The aim is to determine a control law that ensures the overall stabilization of the system. This is done in several steps. First, we try to stabilize x_n through u. Taking a function of Lyapunov V_n which is positive definite on R, one seeks to find a return of state $u = a_n(x_n)$ to stabilize x_n. The next step is to increase the system from x_n to x_{n-1}, and replace the control u, with $u = a_n(x_n) + V_{n-1}$. By taking a function of Lyapunov V_{n-1} being positive definite on R^2, one seeks to find a return of state $V_{n-1} = a_{n-1}$ which stabilizes the augmented system. The same procedure is repeated until the last step, where a control u which makes it possible to have an overall stabilization of the system, will be calculated.

8 The Steps of the Synthesis

The synthesis of the control law for the system is carried out in several steps [14].

8.1 First step

We take the system $\dot{x}_n = u$. Let the function of Lyapunov be $V_n = \frac{1}{2}x_n^2$. Its derivative is given by $\dot{V}_n = x_n u$. To make $\dot{V}_n$ negative definite, we can take $u = a_n(x_n) = -\lambda_n LgV_n = -\lambda_n x_n$ with $\lambda_n \geq 0$, which gives $\dot{V}_n = -\lambda_n x_n^2$ which is negative definite on R.

8.2 Second step

Let the system be increased:

$$\begin{cases} \dot{x}_{n-1} = f_{n-1}(x_n) + g_{n-1}(x_n)u, \\ \dot{x}_n = u. \end{cases} \tag{9}$$

For $u = a_n(x_n)$, the system becomes

$$\begin{cases} \dot{x}_{n-1} = \phi_{n-1}(x_n), \\ \dot{x}_n = -\lambda_n(x_n), \\ \phi_{n-1}(x_n) = f_{n-1}(x_n) - \lambda_n g_{n-1}(x_n)x_n. \end{cases} \tag{10}$$

The term ϕ_{n-1} is called the interconnection term. The temporal solution of the previous system will be : $\widetilde{x}_n(t) = x_n(0)e^{-\lambda_n t}$, $\widetilde{x}_{n-1}(t) = \int_0^t (f_{n-1}(\widetilde{x}_n) - \lambda_n g_{n-1}(\widetilde{x}_n)\widetilde{x}_n)dt + x_{n-1}(0)$. The new control input for the system (9) will be [14]: $u = a_n(x_n) + v_{n-1}$.

The function of Lyapunov for the system is given by [14]

$$\begin{aligned} V_{n-1} &= V_n + \frac{1}{2}x_{n-1}^2 + \int_0^\infty \widetilde{x}_{n-1}(t)\phi_{n-1}(\widetilde{x}_n(t))dt \\ &= V_n + \frac{1}{2}lim_{t\longrightarrow\infty}\widetilde{x}_{n-1}^2(t). \end{aligned} \tag{11}$$

9 Synthesis of the Control Laws for the Quadrotor

All tracking errors are written in the following form:

$$e_i = \begin{cases} x_i - x_{id}, & i \in 1,2,3,7,8,9; \\ x_i - \dot{x}_{(i-1)d}, & i \in 4,5,6,10,11,12. \end{cases} \tag{12}$$

All the functions of Lyapunov take the form

$$V_i = \begin{cases} \frac{1}{2}e_i^2, & i \in \{4,5,6,10,11,12\}; \\ V_{i+1} + \frac{1}{2}lim_{t\longrightarrow\infty}\widetilde{e}_i^2, & i \in \{1,2,3,7,8,9\}, \end{cases} \tag{13}$$

where e_i presents the temporal solution of the system. The steps of the order summary will be shown for the subsystem

$$\begin{cases} \dot{x}_7 = x_{10}, \\ \dot{x}_{10} = a_1x_{11}x_{12} + b_1U_2. \end{cases} \tag{14}$$

Let the tracking error be defined by $e_7 = x_7 - x_{7d}$. Its dynamics is described by $\dot{e}_7 = x_{10} - \dot{x}_{7d}$. If we set $e_{10} = x_{10} - \dot{x}_{7d}$, the system (14) becomes

$$\dot{e}_7 = e_{10}, \quad \dot{e}_{10} = a_1x_{11}x_{12} - \ddot{x}_{7d} + b_1U_2. \tag{15}$$

The synthesis is done in two steps.

9.1 The first step

We consider only the second equation of the system (15). Let the function of Lyapunov be $V_{10} = \frac{1}{2}e_{10}^2$. Its derivative is $\dot{V}_{10} = e_{10}\dot{e}_{10}$. To make $\dot{V}_{10}$ negative definite, we can choose the control

$$U_2 = a_{10}(x) = \frac{1}{b_1}(-a_1x_{11}x_{12} - \lambda_{10}e_{10} + \ddot{x}_{7d})\lambda_{10} \geq 0. \tag{16}$$

We will have $\dot{V}_{10} = -\lambda_{10}e_{10}^2$ which is negative on R. This control will ensure the convergence of e_{10} towards the origin.

9.2 The second step

By replacing the control found in the previous step in the system (15), we get $\dot{e}_7 = e_{10}$ and $\dot{e}_{10} = -\lambda_{10} e_{10}$. The temporal solution of this system is $\widetilde{e}_7(t) = -\frac{1}{\lambda_{10}} e_{10}(0) e^{\lambda_{10} t} + e_7(0) + \frac{1}{\lambda_{10}} e_{10}(0)$, $\widetilde{e}_{10}(t) = e_{10}(0) e^{-\lambda_{10} t}$. We put $e_{10}(0) = e_{10}$ and $e_7(0) = e_7$. Let the function of Lyapunov be $V_7 = V_{10} + \frac{1}{2} lim_{t \longrightarrow \infty} \widetilde{e}_7^2(t)$. So, $V_7 = \frac{1}{2}[e_{10}^2 + (e_7 + \frac{1}{\lambda_{10}} e_{10})^2]$. If we put $U_2 = a_{10}(x) + w_{10}$, the derivative of V_7 will be $\dot{V}_7 = -\lambda_{10} e_{10}^2 + [\frac{e_7}{\lambda_{10}} + (\frac{1}{\lambda_{10}^2} + 1) e_{10}] b_1 w_{10}$. To make $\dot{V}_7$ negative, we can take

$$w_{10} = -\frac{\lambda_7}{b_1}[\frac{e_7}{\lambda_{10}} + (\frac{1}{\lambda_{10}^2} + 1) e_{10}], \quad \lambda_7 \geq 0. \tag{17}$$

The control U_2 becomes $U_2 = \frac{1}{b_1}(-a_1 x_{11} x_{12} - \frac{\lambda_7}{\lambda_{10}} e_7 - (\frac{\lambda_7}{\lambda_{10}^2} + \lambda_7 + \lambda_{10}) e_{10} + \ddot{x}_{7d}$.

This control will stabilize the system (14). The same steps are taken to determine U_1, U_3, U_4, U_x, U_y. The system controls with forwarding are given as follows:

$$\begin{cases} U_2 = \dfrac{1}{b_1}(-a_1 x_{11} x_{12} - \dfrac{\lambda_7}{\lambda_{10}} e_7 - (\dfrac{\lambda_7}{\lambda_{10}^2} + \lambda_7 + \lambda_{10}) e_{10} + \ddot{x}_{7d}, \\ U_3 = \dfrac{1}{b_2}(-a_1 x_{11} x_{12} - \dfrac{\lambda_8}{\lambda_{11}} e_8 - (\dfrac{\lambda_8}{\lambda_{11}^2} + \lambda_8 + \lambda_{11}) e_{11} + \ddot{x}_{8d}, \\ U_4 = \dfrac{1}{b_3}(-a_5 x_{10} x_{11} - \dfrac{\lambda_9}{\lambda_{12}} e_9 - (\dfrac{\lambda_9}{\lambda_{12}^2} + \lambda_9 + \lambda_{12}) e_{12} + \ddot{x}_{9d}, \\ U_1 = \dfrac{m}{cx_7 cx_8}(g - \dfrac{\lambda_3}{\lambda_6} e_3 - (\dfrac{\lambda_3}{\lambda_6^2} + \lambda_3 + \lambda_6) e_6 + \ddot{x}_{3d}, \\ U_x = \dfrac{m}{U_1}(-\dfrac{\lambda_1}{\lambda_4} e_1 - (\dfrac{\lambda_1}{\lambda_4^2} + \lambda_1 + \lambda_4) e_4 + \ddot{x}_{1d}, \\ U_y = \dfrac{m}{U_1}(-\dfrac{\lambda_2}{\lambda_5} e_2 - (\dfrac{\lambda_2}{\lambda_5^2} + \lambda_2 + \lambda_5) e_5 + \ddot{x}_{2d}, \end{cases} \tag{18}$$

with $\lambda_i \geq 0$ for $i \in$ [1,12].

10 The PID-Backstepping Controller

The PID regulator (proportional-integral-derivative) serves to reduce the error between the measurement and the set point; it is used in most industrial processors thanks to its simplicity and efficiency especially in linear systems. It is based on three operations [12]: proportional action (P): in which a gain K_p is applied to the error, an integral action (I): in which we integrate the error, and we multiply the result by a gain K_i, and a derivative action (D): in which one derives the error, and one multiplies the result by a gain K_d.

The architecture of the cascade PID has been extended to the non-linear case by separating the translation and rotation dynamics from the equations of motion. A linear control is then applied to the dynamics of rotation and a nonlinear control has been applied to the dynamics translation by the backstepping control.

11 Attitude and Altitude Control by PID:

The linear control by a PID is well adapted to the quasi-stationary flight, for which the angles of inclination of the vehicle are small. This makes it possible to obtain a decoupled model in several SISO (Single Input Single Output) chains of the dynamics of the drone. Let $e_\phi = \phi_d - \phi$, and $U_2 = K_{p\phi}e_\phi(t) + K_{I\phi}\int_0^t e_\phi(\tau)d\tau + K_{D\phi}\frac{de_\phi(t)}{dt}$.

The same applies to the pitch and yaw angle and altitude control

$$\begin{cases} e_\theta = \theta_d - \theta, \\ U_3 = K_{p\theta}e_\theta(t) + K_{I\theta}\displaystyle\int_0^t e_\theta(\tau)d\tau + K_{D\theta}\frac{de_\theta(t)}{dt}, \end{cases} \tag{19}$$

and

$$\begin{cases} e = \Psi_d - \Psi, \\ U_4 = K_{p\Psi}e_\Psi(t) + K_{I\Psi}\displaystyle\int_0^t e_\Psi(\tau)d\tau + K_{D\Psi}\frac{de_\Psi(t)}{dt}, \end{cases} \tag{20}$$

and

$$\begin{cases} e = z_d - z, \\ U_1 = K_{pz}e_z(t) + K_{Iz}\displaystyle\int_0^t e_z(\tau)d\tau + K_{Dz}\frac{de_z(t)}{dt}. \end{cases} \tag{21}$$

The parameters K_{px}, K_{Ix}, K_{Dx}, respectively, define the proportional, integral and derivative gains of the angles ϕ, θ, Ψ and altitude Z.

12 Control in Position with the Backstepping Controller

The backstepping represents a recursive method that allows to build a control law which guarantees, at any time, the stability of the system. Writing states in pure parametric form highlights the subsystems. For each of these parts, it is necessary to find, using a Lyapunov function, a control which makes it possible to stabilize this subsystem [11].

To do this, the next state is considered as the new controller input (the virtual control). The order of the subsystem is then increased and the previous development is restarted. At the end, a control law is obtained [5].

13 The Steps of the Synthesis

The backstepping is, in fact, only the construction of the Lyapunov function as well as the step-by-step control for a system that can be written in the form, called cascade, as follows:

$$\begin{cases} \dot{x}_1 = x_2 + \phi_1(x_1), \\ \dot{x}_2 = x_3 + \phi_2(x_1, x_2), \\ \vdots \\ \dot{x}_{n-1} = x_n + \phi_{n-1}(x_1, x_2, \cdots, x_{n-1}), \\ \dot{x}_n = u + \phi_n(x_1, x_2, \cdots, x_n), \\ y = x_1, \end{cases} \tag{22}$$

where $x \in R^n$ and $u \in R$.

The aim is to find a law of control which ensures the continuation of a reference y_d. It is carried out in several stages. In our case, we applied the backstepping control to stabilize the position of the quadrotor along the x and y axes, so we apply it to these two subsystems in cascade [16]. We used backstepping control to stabilize the outer loop and the PID to stabilize the inner loop. The steps of the synthesis are: The first subsystem is given by the following equations:

$$\begin{cases} \dot{x}_1 = x_4, \\ \dot{x}_4 = C_{x7} * S_{x8} * \dfrac{U_1}{m}. \end{cases} \tag{23}$$

Consider $e_1 = x_d - x_1$ as a tracking error of the x-axis, and its time derivative $\dot{e}_1 = \dot{x}_d - \dot{x}_1$. The analysis of stability is treated by Lyapunov's theorem considering a positive definite function $V(e_1)$ and its negative semi-definite temporal derivative. Put $V_1 = \frac{1}{2}(x_d - x_1)^2$ and $\dot{V}_1 = (x_d - x_1)(\dot{x}_d - \dot{x}_1) = (x_d - x_1)(\dot{x}_d - x_4)$. To ensure stability for this equation let us take $x_4 = \phi_4 = -\lambda(x_d - x_1) + \dot{x}_{1d}$. The desired new reference will be the control variable for the preceding subsystem $x_4 = \phi_4$.

The regulation error is $e_2 = \phi_4 - x_4$. Its derivative is $\dot{e}_2 = \dot{\phi}_4 - \dot{x}_4$. The extended Lyapunov function for this system is $V_4 = V_1 + \frac{1}{2}e_2^2$, and

$$V_4 = \frac{1}{2}(e_1^2 + e_2^2). \tag{24}$$

Its derivative is : $\dot{V}_4 = \dot{V}_1 + e_2\dot{e}_2$ and

$$\begin{aligned} \dot{V}_4 &= (x_d - x_1)(\dot{x}_d - \dot{x}_1) + (\phi_4 - x_4)(\dot{\phi}_4 - \dot{x}_4) \\ &= (x_d - x_1)(\dot{x}_d - \phi_4) + (\phi_4 - x_4 - \dot{\phi}_4 - C_{x7} * S_{x8} * \frac{U_1}{m}) \end{aligned} \tag{25}$$

and $C_{x7} * S_{x8} * \frac{U_1}{m} = -\lambda_4(\phi_4 - x_4) + (x_{1d} - x_1) + \dot{\phi}_4 - x_1 + x_{1d}$, in which $\theta_d = x_8^* = arcsin[([-\lambda_4(\phi_4 - x_4) + (x_d - x_1) + \dot{\phi}_4 - x_1 + x_{1d}] * m)/(C_{x7} * U_1)]$. In the same way for the subsystem

$$\begin{cases} \dot{x}_2 = x_5, \\ \dot{x}_5 = -S_{x8} * \dfrac{U_1}{m}. \end{cases} \tag{26}$$

We end up with the control that stabilizes x_5, which is $\phi_5 = -\lambda_2(x_{2d} - x_2) + \dot{x}_2$, and $\phi_d = x_7^* = arcsin[(-m[-\lambda_5(\phi_5 - x_5) + \dot{\phi}_5 - x_2 + x_{2d}])/(U_1)]$.

14 Simulation Results

The parameters of the quadrotor are given as follows: mass (m = 0.650 Kg), moment of inertia of the quadrotor compared to x (j_x = 7.5e-3 $Kg.m^2$), moment of inertia of the quadrotor compared to y (j_y = 7.5e-3 $Kg.m^2$), moment of inertia of the quadrotor compared to z (j_z = 1.3e-2 $Kg.m^2$), coefficient of lift (b = 3.13e-5 Ns^2), coefficient of drag (k = 7.5e-5 Nms^2), moment of inertia of the rotor compared to z (j_{rz} = 6e-5 $Kg.m^2$), distance between the rotor and the center of gravity (d = 0.23 m).

In order to validate the robustness and disturbance resistance of the proposed controls schemes for stabilizing the quadrotor at trajectory tracking, the simulation is conducted in MATLAB SIMULINK 2016, a programming environment on an Intel Core i5 - PC running under Windows 10. Similarly, linear and aleatory disturbances are added to the outputs of the quadrotor.

The two control techniques, the forwarding controller and the PID-backstepping hybrid controller, were implemented on a non-linear model of the quadcopter. Numerical simulation was performed using MATLAB SIMULINK software. The resolution of the systems of differential equations was made by the Runge-Kutta method of order 4 with a simulation step t = 0.0001 sec, and a final time tf = 20 sec.

The control parameters of the PID controllers are tuned by the tune function of SIMULINK.

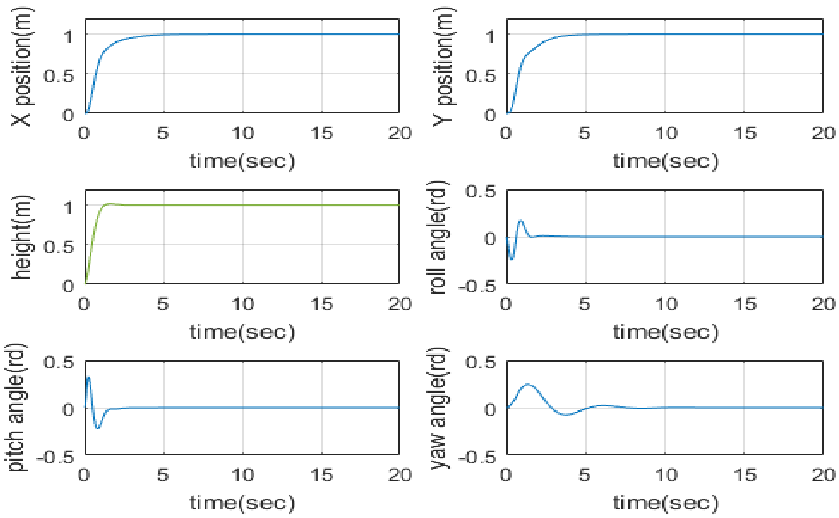

Figure 3: The forwarding system response.

15 Disturbance Rejection with the Forwarding Control

We introduced a disturbance of the order of 10% on the outputs x, y, z and Ψ and the results are given in the figures below. We note that this control was able to eliminate the effect of the disturbance which confirms its robustness. The control parameters of the PID controllers are tuned by the tune function of SIMULINK. The tuned parameters are given as follows using a unit step response case: altitude : (K_{Pz}=132.3356, K_{Iz}=205.1444, K_{Dz}=20.9622), roll : ($K_{P\phi}$ =0.6054, $K_{I\phi}$=0.2834, $K_{D\phi}$ =0.3175), pitch : ($K_{P\theta}$=0.6054, $K_{I\theta}$=0.2834, $K_{D\theta}$=0.3175), yaw : ($K_{P\psi}$=24.1383, $K_{I\psi}$ =113.0029, $K_{D\psi}$ =1.2660.

The responses of the quadrotor to the PID control and backstepping are given in the following figures. To test the robustness of this control, we introduced a disturbance of the order of 10% of the input signal on the outputs x, y, z and Ψ from the 10th second, we obtain the following response. In the comparative simulation, the quadrotor follows a referenced trajectory of x, y, z and Φ and θ are equal to zero, which is shown in the figures above. Linear and noise signals are applied at the 10^{th} second. From the result, the response of the forwarding is slightly faster than that of the PID-backstepping. As the noise signal increases, the tracking error becomes bigger. Nevertheless, it is obvious

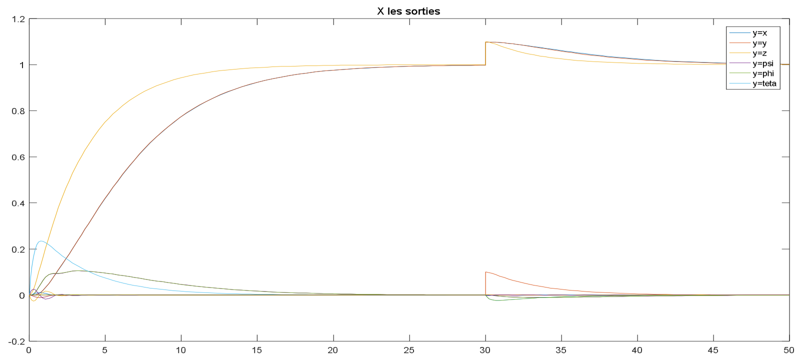

Figure 4: Rejection of the disturbance applied at the moment of 30 sec.

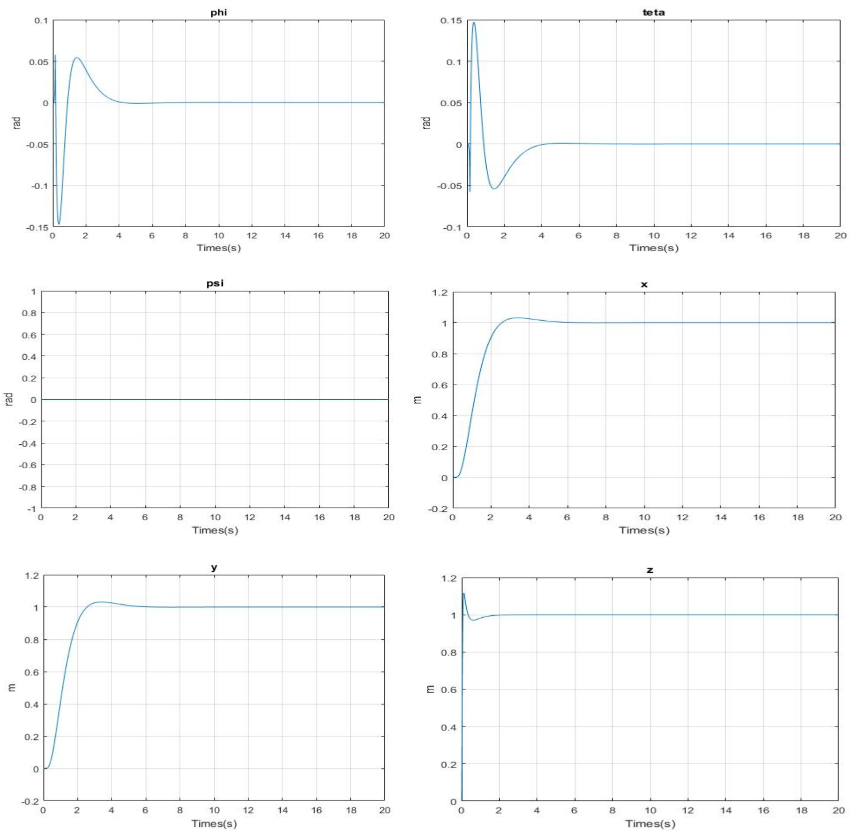

Figure 5: The system response with the PID-backstepping control.

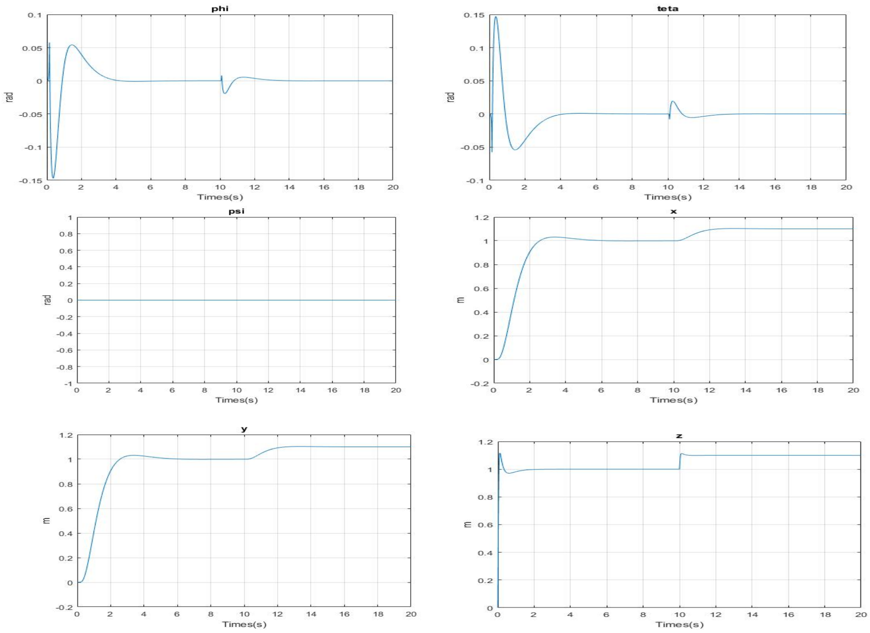

Figure 6: The system response with the PID-backstepping control with disturbances.

that the steady state error of forwarding is smaller than that of PID-backsteppig. In conclusion, the forwarding has a strong ability of disturbance compensation.

16 Conclusion

In this paper, we presented a control law of stabilization of the trajectory of a quadrotor synthesized by the forwarding control based primarily on the development of the dynamic model of the quadrotor, all this by taking into account the different forces and couples that can influence the evolution of this drone and the development of non-holonomic constraints of high order imposed on the movements of the system. These control laws allowed the follow-up of the different desired trajectories expressed in terms of coordinates of the center of mass of the system in spite of the complexity of the proposed model. The forwarding control gave satisfactory results of the follow-up of the imposed trajectories and the rejects of disturbances. The simulations show the good performance of the proposed controller.

To ensure the robustness of the forwarding control we compared its performance with another hybrid control that is based on the PID-backstepping control. Unlike the forwarding control, the PID-backstepping control gives satisfactory results as long as we are close to the operating point where the effect of disturbances on system responses is studied by different simulations. The proposed approaches have proved their robustness and efficiency in simulation for the stability of the position and attitude of the system. Our perspectives are to test the effectiveness of these strategies on the real system.

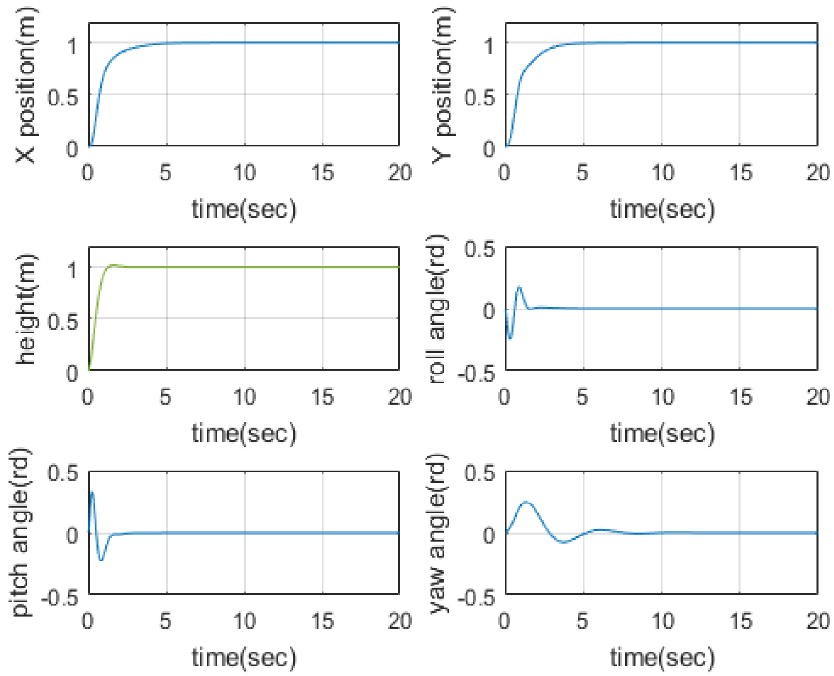

Figure 7: The system response with the forwarding control with variation of parameters.

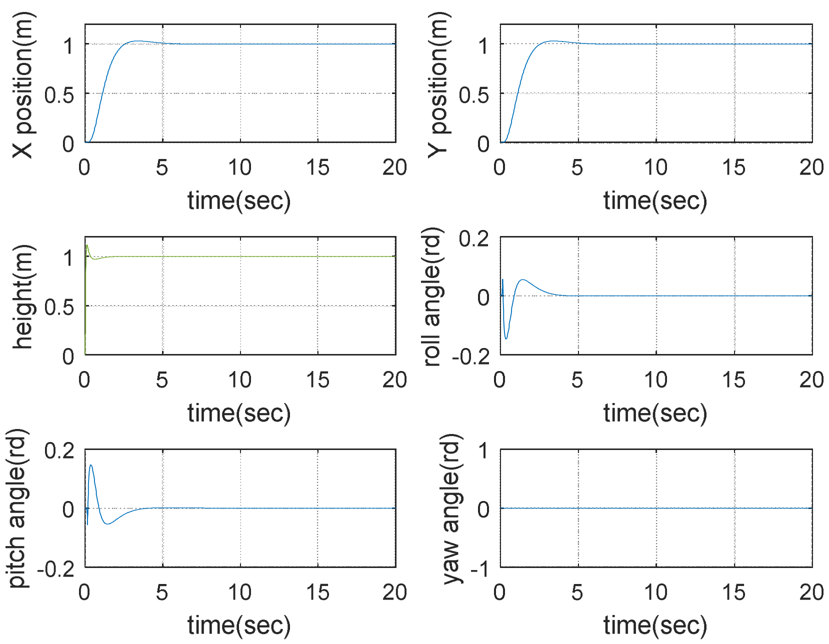

Figure 8: The system response with the PID-backstepping control with variation of parameters.

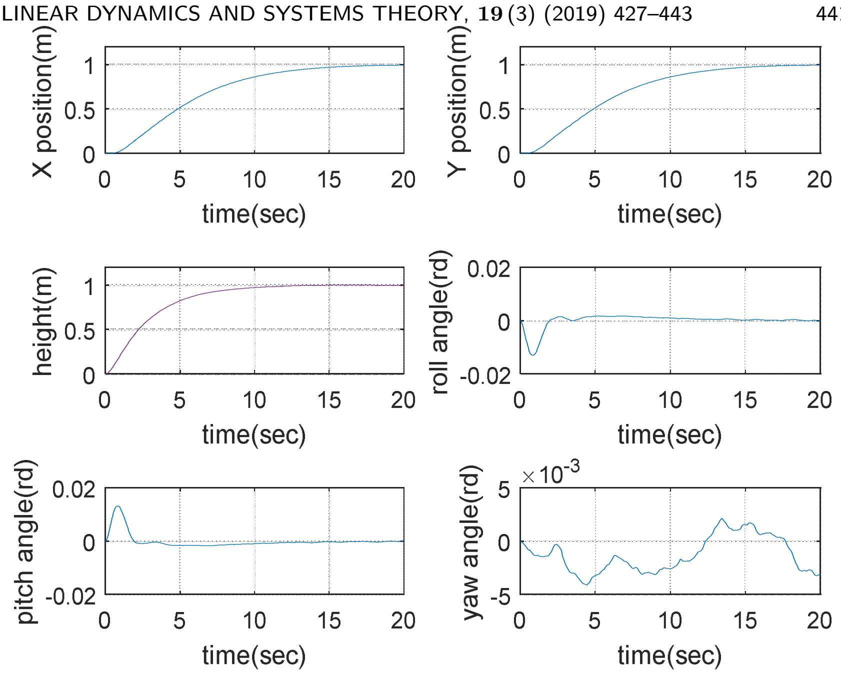

Figure 9: The system response with the forwarding control with aleatory disturbances.

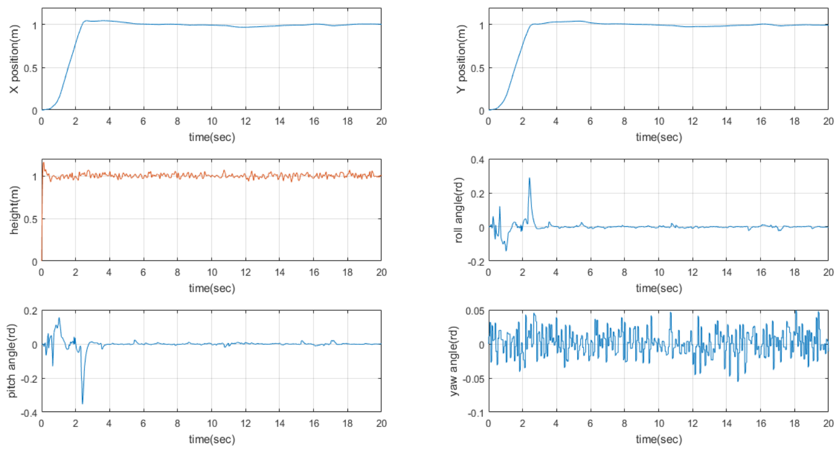

Figure 10: The system response with the PID-backstepping control with aleatory disturbances.

References

[1] A. Y. Aleksandrov, E. B. Aleksandrova, A. V. Platonov, and M. V. Voloshin. On the global asymptotic stability of a class of nonlinear switched systems. *Nonlinear Dynamics and*

Systems Theory **2** (17) (2017) 107–120.

[2] R. Ashton & M. Maggiore. A class of position controllers for underactuated VTOL vehicles. *IEEE Transactions on Automatic Control* **59** (9) (2014) 2580–2585.

[3] M. A. M. Basri, A. R. Husain and K. A. Danapalasingam, A hybrid optimal backstepping and adaptive fuzzy control for autonomous quadrotor helicopter with time-varying disturbance. *J. Aerospace Engineering* **229** (12) (2015) 2178–2195.

[4] H. Bolandi, M. Rezaei, R. Mohsenipour, H. Nemati and S. M. Smailzadeh. Attitude control of a quadrotor with optimized pid controller. *Intelligent Control and Automation* **04** (3) (2013) 342–349.

[5] S. Bouabdallah, A. Noth and R. Siegwart. PID vs LQ control techniques applied to an indoor micro quadrotor. In: *Proc. of the IEEE International Conference on Intelligent Robots and Systems (IROS)* (2004) 2451–2456.

[6] H. Bouadi, M., Bouchoucha, and M. Tadjine. Modelling and Stabilizing Control Laws Design Based on Sliding Mode for an UAV Type-Quadrotor. *Engineering Letters* **15** (2) (2007) 15–24.

[7] F. Chen, R. Jiang, K. Zhang, B. Jiang, G. Tao. Robust Backstepping Sliding-Mode Control and Observer-Based Fault Estimation for a Quadrotor UAV. *IEEE Transactions on Industrial Electronics* **63** (8) (2016) 5044–5056.

[8] Z. Dong, H. Fan, Y. Wang, L. Xu and W. Wang. Adaptive backstepping controller design for quadrotor aircraft with unknown disturbance. In: *Control, Automation, Robotics and Vision (ICARCV), 2016 14th International Conference*, (2016, November), 1–5.

[9] M. Idres, O., Mustapha and M. Okasha. Quadrotor trajectory tracking using PID cascade control. In: *IOP Conference Series: Materials Science and Engineering*, IOP Publishing **270** (1) (2017, December), 10–12.

[10] M. Islam, M. Okasha and M. M. Idres, Trajectory tracking in quadrotor platform by using PD controller and LQR control approach. In: *IOP Conference Series: Materials Science and Engineering*, IOP Publishing **260** (1) (2017, November), 012–026.

[11] A. G. Mazko. Robust Output Feedback Stabilization and Optimization of Control Systems *Nonlinear Dynamics and Systems Theory* **17** (1) (2017) 42–59.

[12] J. P. Ortiz, L. I. Minchala, and M. J. Reinoso. Nonlinear robust h-infnity PID controller for the multivariable system quadrotor. *IEEE Latin America Transactions* **14** (3) (2016) 1176–1183.

[13] J. A. Paredes, C. Jacinto, R. Ramrez, I. Vargas and L. Trujillano. Simplified fuzzy-PD controller for behavior mixing and improved performance in quadcopter attitude control systems. In: *ANDESCON, 2016 IEEE*, (2016, October), 1–4.

[14] R. Sepulchre, M. Jankovic, and P. Kokotovic. *Constructive Nonlinear Control.* Springer-Verlag. London, 1997.

[15] X. Shi, B., Hu, C., Yin, Y., Cheng and X. Huang. Design of trajectory tracking controller with backstepping method for quadrotor unmanned aerial vehicles. Chinese Control and Decision Conference (CCDC), (2018, June), 3124–3128.

[16] L. Wang, H. Jia. The trajectory tracking problem of quadrotor UAV: Global stability analysis and control design based on the cascade theory. *Asian Journal of Control* **16** (2) (2014) 574–588.

[17] X. Wang, X. Lin, Y. Yu, Q. Wang and C. Sunl. Backstepping Control for Quadrotor with BP Nerual Network Based Thrust Model. *32nd Youth Academic Annual Conference of Chinese Association of Automation (YAC)* (2017) 292–297.

[18] J. Xiong and E. H. Zheng. Position and attitude tracking control for a quadrotor UAV. *ISA transactions* **53** (3) (2014) 725–731.

[19] C. Yin, X. Huang, Y. Chen, S. Dadras, S. M. Zhong and Y. Cheng. Fractional-order exponential switching technique to enhance sliding mode control. *Applied Mathematical Modelling* **44** (2017) 705–726.

[20] X. Zhang, X. Huang, and H. Lu. Forwarding-based Trajectory Tracking Control for Nonlinear Systems with Bounded Unknown Disturbances. *International Journal of Control, Automation and Systems* **14** (5) (2016) 1–13.