Nonlinear Dynamics and Systems Theory, **20** (1) (2020) 38–50

Adaptive Sliding Mode Control Synchronization of a Novel, Highly Chaotic 3-D System with Two Exponential Nonlinearities

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Received: May 4, 2019; Revised: December 24, 2019

Abstract: In this paper, a new 3D chaotic system with three nonlinearities is introduced. Basic dynamical properties of this new chaotic system are studied such as equilibrium points and their stability, dissipativity and Lyapunov exponent, Lyapunov exponent spectrum, Kaplan-Yorke dimension. Also, an adaptive integral sliding mode control scheme is proposed for synchronization of the new chaotic system with unknown system parameters based on the Lyapunov stability theory and adaptive control theory of this new chaotic system with unknown system parameters. Finally, numerical simulations are presented to show the effectiveness of the proposed chaos synchronization scheme using Matlab.

Keywords: *chaotic system; strange attractor; Lyapunov exponent; Lyapunov stability theory; adaptive control; synchronization.*

Mathematics Subject Classification (2010): 37B55, 34C28, 34D08, 37B25, 37D45, 93C40, 93D05.

1 Introduction

Chaos as an important nonlinear phenomenon has been studied in mathematics, engineering and in many other disciplines. Synchronization of chaotic systems has become an active research area because of its potential applications in different industrial areas [1, 2, 3]. For the first time chaotic synchronization was illustrated by Fujiska and Yamada [2] in 1983, then, Pecora and Carroll [3] in 1990, reported a new and very effective method for the synchronization of two chaotic systems with different initial conditions. The control

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scheme has been applied in the recent decade for the synchronization of chaotic or hyperchaotic system, for example, the OYG method [4], adaptive control [5, 13, 14, 15, 19, 20], backstepping design method [6], sliding mode control [7, 20], PC synchronization method [3], passive control [8], fuzzy control [9], nonlinear active control [10], etc. The adaptive control scheme is used when parameters are unknown or initially uncertain. The sliding mode control method is often used because of its inherent advantages of easy realization, fast response and good transient performance, as well as its insensitivity to parameter uncertainties and external disturbances. Also, in the adaptive method, the control law and parameter update law are designed in such a way that the chaotic response system to behave like chaotic drive systems. As a result, the adaptive scheme maintains consistent performance of a system in the presence of uncertainty as well as variations in plant parameters. The adaptive control technique is different from other control methods since it does not need a priori information about the bounds on these uncertain or time varying parameters because this method of control is concerned with the control law changing themselves. Recently, many papers are available on synchronization of chaotic systems using this method of control.

In this paper a new chaotic system is considered for synchronization using the sliding mode control method and adaptive sliding mode control method when system parameters are unknown. Stabilization and convergence of error dynamics are achieved using the Lyapunov stability theory [11, 12]. This paper is organized as follows. The first section deals with the description and some properties of the novel chaotic system. The next two sections deal with the synchronization problem for globally and exponentially synchronizing the identical 3-D novel chaotic systems using the integral sliding mode control and adaptive integral sliding mode control law with unknown system parameters, respectively. Finally, numerical simulations using MATLAB have been shown to illustrate our results for the new chaotic system with unknown parameters.

1.1 Description of the novel chaotic system

A novel 3D autonomous chaotic system is expressed as follows:

$$\begin{cases} \frac{dx}{dt} = a(y-x), \\ \frac{dy}{dt} = cx - y - xz - e^x, \\ \frac{dz}{dt} = e^{xy} - dy - bz, \end{cases} \tag{1}$$

where x, y, z are the state variables and a, b, c are positive real parameters.

There are nine terms on the right-hand side but it mainly relies on three nonlinearities, namely, e^{xy}, $e^x$and xz , respectively.

System (1) can generate a new strange attractor for the parameters $a = 15, b = 3, c = 300, d = 1$ with the initial conditions $(x(0), y(0), z(0)) = (1, 1, 1)$. The chaotic attractor is displayed in Figure 1. It appears that the new attractor exhibits the interesting complex and abundant chaotic dynamics behavior, which is similar to the Lorenz chaotic attractor, but is different from that of the Lorenz system or any existing systems.

1.2 Basic properties

In this section, some basic properties of the system (1) are given. We start with the equilibrium points of the system and check their stability at the initial values of the parameters a, b, c.

1.3 Equilibrium points

Putting equations of the system (1) equal to zero, i.e.,

$$\begin{cases} a(y-x)=0, \\ cx-y-xz-e^{x}=0, \\ e^{xy}-dy-bz=0, \end{cases} \tag{2}$$

gives numerically the only equilibrium point

$$p^{*}=\left(3.359\,5\times10^{-3},3.359\,5\times10^{-3},0.332\,22\right).$$

1.4 Stability

In order to check the stability of the equilibrium points we derive the Jacobian matrix at a point $p\,(x,y,z)$ of the system (1)

$$J(p)=\begin{pmatrix} -a & a & 0 \\ c-z-e^{x} & -1 & -x \\ ye^{xy} & -d+xe^{xy} & -b \end{pmatrix}. \tag{3}$$

For p^{*}, we obtain three eigenvalues

$$\lambda_1=59.297,\lambda_2=-3.0,\lambda_3=-75.297. \tag{4}$$

Since all the eigenvalues are real, Hartman-Grobman theorem implies that p is a saddle point which is unstable according to the Lyapunov theorem of stability.

1.4.1 Dissipativity

In vector notation, we may express the system (1) as

$$\dot{X}=f\,(X)=\begin{pmatrix} f_1(x,y,z) \\ f_2(x,y,z) \\ f_3(x,y,z) \end{pmatrix}.$$

Let Ω be any region in R^3 with a smooth boundary and also, $\Omega(t)=\phi_t(\Omega)$, where ϕ_t is the flow of f. Furthermore, let $V(t)$ denote the volume of $\Omega(t)$. By Liouville's theorem, we have

$$\dot{V}(t)=\int\limits_{\Omega(t)}(\nabla.f)\,dxdydz \tag{5}$$

with

$$\nabla.f=\frac{\partial\dot{f}_1}{\partial x}+\frac{\partial\dot{f}_2}{\partial y}+\frac{\partial\dot{f}_3}{\partial z}=-(a+b+1)<0 \tag{6}$$

and therefore

$$\dot{V}(t)=\int\limits_{\Omega(t)}(-19)\,dxdydz=-19V(t).$$

By integration, we get

$$V(t)=e^{-19t}V(0), \tag{7}$$

then, $V(t)\to0$ as $t\to\infty$. This shows that the novel chaotic system (1) is dissipative.

1.4.2 Lyapunov exponents and Kaplan-Yorke dimension

Lyapunov exponents are used to measure the exponential rates of divergence and convergence of nearby trajectories, which is an important characteristic to judge the system whether it is chaotic or not. The existence of at least one positive Lyapunov exponent implies that the system is chaotic.

For the chosen parameter values of a, b, c, d, the Lyapunov exponents of the novel chaotic system (1) are obtained using Matlab with the initial conditions $(x(0), y(0), z(0)) = (1, 1, 1)$

$$L_1 = 6.6231, L_2 = -0.00206431, L_3 = -20.621. \tag{8}$$

The Lyapunov exponents spectrum is shown in Fig. 1.

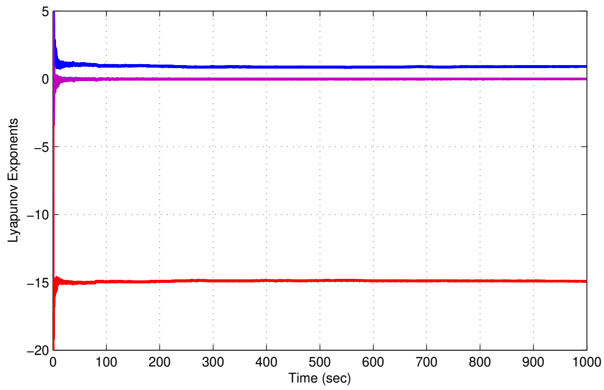

Figure 1: Lyapunov exponents spectrum.

Since the spectrum of Lyapunov exponents (8) has a maximal positive value L_1, it follows that the 3-D novel system (1) is a highly chaotic . The Kaplan-Yorke dimension of system (1) is calculated as

$$D_{KL} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.3211. \tag{9}$$

In Figs. 2-6, the 2-D projections of the strange chaotic attractor of the novel chaotic system (1) on the (x; y), (x; z), (y; z), (z; x), (z; y) planes are shown, respectively.

1.5 Synchronizing of the identical 3-D novel chaotic systems using integral sliding mode control

In this section, an integral sliding mode controller will be designed for globally and exponentially synchronizing the identical 3-D novel chaotic systems.

Thus, the master system is given by the novel chaotic system dynamics

$$\begin{cases} \frac{dx_1}{dt} = a(x_2 - x_1), \\ \frac{dx_2}{dt} = cx_1 - x_2 - x_1x_3 - e^{x_1}, \\ \frac{dx_3}{dt} = e^{x_1x_2} - dx_2 - bx_3. \end{cases} \tag{10}$$

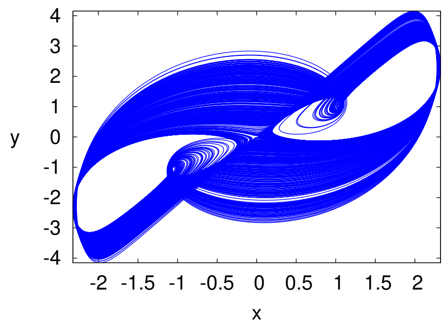

Figure 2: Projection on the $x - y$ plane of the chaotic attractor of system (1).

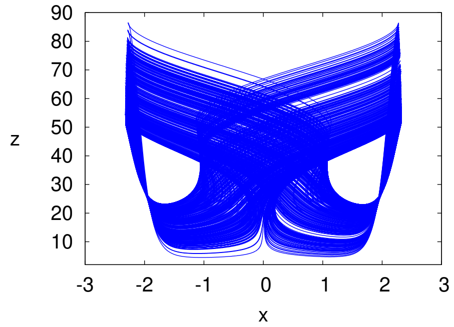

Figure 3: Projection on the $x - z$ plane of the chaotic attractor of system (1).

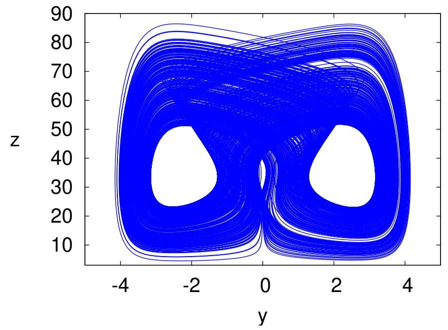

Figure 4: Projection on the $y - z$ plane of the chaotic attractor of system (1).

Also, the slave system is given by the novel chaotic system dynamics

$$\begin{cases} \frac{dy_1}{dt} = a(y_2 - y_1) + u_1, \\ \frac{dy_2}{dt} = cy_1 - y_2 - y_1 y_3 - e^{y_1} + u_2, \\ \frac{dy_3}{dt} = e^{y_1 y_2} - dy_2 - by_3 + u_3. \end{cases} \tag{11}$$

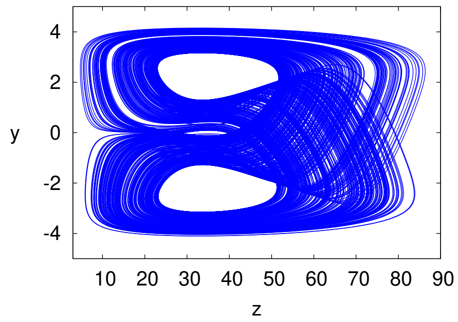

Figure 5: Projection on the $z-y$ plane of the chaotic attractor of system (1).

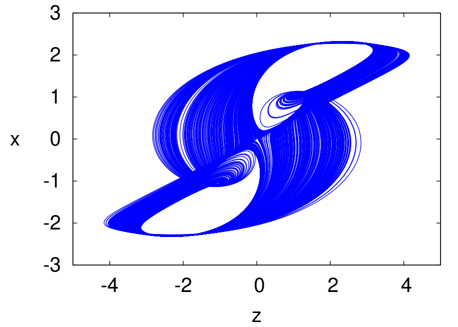

Figure 6: Projection on the $z-x$ plane of the chaotic attractor of system (1).

In (10) and (11), the system parameters a, b, c, d are $a = 15, b = 3, c = 300, d = 1$ and the main objective here is to design the controllers u_1, u_2, u_3 to synchronize two of the identical 3-D novel chaotic systems in equation (11) with equation (10), respectively.

The synchronization error between the novel chaotic systems (10) and (11) is defined as

$$\begin{cases} e_1 = y_1 - x_1, \\ e_2 = y_2 - x_2, \\ e_3 = y_3 - x_3, \end{cases} \tag{12}$$

(12) implies

$$\begin{cases} \dot{e}_1 = \dot{y}_1 - \dot{x}_1, \\ \dot{e}_2 = \dot{y}_2 - \dot{x}_2, \\ \dot{e}_3 = \dot{y}_3 - \dot{x}_3. \end{cases} \tag{13}$$

The sliding surface of the integral sliding mode controller is defined as

$$s_i = \left(\frac{d}{dt} + \lambda_i\right)\left(\int_0^t e_i(\tau)\, d\tau\right) = e_i + \lambda_i \int_0^t e_i(\tau)\, d\tau \tag{14}$$

and the reaching law is given by

$$\dot{s_i} = -\eta_i sgn(s_i) - k_i s_i, \quad i = 1, 2, 3, \tag{15}$$

where $\eta_i > 0$, which indicates that the rate of the system reaching the switching surface $s_i = 0$, and the exponential reaching term, $-k_i s_i$, can guarantee that the system state can tend to the sliding mode with a large rate when s_i is bigger.

The derivative of equation in equation (14) results

$$\dot{s_i} = \dot{e}_i + \lambda_i e_i. \tag{16}$$

The Hurwitz condition is realized if $\lambda_i > 0$ for $i = 1, 2, 3$.

Equation (16) by considering the exponential reaching law presented by equation (15) gives

$$\begin{cases} \dot{e_1} + \lambda_1 e_1 = -\eta_1 sgn(s_1) - k_1 s_1, \\ \dot{e_2} + \lambda_2 e_2 = -\eta_2 sgn(s_2) - k_2 s_2, \\ \dot{e_3} + \lambda_3 e_3 = -\eta_3 sgn(s_3) - k_3 s_3. \end{cases} \tag{17}$$

Writing equation (17) with the provision of equations (12) and (13) yields

$$\begin{cases} a(e_2 - e_1) + u_1 + \lambda_1 e_1 = -\eta_1 sgn(s_1) - k_1 s_1, \\ ce_1 - e_2 - y_1 y_3 + x_1 x_3 - e^{y_1} + e^{x_1} + u_2 + \lambda_2 e_2 = -\eta_2 sgn(s_2) - k_2 s_2, \\ -de_2 - be_3 + e^{y_1 y_2} - e^{x_1 x_2} + u_3 + \lambda_3 e_3 = -\eta_3 sgn(s_3) - k_3 s_3. \end{cases} \tag{18}$$

Then, the following control laws result in

$$\begin{cases} u_1 = -a(e_2 - e_1) - \lambda_1 e_1 - \eta_1 sgn(s_1) - k_1 s_1, \\ u_2 = -ce_1 + e_2 + y_1 y_3 - x_1 x_3 + e^{y_1} - e^{x_1} - \lambda_2 e_2 - \eta_2 sgn(s_2) - k_2 s_2, \\ u_3 = de_2 + be_3 - e^{y_1 y_2} + e^{x_1 x_2} - \lambda_3 e_3 - \eta_3 sgn(s_3) - k_3 s_3. \end{cases} \tag{19}$$

Theorem 1.1 *The response of the system in equation (11) with the arbitrary initial condition $y(0) \in R^3$, using the control laws in equation (19) and with η_i, λ_i and $k_i > 0$, is same as the response of the system in equation (10). This means equation (12) is globally asymptotically stable.*

Proof. We consider the quadratic Lyapunov function given by

$$V(s_1, s_2, s_3) = \frac{1}{2}\left(s_1^2 + s_2^2 + s_3^2\right), \tag{20}$$

where $s_i, i = 1, 2, 3.$, are the same as the ones in equation (14). Then, the derivative of equation (20) gives

$$\dot{V} = s_1 \dot{s}_1 + s_2 \dot{s}_2 + s_3 \dot{s}_3. \tag{21}$$

By substituting equation (15) into equation (21) we get

$$\begin{aligned} \dot{V} &= s_1(-\eta_1 sgn(s_1) - k_1 s_1) + s_2(-\eta_2 sgn(s_2) - k_2 s_2) + s_3(-\eta_3 sgn(s_3) - k_3 s_3) \\ &= -\eta_1 |s_1| - k_1 s_1^2 - \eta_2 |s_2| - k_2 s_2^2 - \eta_3 |s_3| - k_3 s_3^2, \end{aligned} \tag{22}$$

which is a negative definite function on $\mathbb{R}^3$ for $\eta_i, k_i > 0$, $i = 1, 2, 3$. Hence, by the Lyapunov stability theory [11, 12], it follows that $e_i(t) \longrightarrow 0$ as $t \longrightarrow \infty$ for $i = 1, 2, 3$. Hence, the proof is complete.

2 Adaptive Synchronization of the Identical 3-D Novel Chaotic Systems

In this section, we derive an adaptive integral sliding mode control law for globally and exponentially synchronizing the identical 3-D novel chaotic systems with unknown system parameters.

Thus, the master system is given by the novel chaotic system dynamics

$$\begin{cases} \frac{dx_1}{dt} = a(x_2 - x_1), \\ \frac{dx_2}{dt} = cx_1 - x_2 - x_1x_3 - e^{x_1}, \\ \frac{dx_3}{dt} = e^{x_1x_2} - dx_2 - bx_3. \end{cases} \tag{23}$$

Also, the slave system is given by the novel chaotic system dynamics

$$\begin{cases} \frac{dy_1}{dt} = a(y_2 - y_1) + u_1, \\ \frac{dy_2}{dt} = cy_1 - y_2 - y_1y_3 - e^{y_1} + u_2, \\ \frac{dy_3}{dt} = e^{y_1y_2} - dy_2 - by_3 + u_3. \end{cases} \tag{24}$$

In (23) and (24), the system parameters a, b, c, d are unknown and the design goal is to find the adaptive feedback controls u_1, u_2, u_3 using the states x_1, x_2, x_3, y_1, y_2, y_3 and the estimates $a_1(t), b_1(t), c_1(t), d_1(t)$ of the unknown parameters a, b, c, d, respectively.

The synchronization error between the novel chaotic systems (23) and (24) is defined as

$$\begin{cases} e_1 = y_1 - x_1, \\ e_2 = y_2 - x_2, \\ e_3 = y_3 - x_3, \end{cases} \tag{25}$$

(25) implies

$$\begin{cases} \dot{e}_1 = \dot{y}_1 - \dot{x}_1, \\ \dot{e}_2 = \dot{y}_2 - \dot{x}_2, \\ \dot{e}_3 = \dot{y}_3 - \dot{x}_3. \end{cases} \tag{26}$$

Thus, the synchronization error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + u_1, \\ \dot{e}_2 = ce_1 - e_2 - y_1y_3 + x_1x_3 - e^{y_1} + e^{x_1} + u_2, \\ \dot{e}_3 = -de_2 - be_3 + e^{y_1y_2} - e^{x_1x_2} + u_3. \end{cases} \tag{27}$$

We take the adaptive control law defined by

$$\begin{cases} u_1 = -a_1(e_2 - e_1) - \lambda_1 e_1 - \eta_1 sgn(s_1) - k_1 s_1, \\ u_2 = -c_1e_1 + e_2 + y_1y_3 - x_1x_3 + e^{y_1} - e^{x_1} - \lambda_2 e_2 - \eta_2 sgn(s_2) - k_2 s_2, \\ u_3 = d_1e_2 + b_1e_3 - e^{y_1y_2} + e^{x_1x_2} - \lambda_3 e_3 - \eta_3 sgn(s_3) - k_3 s_3, \end{cases} \tag{28}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (28) into (27), we obtain the closed-loop error dynamics as

$$\begin{cases} \dot{e}_1 = (a - a_1)(e_2 - e_1) - \lambda_1 e_1 - \eta_1 sgn(s_1) - k_1 s_1, \\ \dot{e}_2 = (c - c_1)e_1 - \lambda_2 e_2 - \eta_2 sgn(s_2) - k_2 s_2, \\ \dot{e}_3 = (d_1 - d)\, e_2 - (b - b_1)e_3 - \lambda_3 e_3 - \eta_3 sgn(s_3) - k_3 s_3. \end{cases} \tag{29}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) = a - a_1(t), \\ e_c(t) = c - c_1(t), \\ e_b(t) = b - b_1(t), \\ e_d(t) = d - d_1(t). \end{cases} \tag{30}$$

Differentiating (30) with respect to t, we obtain

$$\begin{cases} \frac{de_a(t)}{dt} = -\frac{da_1(t)}{dt}, \\ \frac{de_c(t)}{dt} = -\frac{dc_1(t)}{dt}, \\ \frac{de_b(t)}{dt} = -\frac{db_1(t)}{dt}, \\ \frac{de_d(t)}{dt} = -\frac{dd_1(t)}{dt}. \end{cases} \tag{31}$$

By using (31), we rewrite the closed-loop system (29) as

$$\begin{cases} \dot{e}_1 = e_a(e_2 - e_1) - \lambda_1 e_1 - \eta_1 sgn(s_1) - k_1 s_1, \\ \dot{e}_2 = e_c e_1 - \lambda_2 e_2 - \eta_2 sgn(s_2) - k_2 s_2, \\ \dot{e}_3 = -e_d e_2 - e_b e_3 - \lambda_3 e_3 - \eta_3 sgn(s_3) - k_3 s_3. \end{cases} \tag{32}$$

We consider the quadratic Lyapunov function given by

$$V(s_1, s_2, s_3, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(s_1^2 + s_2^2 + s_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right), \tag{33}$$

which is a positive definite function on $\mathbb{R}^6$.

Differentiating V along the trajectories of the systems (31) and (32), we obtain the following:

$$\begin{cases} \dot{V} = -\sum_{i=1}^{3} k_i s_i^2 - (\eta_1 |s_1| + \eta_2 |s_2| + \eta_3 |s_3|) + e_a\left(s_1(e_2 - e_1) - \frac{da_1(t)}{dt}\right), \\ \qquad -e_b\left(s_3 e_3 + \frac{db_1(t)}{dt}\right) + e_c\left(s_2 e_1 - \frac{dc_1(t)}{dt}\right) - e_d\left(s_3 e_2 + \frac{dd_1(t)}{dt}\right). \end{cases} \tag{34}$$

In view of (34), we take the parameter update law as follows:

$$\begin{cases} \frac{da_1(t)}{dt} = s_1(e_2 - e_1), \\ \frac{db_1(t)}{dt} = -s_3 e_3, \\ \frac{dc_1(t)}{dt} = s_2 e_1, \\ \frac{dd_1(t)}{dt} = -s_3 e_2. \end{cases} \tag{35}$$

Substituting (35) into (34), we obtain

$$\dot{V} = -\sum_{i=1}^{3} k_i s_i^2,$$

which is a negative definite function on $\mathbb{R}^3$. Hence, by the Lyapunov stability theory [11, 12], it follows that $e_i(t) \longrightarrow 0$ as $t \longrightarrow \infty$ for $i = 1, 2, 3$. Hence, we have proved the following theorem.

Theorem 2.1 *The 3-D novel chaotic systems (23) and (24) with unknown parameters are globally and exponentially synchronized for all initial conditions by the adaptive feedback control law (28) and the parameter update law (35), where k_1, k_2, k_3 are positive constants.*

2.1 Numerical simulations

We used the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ to solve the system of differential equations (23), (24) and (35) when the adaptive control law (28) is applied.

The parameter values of the novel 3-D chaotic system (23) are chosen as in the chaotic case, i.e., $a = 15, b = 3, c = 300, d = 1$. The positive gain constants are taken as $k_i = 5$, for $i = 1, 2, 3$.

The initial conditions of the drive system (23) are chosen as: $x_1(0) = 2, x_2(0) = -5, x_3(0) = 7$ and $y_1(0) = 12, y_2(0) = 6, y_3(0) = 10$ for the slave system (24). Furthermore, as initial conditions of the parameter estimates of the unknown parameters, we have chosen $a_1(0) = 20, b_1(t) = 5, c_1(t) = 25, d_1(t) = 3$.

In Figs. 7-9, the synchronization of the states of the master system (23) and slave system (24) is depicted, when the adaptive control law (28) and parameter update law (35) are implemented.

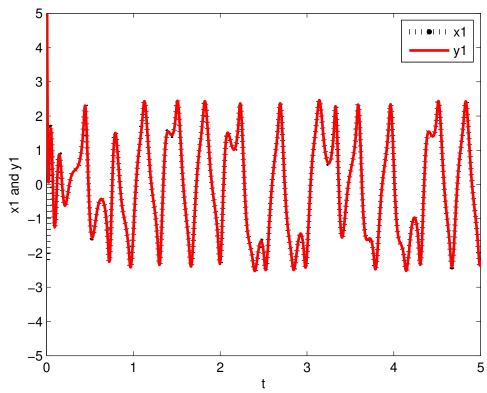

Figure 7: Synchronization of the states $x_1(t)$ and $y_1(t)$.

3 Conclusion

In this paper, a new chaotic system is introduced. Basic properties of this system are studied such as equilibrium points and their stability and the Lyapunov exponent and Kaplan-Yorke dimension. Moreover, the synchronization problem for globally and exponentially synchronizing the identical 3-D novel chaotic systems is solved using the integral sliding mode control and adaptive integral sliding mode control law with unknown system parameter, respectively. Numerical simulations using MATLAB have been shown to illustrate our results for the new chaotic system with unknown parameters. The results of this work are very important and have many applications in many fields such as security and communication. Therefore, further research on the system is still important and insightful and will be taken into consideration in a future work.

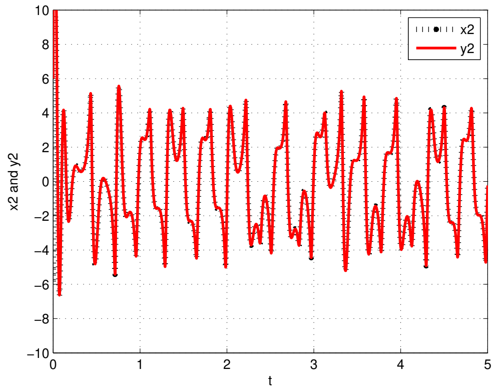

Figure 8: Synchronization of the states $x_2(t)$ and $y_2(t)$.

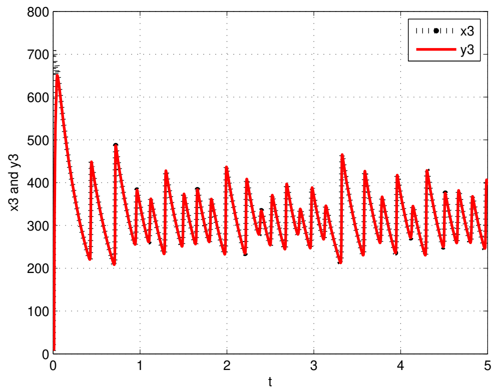

Figure 9: Synchronization of the states $x_3(t)$ and $y_3(t)$.

Acknowledgment

The author would like to thank the editor in chief and the referees for their valuable suggestions and comments.

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