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Control of a Novel Class of Uncertain Fractional-Order Hyperchaotic Systems with External Disturbances via Sliding Mode Controller

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Abstract: In this paper, a novel class of fractional-order hyperchaotic systems is proposed. In order to control hyperchaos in these systems, an appropriate sliding mode controller is also designed. Based on the Lyapunov stability theory, the control scheme guarantes the asymptotic stability of the fractional-order hyperchaotic systems in the presence of uncertainty and external disturbance. Simulation results of control design of fractional-order Liu and Lorenz hyperchaotic systems are presented to show the effectiveness of the proposed scheme and stabilization of the systems on the sliding surface.

Keywords: *hyperchaotic systems; fractional-order system; sliding mode control; Lyapunov stability.*

Mathematics Subject Classification (2010): 34A34, 37B25, 37B55, 93C55, 37C25, 93D05.

1 Introduction

The concepts of derivation and fractional integration are often associated with the names of Riemann and Liouville, while the question about the generalization of the notion of fractional-order derivative is older. Indeed, the history of fractional calculus goes back more than three centuries. Recently, fractional calculus has attracted the increasing attention of physicists as well as engineers in several fields of engineering science [1].

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On the other hand, the chaos theory as a very interesting nonlinear phenomenon has been intensively investigated due to its great importance for applications in several areas of science and technology [2]. It is well known that chaotic systems are defined as nonlinear dynamical systems which are very sensitive to initial conditions. The principal feature used to identify a chaotic behaviour is the well-known Lyapunov exponent criteria. In fact, a system that has one positive Lyapunov exponent is known as a chaotic system. However, a hyperchaotic system is defined as a chaotic system with more than one positive Lyapunov exponent. It is worth mentioning that hyperchaotic systems can show more complex dynamical behaviors than a chaotic system. Thus, the behavior of a hyperchaotic system has the characteristics of high security and it is widely used in secure communication [3], encryption [4] and so on.

The chaos control is an important research problem in the chaos theory. Many control strategies have been developed in the literature for the stabilization of nonlinear fractional-order chaotic and hyperchaotic systems such as the active control [5, 6], the adaptive control [7], the backstepping control [8], the fuzzy adaptive control [9], and the sliding mode control (SMC) [10].

A SMC is a robust nonlinear control. The main feature of the SMC is that it can switch the control law very quickly to drive the states of the system from any initial states onto some predefined sliding surface.

Recently, the SMC has been considered as a challenging research topic for the control and synchronization of fractional-order chaotic systems. For example, in [11], Roopaei et al. have introduced a class of integer-order chaotic systems covering about half of the recently published integer-order chaotic models. In [12], Yin et al. have presented a SMC law for a novel class of three different fractional-order nonlinear systems to realize the chaos control.
Motivated by the above two contributions, in this paper, we first introduce a novel class of fractional-order hyperchaotic systems. Then, we propose a SMC law to control hyperchaos in such fractional-order systems. The controller is used to stabilize the novel fractional-order hyperchaotic systems, even the fractional-order systems with uncertainty and external disturbance. Numerical simulations show that the proposed method can easily stabilize the system on the sliding surface.

The present manuscript is organized as follows. In Section 2 we present our novel class of fractional-order hyperchaotic systems. Section 3 presents the employment of the sliding mode control design of fractional hyperchaotic systems. Numerical simulations are presented to show the viability and efficiency of the proposed method in Section 4. Finally, the paper is concluded in Section 5.

2 Description of a Novel Class of Hyperchaotic Systems

Our proposed class of the fractional-order hyperchaotic systems is described as

$$\begin{cases} D^{\alpha_1} x_1 = x_2 f(x) - \xi_1(x), \\ D^{\alpha_2} x_2 = g(x) - \beta x_2, \\ D^{\alpha_3} x_3 = x_2 h(x) - \xi_2(x), \\ D^{\alpha_4} x_4 = x_2 k(x) - \xi_3(x), \end{cases} \tag{1}$$

where $x = (x_1, x_2, x_3, x_4)^T \in \mathbb{R}^4$ is the state variable, f, g, k, h, ξ_j, $j = 1, 2, 3$ are considered as a continuation of the nonlinear vector functions, which belong to $\mathbb{R}^4 \mapsto \mathbb{R}$ space,

β is the known parameter for any negative or non-negative value, $\alpha_i \in]0,1[$, $i = 1,2,3,4$, are the fractional orders, and D^α is the Caputo derivative which is defined as

$$D^\alpha x(t) = J^{n-\alpha} x^{(n)}(t), \quad \alpha \in (0,1), \tag{2}$$

where $n = \lceil \alpha \rceil$, i.e., n is the first integer which is not less than α; $x^{(n)}$ is the general n-order derivative and J^γ is the γ-order Riemann–Liouville integral operator expressed as follows:

$$J^\gamma y = \frac{1}{\Gamma(\gamma)} \int_0^t (t-\tau)^{\gamma-1} y(\tau) d\tau, \tag{3}$$

where $\Gamma(.)$ is the gamma function.

Remark 2.1 The major advantage of the Caputo definition is that the initial conditions for fractional-order differential equations take the same form as for integer-order differential equations.

Remark 2.2 In system (1), the fractional-order system is called a commensurate fractional-order system if $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$, otherwise the system is called an incommensurate fractional-order system.

Remark 2.3 Note that many hyperchaotic systems can be described by the proposed class (1). Table 1 details this class of fractional-order hyperchaotic systems.

3 Sliding Mode Control of a Fractional-Order Hyperchaotic System and Stability Analysis

In the following context, we shall design a sliding mode controller to establish the asymptotic stability of the fractional-order hyperchaotic system in question.

3.1 Control design via the sliding mode methodology

Let us consider the fractional-order hyperchaotic system (1), which is perturbed by the uncertainty $\Delta g(x)$ of $g(x)$ and the external disturbance $d(t)$.
Now, the control technique will be employed as

$$\begin{cases} D^{\alpha_1} x_1 = x_2 f(x) - \xi_1(x), \\ D^{\alpha_2} x_2 = g(x) - \beta x_2 + \Delta g(x) + d(t) + u, \\ D^{\alpha_3} x_3 = x_2 h(x) - \xi_2(x), \\ D^{\alpha_4} x_4 = x_2 k(x) - \xi_3(x). \end{cases} \tag{4}$$

In the sequel, the following assumptions are required.

Assumptions.

* Suppose that f, g, k , h and ξ_j, $j = 1,2,3$ are required to ensure the existence and uniqunes of the system (4) in the presence of the uncertainty $\Delta g(x)$ and the external disturbance $d(t)$ under the controller u in the interval $[t_0, +\infty[$, $t_0 > 0$ for any given initial condition.

* The uncertainties $\Delta g(x)$ and the external perturbation $d(t)$ are always bounded. Suppose that m_1, m_2 are the upper bound of $\Delta g(x)$ and $d(t)$, respectively, i.e.,

$$\begin{cases} \|\Delta g(x)\| \le m_1, \\ \|d(t)\| \le m_2. \end{cases} \tag{5}$$

Name and Mode	$f(.)$, $g(.)$,$h(.)$ and $k(.)$	$\xi_1(.)$, $\xi_2(.)$ and $\xi_3(.)$
Lorenz's system [14] $\begin{cases} D^{\alpha_1}x_1 = a(x_2 - x_1) + x_4, \\ D^{\alpha_2}x_2 = cx_1 - x_1x_3 - x_2, \\ D^{\alpha_3}x_3 = x_1x_2 - bx_3, \\ D^{\alpha_4}x_4 = -x_2x_3 + rx_4 \end{cases}$	$\begin{cases} f(x) = a, \\ g(x) = cx_1 - x_1x_3, \\ h(x) = x_1, \\ k(x) = -x_3 \end{cases}$	$\begin{cases} \xi_1(x) = -ax_1 + x_4, \\ \xi_2(x) = -bx_3, \\ \xi_3(x) = rx_4 \end{cases}$
Chen's system [15] $\begin{cases} D^{\alpha_1}x_1 = a(x_2 - x_1) + x_4, \\ D^{\alpha_2}x_2 = dx_1 + cx_2 - x_1x_3, \\ D^{\alpha_3}x_3 = x_1x_2 - bx_3, \\ D^{\alpha_4}x_4 = x_2x_3 + kx_4. \end{cases}$	$\begin{cases} f(x) = a, \\ g(x) = cx_1 - x_1x_3, \\ h(x) = x_1, \\ k(x) = -x_3 \end{cases}$	$\begin{cases} \xi_1(x) = -ax_1 + x_4, \\ \xi_2(x) = -bx_3, \\ \xi_3(x) = rx_4 \end{cases}$
Liu's system [16] $\begin{cases} D^{\alpha_1}x_1 = a(x_2 - x_1), \\ D^{\alpha_2}x_2 = bx_1 - x_4 + x_1x_3, \\ D^{\alpha_3}x_3 = -x_1x_2 - cx_3 + x_4, \\ D^{\alpha_4}x_4 = x_2 + dx_1. \end{cases}$	$\begin{cases} f(x) = a, \\ g(x) = bx_1 - x_4 + x_1x_3, \\ h(x) = -x_1, \\ k(x) = 1 \end{cases}$	$\begin{cases} \xi_1(x) = -ax_1, \\ \xi_2(x) = -cx_3 + x_4, \\ \xi_3(x) = dx_1 \end{cases}$
Finance's system [17] $\begin{cases} D^{\alpha_1}x_1 = x_3 + (x_2 - a)x_1 + x_4, \\ D^{\alpha_2}x_2 = 1 - bx_2 - x_2^2, \\ D^{\alpha_3}x_3 = -x_1 - cx_3, \\ D^{\alpha_4}x_4 = -dx_1x_2 - kx_4. \end{cases}$	$\begin{cases} f(x) = x_1, \\ g(x) = 1 - x_1^2, \\ h(x) = 0, \\ k(x) = -dx_1 \end{cases}$	$\begin{cases} \xi_1(x) = x_3 - ax_1 + x_4, \\ \xi_2(x) = -x_1 - cx_3, \\ \xi_3(x) = -kx_4 \end{cases}$
Lű's system [18] $\begin{cases} D^{\alpha_1}x_1 = a(x_2 - x_1) + x_4, \\ D^{\alpha_2}x_2 = cx_2 - x_1x_3, \\ D^{\alpha_3}x_3 = x_1x_2 - bx_3, \\ D^{\alpha_4}x_4 = x_1x_3 + dx_4. \end{cases}$	$\begin{cases} f(x) = a, \\ g(x) = -x_1x_3, \\ h(x) = x_1, \\ k(x) = 0 \end{cases}$	$\begin{cases} \xi_1(x) = -ax_1 + x_4, \\ \xi_2(x) = -bx_3, \\ \xi_3(x) = x_1x_3 + dx_4 \end{cases}$

Table 1: The class of fractional-order hyperchaotic systems characterized by the class (1).

To ensure the asymptotic stability of the dynamical system (4) on the switching surface, the fractional integral-type sliding mode surface s is selected as

$$s(t) = D^{\alpha_1 - 1}x_2 + \int_0^t \lambda x_2(\tau) + \Psi(\tau)d\tau, \tag{6}$$

where $\Psi(.)$ is a function selected as

$$\Psi(t) = x_1 f(x) + x_3 h(x) + x_4 k(x). \tag{7}$$

The controller gain λ has been introduced in the sliding mode surface s to confirm that the dynamics of the system will be stabilized quickly.

It is well known that for the sliding mode technique, the sliding surface and its derivative must satisfy

$$s(t) = 0, \quad \dot{s}(t) = D^{\alpha_1}x_2 + \lambda x_2 + x_1 f(x) + x_3 h(x) + x_4 k(x) = 0. \tag{8}$$

Therefore, the equivalent control law is obtained by

$$\begin{aligned} u_{eq} &= D^{\alpha_2}x_2 - g(x) - \Delta g(x) - d(t) + \beta x_2 & (9) \\ &= -g(x) - \Delta g(x) - d(t) - x_1 f(x) - x_3 h(x) - x_4 k(x) + (\beta - \lambda)x_2. & (10) \end{aligned}$$

In the real world applications, $\Delta g(x)$ and $d(t)$ are unknown. Therefore the equivalent control input is modified to

$$u_{eq} = -g(x) - x_1 f(x) - x_3 h(x) - x_4 k(x) + (\beta - \lambda) x_2. \tag{11}$$

To design the reaching mode control scheme, which drives the states onto the sliding surface, the reaching law can be selected as

$$u_{ad} = -\eta sign(s), \tag{12}$$

where

$$sgn(s) = \begin{cases} 1, & s > 0, \\ 0, & s = 0, \\ -1, & s < 0 \end{cases} \tag{13}$$

represents the sign function, and η is the reach gain of the controller, which is a positive constant. In this way, the total control law is constructed as

$$\begin{aligned} u &= u_{eq} + u_{ad} \\ &= -x_1 f(x) - x_3 h(x) - x_4 k(x) - g(x) + (\beta - \lambda) x_2 - \eta sign(s). \end{aligned} \tag{14}$$

3.2 Stability analysis

Theorem 3.1 *If the controller u is selected as in the equation(14), then the trajectories of the fractional-order dynamics (4) converge to the sliding surface $s(t) = 0$ for $m_1 + m_2 < \eta$.*

Proof. Define the following Lyapunov functional candidate:

$$V = \frac{1}{2} s^2. \tag{15}$$

The time derivative of V is given by

$$\begin{aligned} \dot{V} &= \dot{s} s = \left\{ D^{\alpha 2} x_2 + \lambda x_2 + x_1 f(x) + x_3 h(x) + x_4 k(x) \right\} s \\ &= \{ g(x) - \beta x_2 + \Delta g(x) + d(t) + u + \lambda x_2 + x_1 f(x) + x_3 h(x) + x_4 k(x) \} s \\ &= \{ \Delta g(x) + d(t) - \eta sign(s) \} s \\ &\leq (m_1 + m_2 - \eta) |s|. \end{aligned} \tag{16}$$

Equation (16) implies that as long as suitable m_1, m_2 and η, which satisfy $m_1 + m_2 < \eta$, are selected, one obtains $\dot{V} < 0$.

In view of Barbalat's lemma [19], it can be concluded that $s, \dot{s} \in L_\infty$. As $t \to \infty$, s approaches zero, which shows that all trajectories of the proposed system will converge to the sliding surface $s(t) = 0$. This completes the proof.

Remark 3.1 In the case when the system uncertainty and external disturbance are ignored and if the controller u is selected as in equation(14), the trajectories of the fractional-order systems (4) converge to the sliding surface $s(t) = 0$ for all $\eta > 0$.

Proof. Define the following Lyapunov functional candidate:

$$V = \frac{1}{2} s^2. \tag{17}$$

The time derivative of V is given by

$$\begin{aligned}
\dot{V} &= \dot{s}s = \left\{D^{\alpha 2}x_2 + \lambda x_2 + x_1 f(x) + x_3 h(x) + x_4 k(x)\right\} s \\
&= \{g(x) - \beta x_2 + \Delta g(x) + d(t) + u + \lambda x_2 + x_1 f(x) + x_3 h(x) + x_4 k(x)\} s \\
&= -\eta sign(s)s \\
&= -\eta |s| < 0. \qquad (18)
\end{aligned}$$

In view of Barbalat's lemma [19], it can be concluded that $s, \dot{s} \in L_\infty$. As $t \to \infty$, s approaches zero, which shows that all trajectories of the proposed system will converge to the sliding surface $s(t) = 0$, for all $\eta > 0$.

4 Simulation Results

To illustrate the performance of the proposed control approach, we present two examples, namely, fractional-order hyperchaotic Liu's system and fractional-order hyperchaotic Lorenz's system. Numerical simulations are implemented using the MATLAB software.

4.1 Sliding mode control design of hyperchaotic Liu's system

Here, we will firstly consider a case when the system uncertainty and external disturbance are ignored. By introducing the control input to the second state equation of fractional-order hyperchaotic Liu's system, the controlled system is derived as

$$\begin{cases} D^{\alpha_1}x_1 = a(x_2 - x_1), \\ D^{\alpha_2}x_2 = bx_1 - x_4 + x_1x_3 + u, \\ D^{\alpha_3}x_3 = -x_1x_2 - cx_3 + x_4, \\ D^{\alpha_4}x_4 = x_2 + dx_1. \end{cases} \qquad (19)$$

For the fractional order values $\alpha_1 = 0.98$, $\alpha_2 = 0.97$, $\alpha_3 = 0.97$ and $\alpha_4 = 0.98$, the system (19) without the controller u exhibits a hyperchaotic behavior, as shown in Figure 1, when the parameters are given by

$$(a, b, c, d, k) = (10, 35, 1.4, 5), \qquad (20)$$

and the initial value is taken as

$$(x_1(0), x_2(0), x_3(0), x_4(0))^T = (10, 15, 1, 1)^T. \qquad (21)$$

The sliding surface is given by

$$S(t) = D^{\alpha_1 - 1}x_2 + \int_0^t \lambda x_2(\tau) + \Psi(\tau)d\tau, \qquad (22)$$

where

$$\Psi(t) = ax_1 - x_1x_3 + x_4. \qquad (23)$$

According to the general control law given by equation(14), the vector controller u can be designed as

$$u = -(a + b)x_1 - \lambda)x_2 - \eta sign(s). \qquad (24)$$

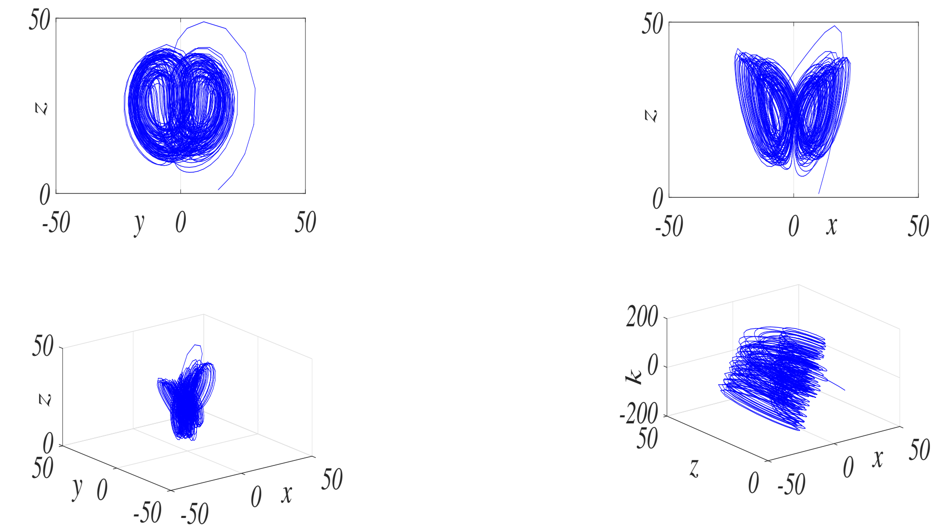

Figure 1: Hyperchaotic attractors of Liu's system (19).

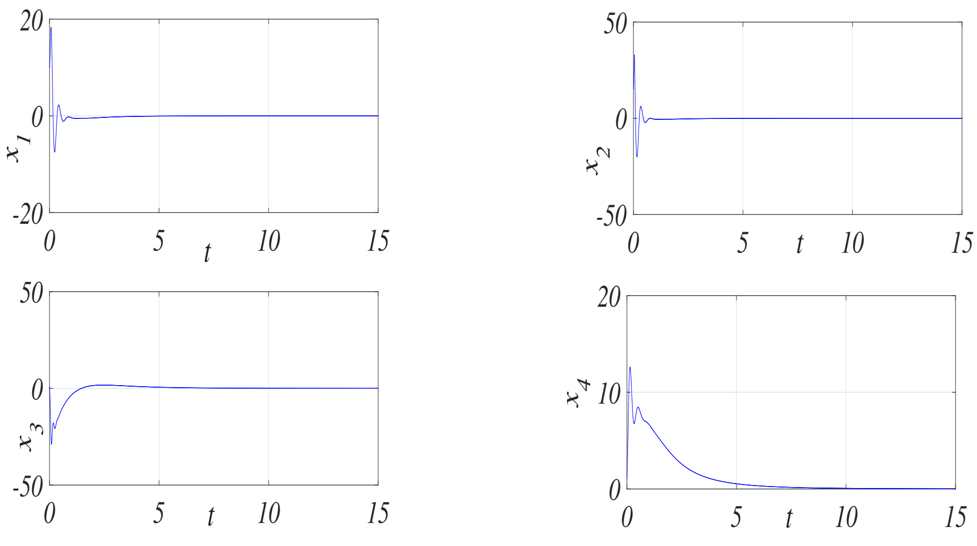

Figure 2: Time-history of the controlled states of equation(19).

With the gain of control law $\eta = 0.02$ and the parameter $\lambda = 0.01$, the states x_1, x_2, x_3 and x_4 of the system (19) with the sliding surface (22) in the presence of the controller (24) are illustrated in Figure 2.

From Figure 2, it is clear that the control law (24) is efficient for controlling fractional-order hyperchaotic Liu's system.

4.2 Sliding mode control design of uncertain hyperchaotic Lorenz's system

In this part, we consider the fractional-order version of hyperchaotic Lorenz's system in the presence of uncertainty and external disturbance, which is expressed as

$$\begin{cases} D^{\alpha_1}x_1 = a(x_2 - x_1) + x_4, \\ D^{\alpha_2}x_2 = cx_1 - x_1x_3 - x_2 + \Delta g(x) + d(t) + u, \\ D^{\alpha_3}x_3 = x_1x_2 - bx_3, \\ D^{\alpha_4}x_4 = -x_2x_3 + rx_4. \end{cases} \tag{25}$$

For the fractional order values $\alpha_1 = 0.95$, $\alpha_2 = 0.96$, $\alpha_3 = 0.96$ and $\alpha_3 = 0.97$, the system

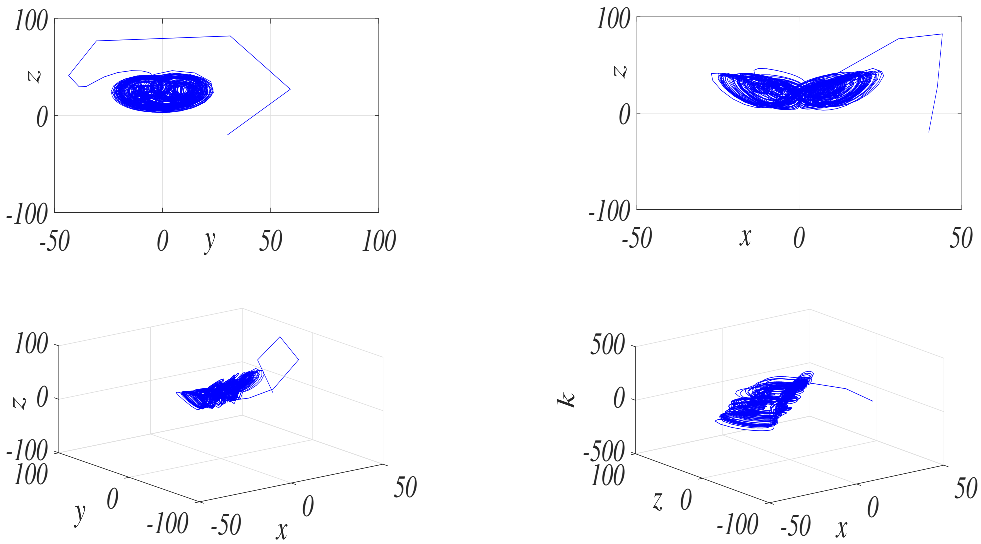

Figure 3: Hyperchaotic attractors of the Lorenz's System (25).

(25) without the uncertainty $\Delta g(x)$, the external disturbance $d(t)$ and the controller u, exhibits a hyperchaotic behavior, as shown in Figure 3, when the parameters of the system are given by

$$(a, b, c, r) = (10, \frac{8}{3}, 28, -1), \tag{26}$$

and the initial value

$$(x_1(0), x_2(0), x_3(0), x_4(0))^T = (40, 30, -20, -50)^T. \tag{27}$$

The uncertainty $\Delta f(x)$ applied to the system is given by

$$\Delta g(x) = 0.05\cos(2x_2). \tag{28}$$

The external disturbances $d(t)$ are defined as

$$d(t) = 0.02\sin(2t). \tag{29}$$

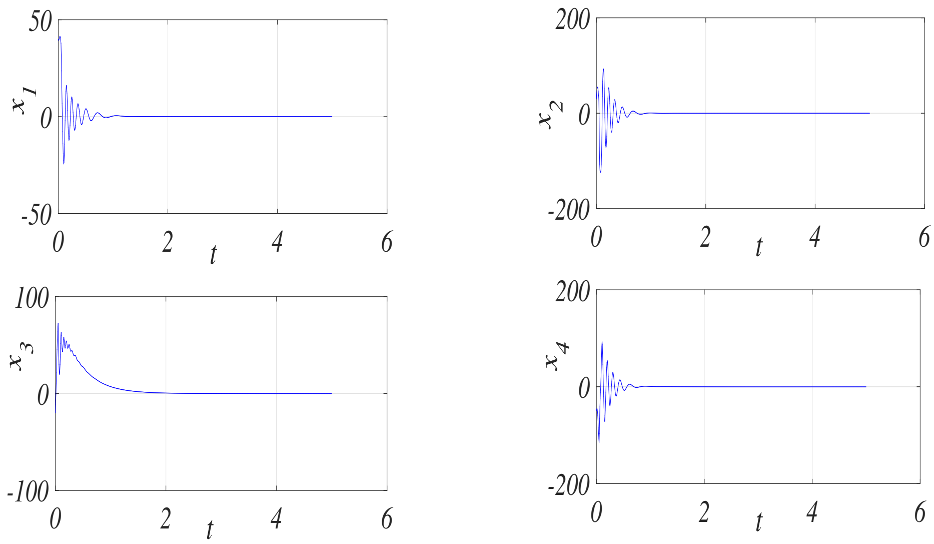

Figure 4: Time-history of the controlled states of equation (25).

The sliding surface is given by

$$s(t) = D^{\alpha_1 - 1} x_2 + \int_0^t \lambda x_2(\tau) + \Psi(\tau) d\tau, \tag{30}$$

where

$$\Psi(t) = a x_1 + x_1 x_3 - x_3 x_4. \tag{31}$$

According to the general control law given by equation(14), the vector controller u can be designed as:

$$u = -(a + c)x_1 + x_3 x_4 + (1 - \lambda)x_2 - \eta sign(s). \tag{32}$$

With the gain of control law $\eta = 0.02$ and the positive parameter $\lambda = 1$, the states x_1, x_2, x_3 and x_4 of the system (25) with the sliding surface (30) in the presence of the controller (32) are illustrated in Figure 4. From Figure 4, the control law (32) is capable of controlling fractional-order hyperchaotic Lorenz's system in the presence of uncertainty and external disturbance.

5 Conclusion

In this paper, a novel class of fractional-order hyperchaotic systems with uncertainty and external disturbance has been proposed. Based on the Lyapunov stability theorem, a sliding mode control law has been designed to control hyperchaos in such fractional-order systems. The sliding mode controller has been shown to guarantee the asymptotic stability of the proposed fractional-order hyperchaotic systems in the presence of uncertainty and external disturbance. From the numerical examples for the class of fractional-order Liu and Lorenz systems, it is obvious that a satisfying control performance can be realised by using the proposed scheme.

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References

[1] S. Sutha, P. Lakshmi and S. Sankaranarayanan. Fractional-order sliding mode controller design for a modified quadruple tank process via multi-level switching. *Comput. Electr. Eng.* **45** (2015) 10–21.

[2] T. T. Hartley, C. F. Lorenzo and H. K. Qammar. Chaos in a fractional order Chua system. *IEEE Trans. Circuits Syst. I.* **42** (8) (1996) 485–490.

[3] I. M. Olga, A. K. Alexey, and E. H. Alexander. Generalized synchronization of chaos for secure communication: remarkable stability to noise. *Physics Letters A* **374** (29) (2010) 2925–2931.

[4] E. Inzunza-Gonzalez and C. Cruz-Hernandez. Double hyperchaotic encryption for security in biometric systems. *Nonlinear Dynamics and Systems Theory* **13** (1) (2013) 55–68.

[5] S. Kaouache and M. S. Abdelouahab. Generalized synchronization between two chaotic fractional non-commensurate order systems with different dimensions. *Nonlinear Dynamics and Systems Theory* **18** (3) (2018) 273–284.

[6] S. Kaouache and M. S. Abdelouahab. Modified projective synchronization between integer order and fractional order hyperchaotic systems *Jour. of Adv. Research in Dynamical and Control Systems* **10** (5) (2018) 96–104.

[7] J. H. Park. Adaptive control for modified projective synchronization of a four-dimensional chaotic system with uncertain parameters. *J. Comput. Appl. Math.* **213** (1) (208) 288–293.

[8] K. S. Ojo, A. N. Njah, S. T. Ogunjo and O. I. Olusola. Reduced Order Function Projective Combination Synchronization of Three Josephson Junctions Using Backstepping Technique. *Nonlinear Dynamics and Systems Theory* **14** (2) (2014) 119–133.

[9] A. Poursamad and A. H. Davaie-Markazi. Robust adaptive fuzzy control of unknown chaotic systems. *Appl. Soft. Comput.* **9** (3) (2009) 970–976.

[10] M. Tavazoei and M. Haeri. Synchronization of chaotic fractional-order systems via active sliding mode controller. *Physica A: Statistical Mechanics and its Applications* **387** (2008) 57–70.

[11] M. Roopaei, B. R. Sahraei and T. C. Lin. Adaptive sliding mode control in a novel class of chaotic systems. *Communications in Nonlinear Science and Numerical Simulation* **15** (12) (2010) 4158–4170.

[12] C. Yin, S. M. Zhong and W. F. Chen. Design of sliding mode controller for a class of fractional-order chaotic systems. *Communications in Nonlinear Science and Numerical Simulation* **17** (1) (2012) 356–366.

[13] I. Podlubny. *Fractional Differential Equations.* Academic Press, San Diego, 1999.

[14] X. Y. Wang and J. M. Song. Synchronization of the fractional order hyperchaos Lorenz systems with activation feedback control. *Commun. Nonlinear Sci. Numer. Simul.* **14** (8) (2009) 3351–3357.

[15] X. Li. Modified projective synchronization of a new hyperchaotic system via nonlinear control. *Chin. Phys. B* **52** (2009) 274–278.

[16] L. Liu, C. Liu and Y. Zhang. Analysis of a novel four-dimensional hyperchaotic system. *Chin. J. Phys.* **46** (2008) 386–393.

[17] H. Yu, G. Cai and Y. Li. Dynamic analysis and control of a new hyperchaotic finance system. *Nonlinear Dyn.* **67** (2012) 2171–2181.

[18] A. Chen, J. Lu, J. Lű and S. Yu. Generating hyperchaotic Lu attractor via state feedback control. *Physica A* **364** (2006) 103–110.

[19] H. K. Khalil. *Nonlinear Systems.* 2nd ed., Prentice Hall, New Jersey, USA, 2001.