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An Enhanced Bi-Directional Chaotic Optimization Algorithm

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Abstract: Based on the improved chaos searching strategy, an enhanced Bi-directional chaotic optimization algorithm (EBCOA)is proposed in this study. A Lozi chaos mapping is used as a chaos generator to produce a chaos variable. In the process of EBCOA, and in order to make the chaos search more efficient, a new sub-step local chaos optimization method is proposed and a global search is done to find the current optimal solution in a certain range, and then a fine search reduces the space of optimized variables. Compared with the algorithm of traditional chaos search, the proposed algorithm is more accurate and can respond quickly. Simulation and experimental results confirm the efficiency of the proposed algorithm.

Keywords: *chaos; global optimization; chaotic map; chaos optimization algorithm.*

Mathematics Subject Classification (2010): 34D45, 70K55.

1 Introduction

In the field of mathematics, physics and engineering science, it is well recognized that chaos theory can be applied as a very useful technique in practical application. Chaos is aperiodic behavior in a deterministic system which exhibits sensitive dependence on initial conditions, and thus provides great diversity based on the ergodic property of the chaos phase, which transits every state without repetition in certain ranges. Chaos is a term used to describe behavior that is seemingly random, but has an underlying mathematical order to it [1–5]. Chaos is very common in nature, but is often mistaken for random behavior. It is generated through a deterministic iteration formula. Due to

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these characteristics, chaos theory can be applied in the optimization algorithm [6, 7]. [9] proposed a chaotic differential evolution algorithm for multi-objective optimization. Many deterministic, stochastic methods for solving the global optimization problem have been proposed which, in turn, employed local moves or local exploitation, i.e., a new candidate point is generated in a neighborhood of the current one. For example, all Multistart-like algorithms generate candidate points in a neighborhood of the current one, Genetic Algorithms use mutation to generate a point in the neighborhood of a member of the current population, etc. The number of local minima is a critical issue for global optimization problems. It is well known that local moves alone are not enough to detect a global minimum because of geting trapped into a local minimum. Therefore, we need to employ other techniques to escape from local minima such random generation of starting points in Multistart-like algorithms; crossover in Genetic Algorithms, chaotic generation of starting points in two-phase algorithms (COA) [10–17].

In this study, an enhanced bi-directional chaos optimization algorithm (EBCOA) based on a new chaos search strategy is proposed in order to deal with premature convergence in later evolution. From the testing results of the benchmark functions, the results of EBCOA are obviously better than those of the standard bi-directional chaos optimization algorithm (BCOA). The rest of the paper is organized as follows. In Section 2, we describe the BCOA presented in the literature and we present a new approach, the EBCOA, based on the nested phases strategy and the use of 2-D chaotic sequences. In Section 3, simulation results are provided to validate the effectiveness of the proposed method. The paper ends with the conclusion as Section 4 followed by the references.

2 Chaos Search Strategy

Chaos occurs in many nonlinear systems, which is generated by deterministic equations. Chaotic systems with their interesting properties such as topologically mixing and dense periodic orbits, ergodicity and intrinsic stochasticity, can be used in various applications such as global optimization. In feature selection, chaos search is more capable of escaping from local optima than random search. One way of application with chaos is a chaotic optimization algorithm (COA) [6, 7, 13, 16, 17], which utilizes the nature of chaos sequence including the quasi-stochastic property and ergodicity. The experimental studies assert that the benefits by chaotic variables instead of random variables are more obvious although the mathematical theory can not be formulated.

2.1 Generation of chaotic sequences

In this section, we present the chaotic maps used, which generate chaotic sequences in the process of evolutionary algorithms [12]. Chaos theory studies the behavior of systems that follow deterministic laws but appear to be random and unpredictable, i.e., dynamical systems. Chaotic variables can go through all states in certain ranges according to their own regularity without repetition [10–12]. A chaotic map is a map that exhibits some type of chaotic behavior. In this work, we applied 2-D chaotic maps that are common in the literature, namely, the Lozi map [18] given by

$$\begin{cases} y_1(k) = 1 - a|(y_1(k-1))| + by(k-1), \\ y(k) = y_1(k-1), \end{cases} \tag{1}$$

$$z(k) = \frac{y(k) - \alpha}{\alpha' - \alpha}, \tag{2}$$

where k is the iteration number. In this work, the values of y are normalized in the range [0;1] to each decision variable in n-dimensional space of the optimisation problem. Therefore, $y_1 \in [-0.6417; 0.6716]$ and $(\alpha; \alpha') = (-0.6418; 0.6716)$.

The parameters used in this study are $a = 1.7$ and $b = 0.5$, see Figure 1, these values are suggested in [13].

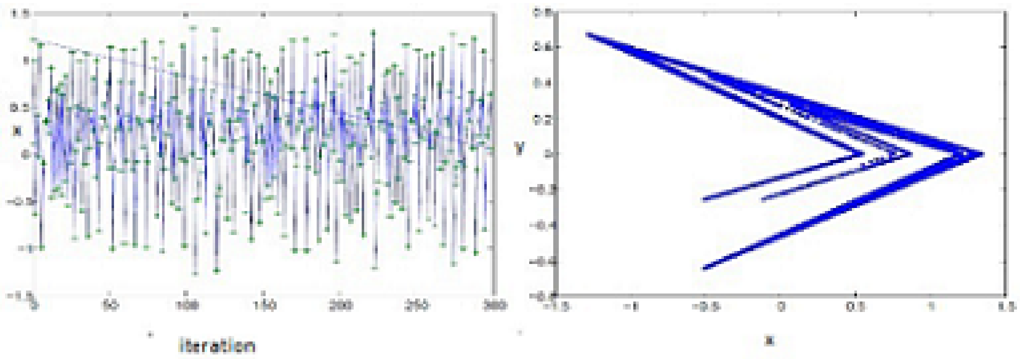

Figure 1: Attractor and temporal series of the Lozi map.

2.2 Two-phase methods and basic BCOA

In this section we briefly recall the BCOA introduced by Ying Song [1]. Many chaotic strategies in global optimization consist of two phases: the global phase and the local phase. During the global phase, chaotic points are drawn from the domain of searches X according to a certain, often uniform, distribution. Then, the objective function is evaluated in these points. During the local phase, the sample points are manipulated by means of local search to yield a candidate global minimum. Consider the following optimization problem for a nonlinear function:

$$minf(X), \quad X = [x_1, x_2, x_3, ..., x_n],$$

$$L_i \leq x_i \leq U_i.$$

The chaotic variables are

$$Z^{(k+1)} = g(Z^k),$$

where Z^k are chaotic states generated by the chaotic equation.

The basic process of the BCOA [1] strategy can be described as follows.

Step 1: also called the first carrier wave. Define a chaotic sequences generator based on the Logistic map. Generate a sequence of the chaotic points and map it to a sequence of decision points in the original decision space. Then, calculate the objective functions with respect to the generated decision points, and choose the point with the minimum objective function as the current optimum.

The ergodic area of chaotic variables to the variance range of optimisation variables is

$$X^k = c + d \cdot Z^k,$$

where c and d are constant vectors such as amplification gains and, respectively, consist of n elements $c_i = L_i$ and $d_i = U_i - L_i$.

Step 2: also called the second carrier wave. The current optimum is assumed to be close to the global optimum after certain iterations, and it is viewed as the center with a little chaotic perturbation and the global optimum is obtained through the fine search. Repeat the above two steps until some specified convergence criterion is satisfied, and the global optimum is obtained.

The approach of the second carrier wave is as follows:

$$X = X^* + \beta X^*(0.5 - Z),$$

so the search is on both two sides of the sub-optimal solution. Here X^* is the so far best solution. β is the parameter of the second carrier.

We have

$$-0.5\beta \leq \beta(0.5 - Z) \leq 0.5\beta \quad as \quad \beta \geq 0, \tag{3}$$

$$0.5\beta \leq \beta(0.5 - Z) \leq -0.5\beta \quad as \quad \beta \leq 0, \tag{4}$$

so the search is on both two sides of the sub-optimal solution.

3 Proposed EBCOA

3.1 Block flow diagram of EBCOA

Applying the local search technique has been hot and can bring two benefits to the whole search procedure. First, the search can be driven into a better area further from local optima. Second, but not less important, the exploitation of some promising areas of the search space can be enhanced so as to speed up the convergence of the search.

The BCOA method [1] is then improved by the local search around every point obtained by the chaotic series. The logistic map [1, 6, 7] is usually adopted in the COA. But the distribution of chaotic sequences produced by the logistic map is uniformly leading to the slow constringent. The Lozi map marked by (1) is a Gaussian map with which we replace the logistic map to accelerate the rate of convergence.

The EBCOA can be illustrated as follows, where M_g, Ml and M_gl are the maximum number of iterations of the chaotic global search, maximum number of iterations of the chaotic local search and maximum number of iterations of the chaotic local search in the global search, respectively. β is the step size in the chaotic local search, $\bar{x_i}$ is the best solution.

3.2 Step-size control

It is well-established that the convergence of a chaotic optimization algorithm directly depends on how it controls the step size. Moreover, the step-size control influences to a large extent the rate at which a chaotic optimization algorithm approaches the optimum. The step-size adaptation mechanisms are all based on the idea that the smaller the step size, the higher the probability of sampling good solutions.

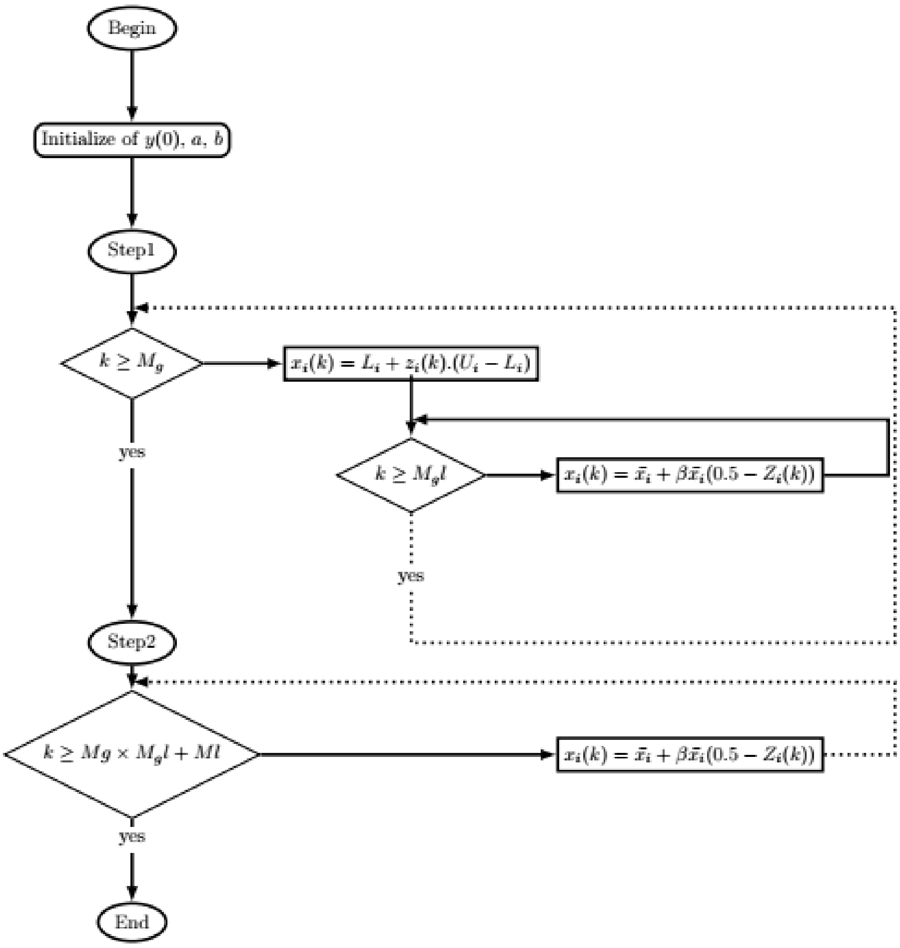

Figure 2: Block flow diagram of the EBCOA.

4 Simulation Results

In applied mathematics, test functions, known as artificial landscapes, are useful to evaluate characteristics of optimization algorithms. For testing our approach, and from the standard set of benchmark problems available in the literature, we use two well known nonlinear benchmark functions [21, 22]. In our study, we overcome this limitation using a number of dimensions 2 and comparing with other heuristic optimization algorithms. The Griewank function has many irregularities but there is only one unique global minimum. The Rastrigin function has many local optimal points and one unique global minimum. Table 1 resumes the global optimum, the function value at global optimum and the search range used for each test function. Figure 2 presents the plot for each test function. All the programs were run on a 2 GHz Pentium IV processor with 2 GB of random access memory in the MATLAB. In each case study, 50 independent runs were made for each of the EBCOA methods. In the tested cases to benchmark problems, the maximum numbers of iterations $maxK$ and $maxK^{'}$ were 10000 and 10000 iterations.

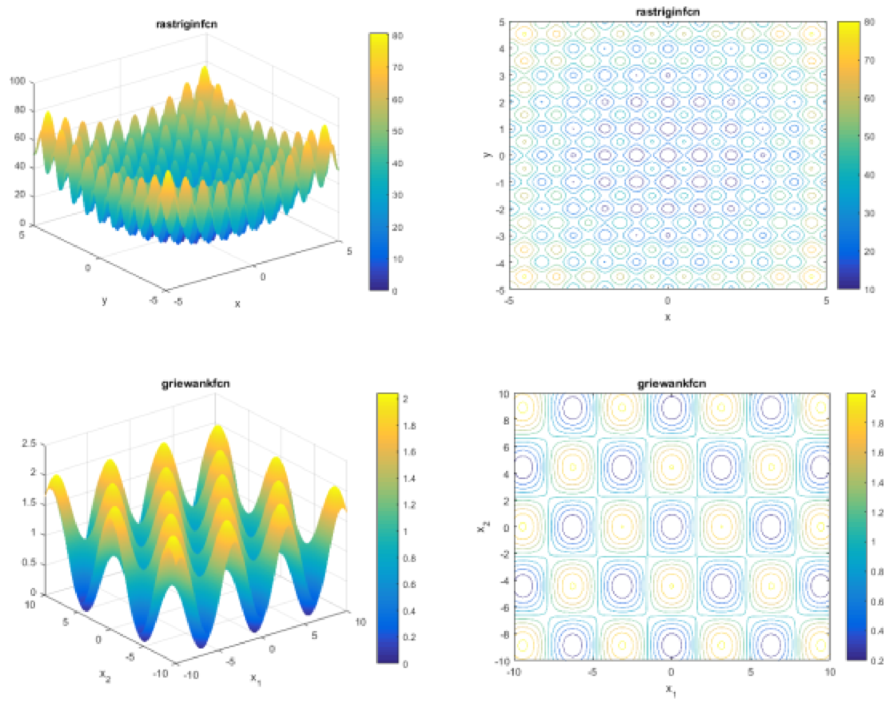

Figure 3: A perspective view and the related contour lines or some of functions when $n = 2$.

4.1 Results for the Rastrigin function

		BCOA	EBCOA
$K^{'}$	β	optimum	optimum
1001	700	4.2752e-6	0
1001	500	4.7997e-9	0
1001	400	1.1219e-11	0
2405	200	3.5527e-15	0
1023	0.1	3.9080e-14	3.90798505 e -14
6965	0.01	4.3343e-13	4.192202141 e -13
$maxK^{'}$	1e -3	2.6392e-5	5.419204974544 e - 6
$maxK^{'}$	1e -4	4.7111e-4	8.3677132572291 e-5
$maxK^{'}$	-(1 e -3)	2.8008e-5	2.1552183152806 e-5
$maxK^{'}$	-(1 e -4)	4.7392e-4	5.0020093164866 e -5

Table 1: Rastrigin optimum for $n = 2$ with different β.

		BCOA	EBCOA
optimum	β	$K^{'}$	$K^{'}$
0	100	1206	10
	10	399	10
	1	368	10
	-100	1918	10
	-10	421	10

Table 2: Number of iterations with different β.

From Table 2, for $\beta \geq 200$, the EBCOA can find the actual optimum 0. Here $|\beta| \in [1e-4, 200] \cup]-120, -(1e-4)]$. The optimum is improved. From Table 3 for $\beta \in]0.1, 100]$, the EBCOA can also find the actual optimum 0 but with the number of iterations less than that in the BCOA.

The optimum value and the convergence speed are better than those in the COA [7] and its improvements, such as the MSCOA [19], COA-BFGS [14] and other evolutionary algorithms (such as the GA, PSO and its improvements) [20–22].

4.2 Results for the Griewank function.

		BCOA	EBCOA
$K^{'}$	β	optimum	optimum
802	11.12	0	0
346	11.10	0.2533	0
904	-9.93	0	0
372	-9.91	0.2516	0

Table 3: Griewank optimum for $n = 2$ with different β.

		BCOA	EBCOA
optimum	β	$K^{'}$	$K^{'}$
0	11.09	802	10
	10	550	10
	1.60	347	10
	-9.90	904	10
	-10	489	10
	-1.33	369	10

Table 4: Number of iterations with different β.

From Table 4 we find that, for $\beta \geq 11.09$ and $\beta \leq -9.90$, the EBCOA can always find the actual optimum 0, and for $\beta \in [1.60, 11.09] \cup [9.90, -1.33]$, the EBCOA can also find the actual optimum 0 but with the number of iterations less than that in the BCOA. The optimum value and the convergence speed are better than those in the COA [7] and its improvements such as the MSCOA [19], COA-BFGS [14] and other evolutionary algorithms (such as the GA, PSO and its improvements) [20–22].

5 Conclusion

Based on the ergodic property, chaos is adopted to enrich the search behavior and prevent solutions from being trapped in the local optimum in optimization problems. This paper focuses on exploring the effects of chaotic maps and giving guidance for improving the Bi-directional chaotic optimization algorithm in solving optimization problems. Through proposing a new algorithm, the EBCOA, we have improved the BCOA doing some modification in the global step of research, we refined the final solution using a second bi-directional method of local search. The presented study allows us to conclude that the proposed method is fast and converges to a good optimum.

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