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Synchronization of Time-Delay Chua's Oscillator with Application to Secure Communication

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Abstract: In this paper, we use a Generalized Hamiltonian systems approach to synchronize the time-delay-feedback Chua's oscillator (hyperchaotic circuit with multiple positive Lyapunov exponents). Synchronization is thus between the transmitter and the receiver dynamics with the receiver being given by an observer. We apply this approach to transmit private analog and binary information signals in which the quality of the recovered signal is higher than in traditional observer techniques while the encoding remains potentially secure.

Keywords: *Synchronization; time-delay-feedback Chua's oscillator; hyperchaos; passivity based observers; secure communication.*

Mathematics Subject Classification (2000): 37N35, 65P20, 68P25, 70K99, 93D20, 94A99.

1 Introduction

Recently, chaotic synchronization has received much attention. Many synchronization methods for chaotic oscillators have been proposed in the literature (see e.g. (Pecora and Carroll 1990; Wu and Chua 1993; Feldmann, *et al.* 1996; Nijmeijer and Mareels 1997; Special Issue, 1997; 2000; Fradkov, *et al.* 1998; Chen and Dong 1998; Cruz and Nijmeijer 1999; 2000; Sira-Ramírez and Cruz 2001; Pikovsky, *et al.* 2001; Aguilar and Cruz 2002) and references therein). Data encryption using chaotic dynamics was reported in the early 1990s as a new approach for signal encoding which differs from the conventional methods using numerical algorithms as the encryption key. As a result,

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chaotic synchronization plays an important role in chaotic communications. Different methods have been developed in order to hide the contents of a message using chaotic synchronization, such as *chaotic additive masking* (Cuomo, *et al.* 1993), *chaotic switching* (Parlitz, *et al.* 1992; Cuomo, *et al.* 1993; Dedieu, *et al.* 1993), and *chaotic parameter modulation* (Yang and Chua 1996). However, it has been shown see e.g., (Short 1994; Pérez and Cerdeira 1995) that encrypted signals by means of comparatively simple chaos with only one positive Lyapunov exponent does not ensure a sufficient level of security. For higher security the hyperchaotic oscillators characterized by more than one positive exponents are advantageous over simple chaotic oscillators. Two factors that are of primordial importance in security considerations related to chaotic communication systems. These are; the dimensionality of the chaotic attractor, and the effort required to obtain the necessary parameters for the matching of a receiver dynamics.

One way to enhance the level of security of the communication system can consist in applying proper cryptographic techniques to the information signal see e.g., (Yang, *et al.* 1997). Another way to solve this security problem is to encode the message by using high dimensional chaotic attractors, or hyperchaotic attractors, which take advantage of the increased randomness and unpredictably of the higher dimensional dynamics. In such option one generally encounters *multiple positive Lyapunov exponents*. However, the synchronization of hyperchaotic oscillators is a much more difficult problem (see e.g. (Brucoli, *et al.* 1999; Peng, *et al.* 1996; Cruz, *et al.* 2002) and Aguilar and Cruz 2002 for the discrete-time context). Most of the previous work done on hyperchaotic synchronization has been concentrated on finite-dimensional oscillators described by ordinary differential equations. Thus, the number of positive Lyapunov exponents is limited by dimension of the state space.

As alternative way of constructing synchronized hyperchaotic oscillators can be based on delay differential equations, such oscillators have an infinite-dimensional state space and can produce hyperchaotic dynamics with an arbitrarily large number of positive Lyapunov exponents. It has been known that even a very simple first-order oscillator with a time-delay-feedback can produce extremely complex hyperchaotic behaviors (Mackey and Glass 1977; Farmer 1982; Lu and He 1996). This property has already stimulated the work on design of systems for secure communication which claimed to have low detectability (Mensour and Longtin 1998; Pyragas 1998).

The objective of this paper is to extend the approach developed in (Sira-Ramírez and Cruz 2001) to the synchronization of time-delay-feedback Chua's oscillator (hyperchaotic circuit with multiple positive Lyapunov exponents) through a Generalized Hamiltonian systems and observer approach. Moreover, we apply this method to transmit and retrieve private/secure analog and binary information signals using hyperchaotic additive masking and hyperchaotic switching, respectively. We can enumerate several advantages over the existing synchronization methods:

- It enables synchronization be achieved in a systematic way;
- It can be successfully applied to several well-known chaotic or hyperchaotic oscillators;
- It does not require the computation of any Lyapunov exponent;
- It does not require initial conditions belonging to the same basin of attraction.

The organization of the paper is as follows. In Section 2, we obtain the synchronization of the time-delay-feedback Chua's oscillator through a Generalized Hamiltonian systems and observer approach. In Section 3, we present the stability analysis related to the synchronization error. In Section 4, we give an application to secure communication of private analog and binary information signals. Finally, Section 5 is devoted to some concluding remarks and suggestions for further work.

2 Synchronization of Time-Delay Chaotic Oscillator

Consider the following dynamical system described by

$$\dot{x} = f\left(x,\, x\left(t-\tau\right)\right), \tag{1}$$

where $x\left(t\right) = \left(x_1, \dots, x_n\right)^T \in \mathbb{R}^n$ is the state vector, f is a nonlinear function, and τ is the time-delay. The system (1) provides an example of infinite-dimensional oscillator with *multiple positive Lyapunov exponents* (generating extremely complex hyperchaotic signals). Following the approach developed in (Sira-Ramírez and Cruz 2001), the time-delay oscillator described by equation (1) can be written in the following *Generalized Hamiltonian canonical form*,

$$\dot{x} = \mathcal{J}(x)\,\frac{\partial H}{\partial x} + \mathcal{S}(x)\,\frac{\partial H}{\partial x} + \mathcal{F}(x, x\left(t-\tau\right)), \qquad x \in \mathbb{R}^n \tag{2}$$

where $H(x)$ denotes a smooth *energy function* which is globally positive definite in $\mathbb{R}^n$. The column *gradient vector* of H, denoted by $\partial H/\partial x$, is assumed to exist everywhere. We use *quadratic* energy function $H(x) = \frac{1}{2}\,x^T\mathcal{M}x$ with $\mathcal{M}$ being a, constant, symmetric positive definite matrix. In such case, $\partial H/\partial x = \mathcal{M}x$. The square matrices, $\mathcal{J}(x)$ and $\mathcal{S}(x)$ satisfy, for all $x \in \mathbb{R}^n$, the following properties, which clearly depict the *energy managing* structure of the system, $\mathcal{J}(x) + \mathcal{J}^T(x) = 0$, and $\mathcal{S}(x) = \mathcal{S}^T(x)$. The vector field $\mathcal{J}(x)\,\partial H/\partial x$ exhibits the *conservative* part of the system and it is also referred to as the *workless* part, or *work-less* forces of the system; and $\mathcal{S}(x)$ depicting the *working* or *nonconservative* part of the system. For certain systems, $\mathcal{S}(x)$ is *negative definite* or *negative semidefinite*. In such cases, the vector field is addressed to as the *dissipative* part of the system. If, on the other hand, $\mathcal{S}(x)$ is positive definite, positive semidefinite, or indefinite, it clearly represents, respectively, the global, semi-global, and local *destabilizing* part of the system. In the last case, we can always (although nonuniquely) discompose such an indefinite symmetric matrix into the sum of a symmetric negative semidefinite matrix $\mathcal{R}(x)$ and a symmetric positive semidefinite matrix $\mathcal{N}(x)$. And where $\mathcal{F}(x, x(t-\tau))$ represents a *locally destabilizing* vector field.

In the context of observer design, we consider a special class of Generalized Hamiltonian systems with destabilizing vector field and linear output map, $y(t)$, given by

$$\begin{aligned} \dot{x} &= \mathcal{J}(y)\,\frac{\partial H}{\partial x} + (\mathcal{I} + \mathcal{S})\,\frac{\partial H}{\partial x} + \mathcal{F}(y, y(t-\tau)), \qquad x \in \mathbb{R}^n, \\ y &= \mathcal{C}\,\frac{\partial H}{\partial x}, \qquad y \in \mathbb{R}^m, \end{aligned} \tag{3}$$

where $\mathcal{S}$ is a constant symmetric matrix, not necessarily of definite sign. The matrix $\mathcal{I}$ is a constant skew symmetric matrix. The vector variable $y(t)$ is referred to as the system *output*. The matrix $\mathcal{C}$ is a constant matrix.

We denote the *estimate* of the state vector $x(t)$ by $\xi(t)$, and consider the Hamiltonian energy function $H(\xi)$ to be the particularization of H in terms of $\xi(t)$. Similarly, we denote by $\eta(t)$ the estimated output, computed in terms of the estimated state $\xi(t)$. The gradient vector $\partial H(\xi)/\partial\xi$ is, naturally, of the form $\mathcal{M}\xi$ with $\mathcal{M}$ being a, constant, symmetric positive definite matrix.

A dynamic *nonlinear state observer* for the Generalized Hamiltonian system (3) is readily obtained as

$$\begin{aligned} \dot{\xi} &= \mathcal{J}(y)\,\frac{\partial H}{\partial \xi} + (\mathcal{I}+\mathcal{S})\,\frac{\partial H}{\partial \xi} + \mathcal{F}(y, y(t-\tau)) + K(y-\eta), \qquad \xi \in \mathbb{R}^n, \\ \eta &= \mathcal{C}\,\frac{\partial H}{\partial \xi}, \end{aligned} \tag{4}$$

K is a constant matrix, known as the *observer gain.*

The *state estimation error*, defined as $e(t) = x(t) - \xi(t)$ and the output estimation error, defined as $e_y(t) = y(t) - \eta(t)$, are governed by

$$\begin{aligned} \dot{e} &= \mathcal{J}(y)\,\frac{\partial H}{\partial e} + (\mathcal{I}+\mathcal{S}-K\mathcal{C})\,\frac{\partial H}{\partial e}, \qquad e \in \mathbb{R}^n, \\ e_y &= \mathcal{C}\,\frac{\partial H}{\partial e}, \qquad e_y \in \mathbb{R}^m, \end{aligned} \tag{5}$$

where the vector, $\partial H/\partial e$ actually stands, with some abuse of notation, for the gradient vector of the *modified* energy function, $\partial H(e)/\partial e = \partial H/\partial x - \partial H/\partial \xi = \mathcal{M}(x-\xi) = \mathcal{M}e$. We set, when needed, $\mathcal{I}+\mathcal{S} = \mathcal{W}$.

Remark 1 Note that the error state dynamics described by equation (5) is independent of time-delay τ, i.e. equation (5) is a simple linear ordinary differential equation.

Definition 1 Synchronization problem: We say that the receiver dynamics (4) *synchronizes* with the transmitter dynamics (3), if

$$\lim_{t\to\infty} \|x(t) - \xi(t)\| = 0, \tag{6}$$

no matter which initial conditions $x(0)$ and $\xi(0)$ have. Where the state estimation error $e(t) = x(t) - \xi(t)$ represents the *synchronization error.*

Example 1 Time-delay-feedback Chua's oscillator The state equations of "standard" or "classic" Chua's oscillator are given by

$$\begin{aligned} C_1\dot{x}_1 &= G(x_2 - x_1) - F(x_1), \\ C_2\dot{x}_2 &= G(x_1 - x_2) + x_3, \\ L\dot{x}_3 &= -x_2 - R_0 x_3, \end{aligned} \tag{7}$$

with nonlinear function

$$F(x_1) = bx_1 + \frac{1}{2}\,(a-b)(|x_1+1| - |x_1-1|), \quad a,\, b < 0. \tag{8}$$

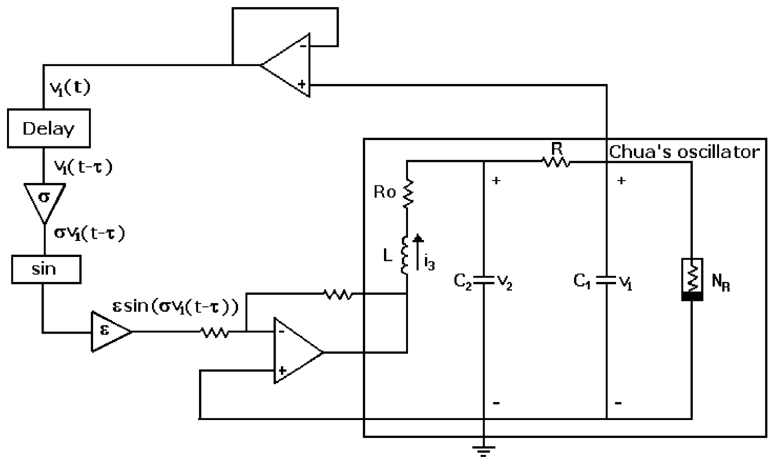

Figure 2.1. Time-delay-feedback Chua's oscillator.

The classic Chua's oscillator (7) – (8) can only produce low-dimensional chaos with one positive Lyapunov exponent. The time-delay-feedback Chua's oscillator considered in this paper is shown in Figure 2.1, and can be described by (Wang, *et al.* 2001):

$$\begin{aligned} C_1\dot{x}_1 &= G(x_2 - x_1) - F(x_1), \\ C_2\dot{x}_2 &= G(x_1 - x_2) + x_3, \\ L\dot{x}_3 &= -x_2 - R_0 x_3 - w(x_1(t-\tau)), \end{aligned} \tag{9}$$

with $F(x_1)$ given by (8) and where the time-delay term is taken as

$$w(x_1(t-\tau)) = \varepsilon \sin(\sigma x_1(t-\tau)), \tag{10}$$

with ε and σ are two positive constants, and τ represents the time-delay. Clearly, the maximum amplitude of the time-delay term is ε, i.e.

$$|w(x_1(t-\tau))| \le \varepsilon. \tag{11}$$

For arbitrarily given $\varepsilon > 0$, the time-delay-feedback Chua's oscillator (9) can be chaotic for sufficiently large σ and τ, even if the corresponding standard Chua's oscillator (7) has stable period orbits.

The time-delay-feedback Chua's oscillator is characterized by the following parameters: $R = 1910\,\Omega,\ R_0 = 16\,\Omega,\ C_1 = 10\,nF,\ C_2 = 100\,nF,\ L = 18.68\,mH,\ G_a = -0.75\,mS,$ $G_b = -0.41\,mS,\ B_p = 1\,V$, and $\tau = 0.001$. These values assure the existence of very complex hyperchaotic behavior. Figure 2.2 shows several different types of attractors from the time-delay-feedback Chua's oscillator for $\tau = 0.001$:

(a) $\varepsilon = 0.07$ and $\sigma = 0.4$;
(b) $\varepsilon = 0.2$ and $\sigma = 0.5$;
(c) $\varepsilon = 0.5$ and $\sigma = 3$;
(d) $\varepsilon = 1$ and $\sigma = 1$.

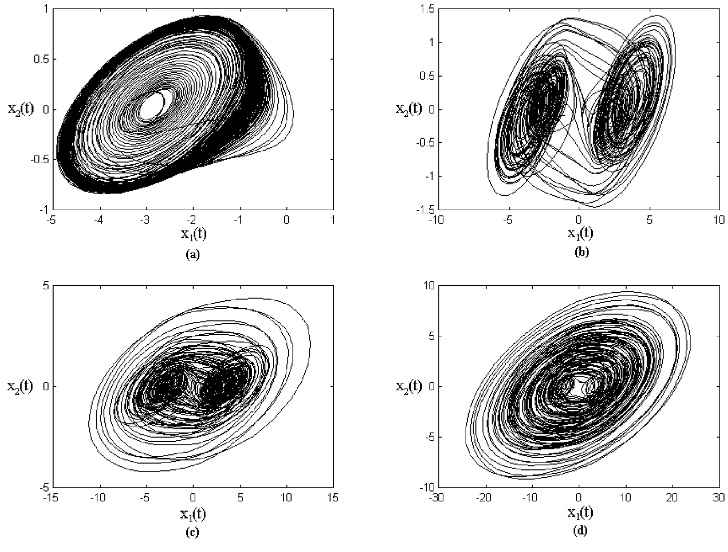

Figure 2.2. Several different types of chaotic attractors from the time-delay-feedback Chua's oscillator for $\tau = 0.001$: (a) $\varepsilon = 0.07$ and $\sigma = 0.4$. (b) $\varepsilon = 0.2$ and $\sigma = 0.5$. (c) $\varepsilon = 0.5$ and $\sigma = 3$. (d) $\varepsilon = 1$ and $\sigma = 1$.

The state equations describing the time-delay-feedback Chua's oscillator (9) in Hamiltonian canonical form with a destabilizing vector field (*transmitter circuit*) is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{LC_2} \\ 0 & -\frac{1}{LC_2} & 0 \end{bmatrix} \frac{\partial H}{\partial x} + \begin{bmatrix} -\frac{G}{C_1^2} & \frac{G}{C_1 C_2} & 0 \\ \frac{G}{C_1 C_2} & -\frac{G}{C_2^2} & 0 \\ 0 & 0 & -\frac{R_0}{L^2} \end{bmatrix} \frac{\partial H}{\partial x} + \begin{bmatrix} -\frac{1}{C_1} F\left(x_1\right) \\ 0 \\ -\frac{1}{L} w\left(x_1\left(t-\tau\right)\right) \end{bmatrix} \tag{12}$$

taking as the Hamiltonian energy function

$$H(x) = \frac{1}{2}\left[C_1 x_1^2 + C_2 x_2^2 + L x_3^2\right] \tag{13}$$

and gradient vector as

$$\frac{\partial H}{\partial x} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & L \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} C_1 x_1 \\ C_2 x_2 \\ L x_3 \end{bmatrix}.$$

The destabilizing vector field evidently calls for $x_1(t)$ to be used as the output, $y_1(t)$, of the transmitter circuit (12). The matrices $\mathcal{C}$, $\mathcal{S}$ and $\mathcal{I}$, are given by

$$\mathcal{C} = \left[\, \frac{1}{C_1} \quad 0 \quad 0 \,\right], \qquad \mathcal{S} = \begin{bmatrix} -\frac{G}{C_1^2} & \frac{G}{C_1 C_2} & 0 \\ \frac{G}{C_1 C_2} & -\frac{G}{C_2^2} & 0 \\ 0 & 0 & -\frac{R_0}{L^2} \end{bmatrix}, \qquad \mathcal{I} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{LC_2} \\ 0 & -\frac{1}{LC_2} & 0 \end{bmatrix}.$$

The pair $(\mathcal{C}, \mathcal{S})$ is neither observable nor detectable. However, the pair $(\mathcal{C}, \mathcal{W})$ is observable. The system lacks damping in the $x_3(t)$ variable, and either in the $x_1(t)$ or

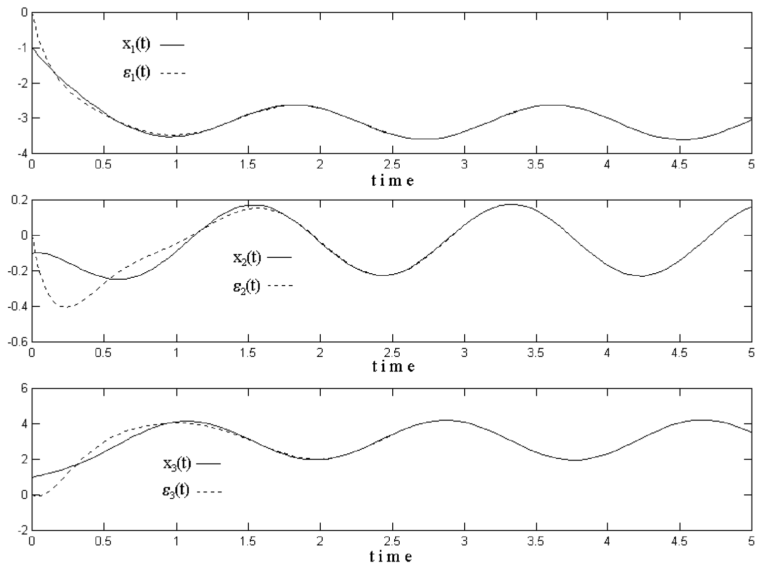

Figure 2.3. Time-delay-feedback Chua's oscillator state trajectories and synchronized receiver trajectories.

the $x_2(t)$ variable as inferred from the negative semi-definite nature of the dissipation structure matrix, $\mathcal{S}$. If $x_1(t)$ is used as output, then the output error injection term can enhance the dissipation in the error state dynamics. The *receiver circuit* is designed as

$$\begin{aligned}\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\\ \dot{\xi}_3\end{bmatrix} &= \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & \frac{1}{LC_2}\\ 0 & -\frac{1}{LC_2} & 0\end{bmatrix}\frac{\partial H}{\partial \xi} + \begin{bmatrix}-\frac{G}{C_1^2} & \frac{G}{C_1C_2} & 0\\ \frac{G}{C_1C_2} & -\frac{G}{C_2^2} & 0\\ 0 & 0 & -\frac{R_0}{L^2}\end{bmatrix}\frac{\partial H}{\partial \xi}\\ &+ \begin{bmatrix}-\frac{1}{C_1}F(y)\\ 0\\ -\frac{1}{L}w(y(t-\tau))\end{bmatrix} + \begin{bmatrix}k_1\\ k_2\\ k_3\end{bmatrix} e_y,\end{aligned} \tag{14}$$

where the gain vector $K=(k_1,k_2,k_3)^T$ is chosen in order to guarantee the asymptotic exponential stability to zero of the state reconstruction error trajectories (synchronization error $e(t)$). From (12) and (14) the synchronization error dynamics is governed by

$$\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\end{bmatrix} = \begin{bmatrix}0 & \frac{k_2}{2C_1} & \frac{k_3}{2C_1}\\ -\frac{k_2}{2C_1} & 0 & \frac{2}{LC_2}\\ -\frac{k_3}{2C_1} & -\frac{2}{LC_2} & 0\end{bmatrix}\frac{\partial H}{\partial e} + \begin{bmatrix}-\frac{G+C_1k_1}{C_1^2} & \frac{2G-C_2k_2}{2C_1C_2} & -\frac{k_3}{2C_1}\\ \frac{2G-C_2k_3}{2C_1C_2} & -\frac{G}{C_2^2} & 0\\ -\frac{k_3}{2C_1} & 0 & -\frac{R_0}{L}\end{bmatrix}\frac{\partial H}{\partial e}. \tag{15}$$

With $x(0)=(-1,-0.1,1)$ and $\xi(0)=(0,0,0)$ we obtain the following numerical results. Figure 2.3 shows the time-delay-feedback Chua's oscillator state trajectories and synchronized receiver trajectories. Figure 2.4 illustrates the time behaviors of the synchronization error trajectories $e_i(t)=x_i(t)-\xi_i(t),\ i=1,2,3$ for $k_1=k_2=k_3=5$. To ease the numerical simulations we resorted the following normalized version of the time-delay-feedback Chua's oscillator:

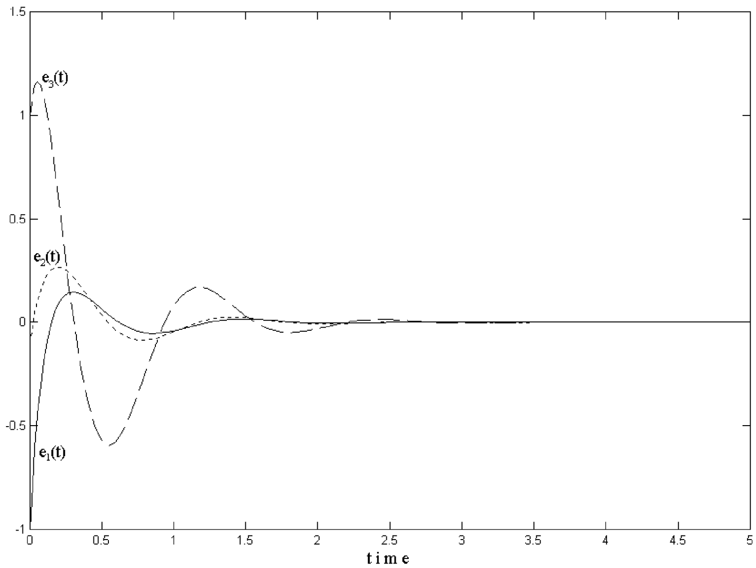

Figure 2.4. Synchronization error trajectories $e_i(t) = x_i(t) - \xi_i(t)$, $i = 1, 2, 3$.

$$\begin{aligned}\dot{x}_1 &= \alpha(x_2 - x_1 - f(x_1)),\\ \dot{x}_2 &= x_1 - x_2 + x_3,\\ \dot{x}_3 &= -\beta x_2 - \gamma x_3 - \beta\,\varepsilon \sin(\sigma x_1(t-\tau)),\end{aligned} \tag{16}$$

where the nonlinear function is given by

$$f(x_1) = bx_1 + \frac{1}{2}\,(a-b)(|x_1+1| - |x_1-1|)$$

with $\alpha = 10$, $\beta = 19.53$, $\gamma = 0.1636$, $a = -1.4325$, $b = -0.7831$, $\sigma = 0.5$, $\varepsilon = 0.2$, and $\tau = 0.001$.

3 Synchronization Stability Analysis

In this section, we examine the stability of the synchronization error (15) between time-delay-feedback Chua's oscillator in Hamiltonian canonical form (12) and nonlinear state observer (14).

Theorem 1 (Sira-Ramírez and Cruz 2001) *The state $x(t)$ of the nonlinear system (12) can be globally, exponentially, asymptotically estimated by the state $\xi(t)$ of an observer of the form (14), if the pair of matrices $(\mathcal{C}, \mathcal{W})$, or the pair $(\mathcal{C}, \mathcal{S})$, is either observable or, at least, detectable.*

An observability condition on either of the pairs $(\mathcal{C}, \mathcal{W})$, or $(\mathcal{C}, \mathcal{S})$, is clearly a sufficient but not necessary condition for asymptotic state reconstruction. A necessary and sufficient condition for global asymptotic stability to zero of the estimation error is given by the following theorem.

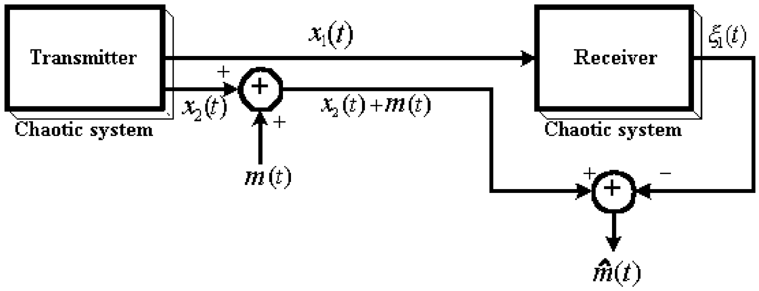

Figure 4.1. Chaotic secure communication system with two transmission channels: $m(t)$ is the private message to be hidden and transmitted. $x_1(t)$ is the synchronizing signal. $x_2(t) + m(t)$ is a hyperchaotic signal, and $\hat{m}(t)$ is the retrieved message at the receiver end.

Theorem 2 (Sira-Ramírez and Cruz 2001) *The state $x(t)$ of the nonlinear system (12) can be globally, exponentially, asymptotically estimated, by the state $\xi(t)$ of the observer (14) if and only if there exists a constant matrix K such that the symmetric matrix*

$$[\mathcal{W} - K\mathcal{C}] + [\mathcal{W} - K\mathcal{C}]^T = [\mathcal{S} - K\mathcal{C}] + [\mathcal{S} - K\mathcal{C}]^T = 2\left[\mathcal{S} - \frac{1}{2}(K\mathcal{C} + \mathcal{C}^T K^T)\right]$$

is negative definite.

4 Application to Chaotic Communications

In this section, we apply the Hamiltonian synchronization of time-delay-feedback Chua's oscillator to send secret messages. In particular, we propose two hyperchaotic communication schemes to transmit analog and binary information signals using two transmission channels and using a single transmission channel, respectively.

4.1 Secret communication using two transmission channels

The secret analog communication system is achieved by using the hyperchaotic additive masking technique. With this scheme, we obtain faster synchronization and higher privacy; one channel is used to send the hyperchaotic synchronizing signal $x_1(t)$ from the transmitter (12), with no connection with the secret message $m(t)$. While the other channel is used to transmit hidden message $m(t)$ which is recovered at the receiver end by means of the comparison between the signals $x_2(t) + m(t)$ and $\xi_2(t)$. Figure 4.1 shows the hyperchaotic secure communication system with two transmission channels. Figure 4.2 shows the secret message communication of an audio message: the private signal information to be hidden and transmitted $m(t)$, audio message (top of figure), the transmitted hyperchaotic signal $x_2(t) + m(t)$ (middle of figure), and the recovered audio message $\hat{m}(t)$ at the receiver end (bottom of figure) which is obtained after a short transient behavior.

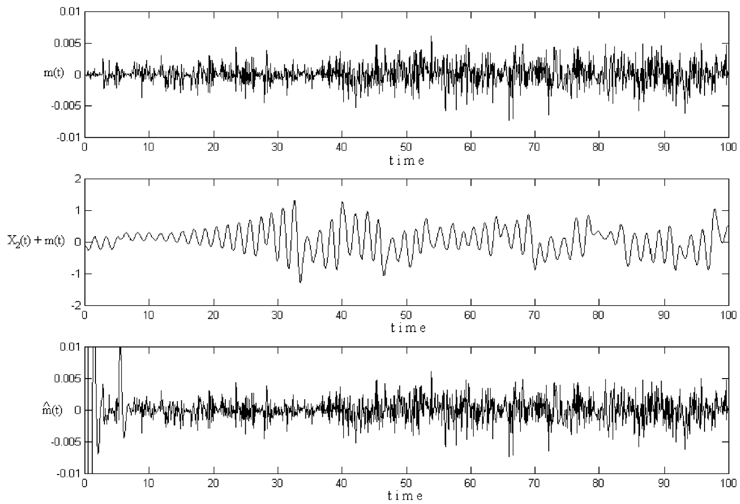

Figure 4.2. Transmission and recovering of an audio message: Private message to be hidden and transmitted (top of figure). Transmitted hyperchaotic signal $x_2(t) + m(t)$ (middle of figure). Recovered audio message at the receiver end $\hat{m}(t)$ (bottom of figure).

4.2 Secret communication using a single transmission channel

As second communication scheme, we propose the secret binary communication system using a single transmission channel, this objective is achieved by hyperchaotic switching technique (see e.g. Parlitz, *et al.* 1992; Cuomo, *et al.* 1993; Dedieu, *et al.* 1993 for chaotic systems). In this technique, the binary message $m(t)$ is used to modulate one or more parameter of the (switching) transmitter, i.e. $m(t)$ controls a switch whose action changes the parameter values of the transmitter. Thus, according to the value of $m(t)$ at any given time t, the transmitter has either the parameter set value p or the parameter set value p'. At the receiver $m(t)$ is decoded by using the synchronization error to decide whether the received signal corresponds to one parameter value, or the other (it can be interpreted as an 'one' or 'zero'). To transmit $m(t)$, let β be the parameter to be modulated in the hyperchaotic Chua transmitter (16), the parameter α and γ were fixed. We use a 'modulation rule' to modulate $m(t)$ as follows

$$\beta(t) = \beta + r \cdot m(t),$$

where $r = 0.41$ while the private message is defined as

$$m(t) = 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 1\ 0 \cdots .$$

The following results illustrate the transmission of $m(t)$ for $t = 10$ sec, when β is switched between $\beta(1) = \beta' = 19.53$ and $\beta(0) = \beta = 19.12$. Figure 4.3 shows the chaotic secure communication system by chaotic switching. While Figure 4.4 shows the transmission and recovering of secret binary message: the private message $m(t)$ (top of figure), the transmitted hyperchaotic signal $x_1(t)$ (middle of figure), and the

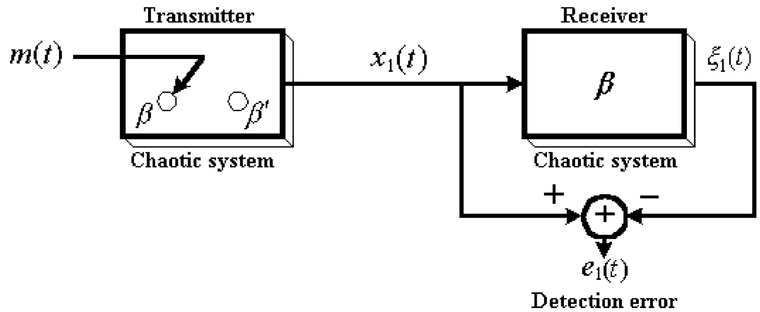

Figure 4.3. Chaotic secure communication system using chaotic switching.

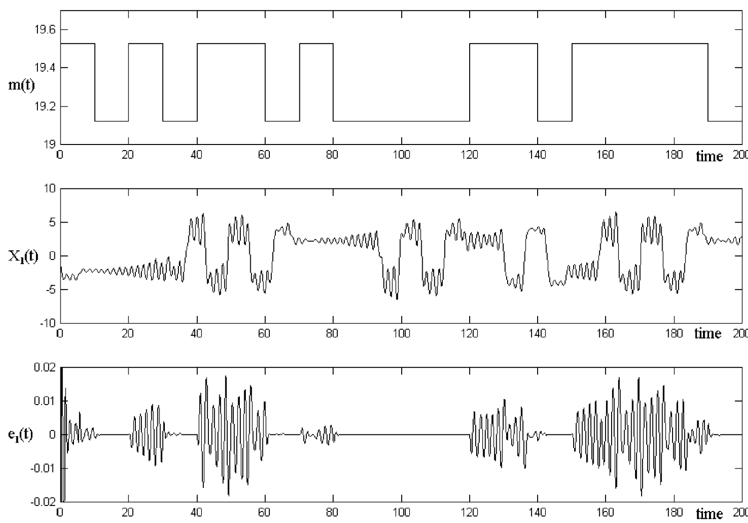

Figure 4.4. Transmission and recovery of a secret binary message: Binary private signal to be hidden and transmitted $m(t)$ (top of figure). Transmitted hyperchaotic signal $x_1(t)$ (middle of figure). Recovered binary message at the receiver end by synchronization error detection $e_1(t) = x_1(t) - \xi_1(t)$ (bottom of figure).

recovered binary message at the receiver end (bottom of figure) by synchronization error detection $e_1(t) = x_1(t) - \xi_1(t)$.

5 Conclusions

In this paper, we have approached the problem of synchronization of time-delay-feedback Chua's circuit from the perspective of Generalized Hamiltonian systems developed in (Sira-Ramírez and Cruz 2001). The approach allows one to give a simple design procedure for the receiver circuit given by a nonlinear observer, and clarifies the issue of deciding on the nature of the output signal to be transmitted. The suggested approach has been successfully applied to a secure communication schemes to transmit analog and binary secret messages. Simulation results have been reported to illustrate the capability of the proposed approach, and shows great potential for actual private/secure communication systems in which the encoding is required to be secure. Because of the increased complexity of the transmitted signal as well as the adoption of infinite-dimensional Chua's oscillator with multiple positive Lyapunov exponents.

In a forthcoming article we will be concerned with a physical implementation of the method with a specific quantization of the degree of safety of the proposal in actual communication systems.

Acknowledgments

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