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Robust Fuzzy Linear Control of a Class of Stochastic Nonlinear Time-Delay Systems

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Abstract: This paper presents the fuzzy linear control design method for a class of stochastic nonlinear time-delay systems with state feedback. First, the Takagi and Sugeno fuzzy linear model is employed to approximate a nonlinear system. Next, based on the fuzzy linear model, a fuzzy linear controller is developed to stabilize the nonlinear system. The control law is obtained to ensure stochastical exponential stability in the mean-square, independent of the time-delay. The sufficient conditions for the existence of such a control are proposed in terms of a certain linear matrix inequality. Finally, a simulation example is given to illustrate the applicability of the proposed design method.

Keywords: *Fuzzy linear control; linear matrix inequality; time-delay systems; stochastic systems; exponential stability.*

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1 Introduction

Most of the systems, which are encountered in control engineering, contain various nonlinearities and are affected by random disturbance signals. Nonlinear systems with time-delay constitute basic mathematical models of real phenomena, for instance in biology, mechanics and economics, see e.g. [8, 18]. Control of time-delay systems has been a subject of great practical importance, which has attracted a great deal of interest for several decades. On the other hand, it turns out that the delayed state is very often the cause for instability and poor performance of systems. Moreover, considerable attention has been given to both the problems of robust stabilization and robust control for linear systems with unavoidable time-varying parameter uncertainties in modelling of dynamical systems and certain types of time-delays [14].

Since the introduction of fuzzy set theory by Zadeh in [30], many people have devoted a great deal of time and effort to both theoretical research and implementation technique for fuzzy logic controllers [15, 22]. With the development of fuzzy systems, it is known that the qualitative knowledge of a system can also be represented in nonlinear functional form. On the basis of this idea, some fuzzy models based control system design methods have appeared in the fuzzy control field [3, 22, 23]. These methods are conceptually simple and straightforward. Fuzzy controllers are usually characterized using Mamdani and T-S type. In general, Mamdani type fuzzy controllers are designed empirically. However, T-S controllers can be designed using the information of several local linearized models of a given system via the so-called parallel-distributed compensation scheme. Various stability conditions of fuzzy systems have been obtained by employing Lyapunov stability theory [4, 9, 10], passivity theory [20], and other methods [5, 12, 22]. Problem of control design based on the state feedback for T-S fuzzy systems using LMI approach has been studied in [28] and the delay-independent stability of T-S fuzzy model for a class of nonlinear time-delay systems was investigated in [7]. Extension of the T-S fuzzy model approach to the stability analysis and control design for both continuous and discrete-time nonlinear systems with time-varying delay has been considered in [2] and also Lee, *et al.* [11] presented design of an output feedback robust H_∞ controller based on T-S fuzzy model for uncertain fuzzy dynamic systems with time-varying delayed state.

Recently, several criteria of input-to-bounded state (IBS) stabilization and bounded-input-bounded-output (BIBO) stabilization in mean-square for nonlinear and quasi-linear stochastic control systems with time-varying uncertainties has been investigated in [6], also, another stability concepts in the mean-square sense such as mean-square stability (MSS) and the internal mean-square stability (IMSS) have been studied in [13]. The stabilization of stochastic systems with multiplicative noise has been studied since the late sixties, particularly in the context of linear quadratic optimal control, see e.g., [17, 24]. Also, a stochastic fuzzy control has been proposed by applying the stochastic control theory, instead of using a traditional fuzzy reasoning in [25] and a class of fuzzy stochastic control systems with random delays investigated in [19].

The main contribution of this paper is to investigate the fuzzy linear control problem for a class of stochastic nonlinear time-delay systems. The attention was focused on the design of state feedback controller which ensures stochastical exponential stability in the mean-square, independent of the time-delay. Finally, the simulation results show that fuzzy linear state feedback controller can achieve the robust stability in the mean-square independent of the time-delay.

Notation The following notations will be used throughout the paper. R^m denotes the m-dimensional Euclidean space and $R^{n\times m}$ denotes the set of all real $n\times m$ matrices. The superscript "T" denotes the transpose and the notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with compatible dimension. $C([-h,0];\, R^n)$ denote the family of continuous functions φ from $[-h,0]$ to R^n with the norm $\|\varphi\| = \sup\limits_{-h\leq\theta\leq 0} |\varphi(\theta)|$, where $|\cdot|$ is the Euclidean norm in R^n. If A is a matrix, denote by $\|A\|$ its operator norm, i.e., $\|A\| = \sup\{|Ax|\colon\, |x| = 1\} = \sqrt{\lambda_{\max}(A^{\mathrm{T}}A)}$, where $\lambda_{\max}(A)$ means the largest eigenvalue of A. $L_2[0,\infty]$ is the space of the square integrable vector. Moreover, let $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ be a complete probability space and $L^P_{F_0}([-h,0];\, R^n)$ denote the family of all F_0-measurable $C([-h,0];\, R^n)$–valued random variables $\zeta = \{\zeta(\theta)\colon\, -h \leq \theta \leq 0\}$ such that $\sup\limits_{-h\leq\theta\leq 0} E|\zeta(\theta)|^P < \infty$ where $E(\cdot)$

stands for the mathematical expectation operator with respect to the given probability measure P.

2 Preliminaries and Problem Formulation

Consider a class of nonlinear continuous-time state delayed stochastic systems described by

$$dx(t) = [A(x(t))x(t) + A_d(x(t))x(t-h) + B(x(t))u(t)]\,dt + E_1\,dw(t), \tag{1}$$

$$x(t) = \varphi(t), \quad t \in [-h, 0], \tag{2}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^{\mathrm{T}} \in R^n$ is the state vector, $u(t) = [u_1(t), u_2(t), \dots, u_m(t)]^{\mathrm{T}} \in R^m$ is the control input, h is the unknown state delay, $\varphi(t)$ is the continuous vector valued initial function and $w(t) = [w_1(t), w_2(t), \dots, w_n(t)]^{\mathrm{T}} \in R^n$ is a scalar Brownian motion defined on the probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$.

A fuzzy dynamic model has been proposed by Takagi and Sugeno [21] to represent local linear input-output relations of nonlinear systems. This fuzzy linear model is described by fuzzy If-Then rules and will be employed here to deal with the control design problem of the nonlinear system (1)–(2). The i-th rule of this fuzzy model for the nonlinear system (1)–(2) is of the following form [9, 21, 23]:

Plant Rule i:

$$\begin{aligned} &\textit{If } z_1(t) \textit{ is } F_{i1} \textit{ and } \dots \textit{ and } z_g(t) \textit{ is } F_{ig},\\ &\textit{then} \quad dx(t) = [A_i x(t) + A_{id} x(t-h) + B_i u(t)]\,dt + E_1\,dw(t) \end{aligned} \tag{3}$$

for $i = 1, 2, \dots, L$, where F_{ij} is the fuzzy set, $A_i \in R^{n\times n}$, $A_{id} \in R^{n\times n}$, $B_i \in R^{n\times m}$, L is the number of If-Then rules, and $z_1(t), z_2(t), \dots, z_g(t)$ are the premise variables.

The overall fuzzy system is inferred as follows [9, 21, 23]:

$$\begin{aligned} dx(t) &= \frac{\left[\sum\limits_{i=1}^{L} \mu_i(z(t))(A_i x(t) + A_{id} x(t-h) + B_i u(t))\right]}{\sum\limits_{i=1}^{L} \mu_i(z(t))}\,dt + E_1\,dw(t)\\ &= \sum_{i=1}^{L} h_i(z(t))(A_i x(t) + A_{id} x(t-h) + B_i u(t))\,dt + E_1\,dw(t) \end{aligned} \tag{4}$$

where

$$z(t) = [z_1(t), z_2(t), \dots, z_g(t)]^{\mathrm{T}}, \tag{5}$$

$$\mu_i(z(t)) = \prod_{j=1}^{g} F_{ij}(z_j(t)), \tag{6}$$

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum\limits_{j=1}^{L} \mu_j(z(t))}, \tag{7}$$

and $F_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in F_{ij}.

Remark 1 In order to consider parametric uncertainties in the T-S fuzzy system (3), we formulate the i-th rule of the fuzzy model as

Plant Rule i:

$$\begin{aligned} &\textit{If } z_1(t) \textit{ is } F_{i1} \textit{ and } \ldots \textit{ and } z_g(t) \textit{ is } F_{ig}, \\ &\textit{then} \quad dx(t) = [(A_i + \Delta A_i^p)x(t) + A_{id}x(t-h) + (B_i + \Delta B_i^p)u(t)]\,dt + E_1\,dw(t) \end{aligned}$$

where ΔA_i^p and ΔB_i^p are assumed norm-bounded matrices with appropriate dimensions, which represent parametric uncertainties in the plant model with the following structure

$$[\Delta A_i^p \ \ \Delta B_i^p] = D_i \Gamma_i(t)\,[F_{1i} \ \ F_{2i}],$$

where D_i, F_{1i} and F_{2i} are known real constant matrices of appropriate dimensions, and $\Gamma_i(t)$ is an unknown matrix function and satisfies $\Gamma_i^{\mathrm{T}}(t)\Gamma_i(t) \le I$ [12].

Assumption 1 We assume $\mu_i(z(t)) \ge 0$ for $i = 1, 2, \ldots, L$ and $\sum_{i=1}^{L} \mu_i(z(t)) > 0$ for all t.

Therefore, we get [9, 23]

$$h_i(z(t)) \ge 0 \tag{8}$$

for $i = 1, 2, \ldots, L$ and

$$\sum_{i=1}^{L} h_i(z(t)) = 1. \tag{9}$$

Therefore, from (1) we get [4]

$$\begin{aligned} dx(t) &= [A(x(t))x(t) + A_d(x(t))x(t-h) + B(x(t))u(t)]\,dt + E_1\,dw(t) \\ &= \Bigg[\sum_{i=1}^{L} h_i(z(t))(A_i x(t) + A_{id}x(t-h) + B_i u(t)) \\ &+ \Bigg\{\Bigg(A(x) - \sum_{i=1}^{L} h_i(z(t))A_i\Bigg)x(t) + \Bigg(A_d(x) - \sum_{i=1}^{L} h_i(z(t))A_{id}\Bigg)x(t-h) \\ &+ \Bigg(B(x) - \sum_{i=1}^{L} h_i(z(t))B_i u(t)\Bigg)\Bigg\}\Bigg]\,dt + E_1 dw(t) \end{aligned} \tag{10}$$

where

$$\begin{aligned} &\Bigg\{\Bigg(A(x) - \sum_{i=1}^{L} h_i(z(t))A_i\Bigg)x(t) + \Bigg(A_d(x) - \sum_{i=1}^{L} h_i(z(t))A_{id}\Bigg)x(t-h) \\ &+ \Bigg(B(x) - \sum_{i=1}^{L} h_i(z(t))B_i u(t)\Bigg)\Bigg\} \end{aligned} \tag{11}$$

denotes the approximation error between the nonlinear system (1) and the fuzzy model (4).

Suppose the following fuzzy controller is employed to deal with the above control system design:

Control Rule j:

$$\begin{aligned} &\textit{If } z_1(t) \textit{ is } F_{j1} \textit{ and } \dots \textit{ and } z_g(t) \textit{ is } F_{jg},\\ &\textit{then} \quad u(t) = K_j x(t) \end{aligned} \tag{12}$$

for $j = 1, 2, \dots, L$. Hence, the overall fuzzy controller is given by

$$u(t) = \frac{\sum_{j=1}^{L} \mu_j(z(t))\,(K_j x(t))}{\sum_{j=1}^{L} \mu_j(z(t))} = \sum_{j=1}^{L} h_j(z(t)) K_j x(t) \tag{13}$$

where $h_j(z(t))$ is defined in (8) and (9) and K_j are the control parameters.

Substituting (13) into (10) yields the closed-loop nonlinear control system as follows:

$$\begin{aligned} dx(t) &= [A(x(t))x(t) + A_d(x(t))x(t-h) + B(x(t))u(t)]\,dt + E_1\,dw(t)\\ &= \Bigg[\Bigg\{ \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))(A_i + B_i K_j)x(t) + A_{id}x(t-h)\Bigg\}\\ &\quad + \Delta A + \Delta A_d + \Delta B\Bigg]\,dt + E_1\,dw(t) \end{aligned} \tag{14}$$

where

$$\Delta A = \left(A(x(t)) - \sum_{i=1}^{L} h_i(z(t))A_i\right)x(t), \tag{15}$$

$$\Delta A_d = \left(A_d(x(t)) - \sum_{i=1}^{L} h_i(z(t))A_{id}\right)x(t-h), \tag{16}$$

$$\Delta B = \sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))(B(x(t)) - B_i)K_j x(t). \tag{17}$$

Assumption 2 There exist bounding matrices ΔA_i, ΔA_{id} and ΔB_i such that for all trajectory $x(t)$

$$\|\Delta A\| \le \left\| \sum_{i=1}^{L} h_i(z(t))\Delta A_i x(t) \right\|, \tag{18}$$

$$\|\Delta A_d\| \le \left\| \sum_{i=1}^{L} h_i(z(t))\Delta A_{id} x(t-h) \right\|, \tag{19}$$

$$\|\Delta B\| \le \left\| \sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))\Delta B_i\, K_j x(t) \right\| \tag{20}$$

and the bounding matrices ΔA_i, ΔA_{id} and ΔB_i can be described by

$$\begin{bmatrix} \Delta A_i \\ \Delta A_{id} \\ \Delta B_i \end{bmatrix} = \begin{bmatrix} \delta_i A_p \\ \delta_{id} A_{pd} \\ \eta_i B_p \end{bmatrix}, \tag{21}$$

where $\|\delta_i\| \le 1$, $\|\delta_{id}\| \le 1$ and $\|\eta_i\| \le 1$, for $i = 1, 2, \dots, L$ [1].

According to Assumption 2, we get

$$\begin{aligned}
(\Delta A)^{\mathrm{T}}(\Delta A) &= \left(\left(A(x(t)) - \sum_{i=1}^{L} h_i(z(t))A_i\right)x(t)\right)^{\mathrm{T}} \\
&\quad \times \left(\left(A(x(t)) - \sum_{i=1}^{L} h_i(z(t))A_i\right)x(t)\right) \\
&\le \left(\sum_{i=1}^{L} h_i(z(t))\,\Delta A_i x(t)\right)^{\mathrm{T}} \left(\sum_{i=1}^{L} h_i(z(t))\,\Delta A_i x(t)\right) \\
&= \left(\sum_{i=1}^{L} h_i(z(t))\,\delta_i A_p x(t)\right)^{\mathrm{T}} \left(\sum_{i=1}^{L} h_i(z(t))\,\delta_i A_p x(t)\right) \le (A_p x(t))^{\mathrm{T}}(A_p x(t)),
\end{aligned} \tag{22}$$

$$\begin{aligned}
(\Delta A_d)^{\mathrm{T}}(\Delta A_d) &= \left(\left(A_d(x(t)) - \sum_{i=1}^{L} h_i(z(t))A_{id}\right)x(t-h)\right)^{\mathrm{T}} \\
&\quad \times \left(\left(A_d(x(t)) - \sum_{i=1}^{L} h_i(z(t))A_{id}\right)x(t-h)\right) \\
&\le \left(\sum_{i=1}^{L} h_i(z(t))\,\Delta A_{id} x(t-h)\right)^{\mathrm{T}} \left(\sum_{i=1}^{L} h_i(z(t))\,\Delta A_{id} x(t-h)\right) \\
&= \left(\sum_{i=1}^{L} h_i(z(t))\,\delta_{id} A_{pd} x(t-h)\right)^{\mathrm{T}} \left(\sum_{i=1}^{L} h_i(z(t))\,\delta_{id} A_{pd} x(t-h)\right) \\
&\le (A_{pd} x(t-h))^{\mathrm{T}}(A_{pd} x(t-h))
\end{aligned} \tag{23}$$

and

$$\begin{aligned}
(\Delta B)^{\mathrm{T}}(\Delta B) &= \left(\sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))(B(x(t)) - B_i)K_j x(t)\right)^{\mathrm{T}} \\
&\quad \times \left(\sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))(B(x(t)) - B_i)K_j x(t)\right) \\
&\le \left(\sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))\,\Delta B_i K_j x(t)\right)^{\mathrm{T}} \left(\sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))\,\Delta B_i K_j x(t)\right)
\end{aligned} \tag{24}$$

$$= \Big(\sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))\, \eta_i B_p\, K_j x(t)\Big)^{\mathrm{T}} \Big(\sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))\, \eta_i B_p\, K_j x(t)\Big)$$

$$\leq \Big(\sum_{j=1}^{L} h_j(z(t)) B_p K_j x(t)\Big)^{\mathrm{T}} \Big(\sum_{j=1}^{L} h_j(z(t)) B_p K_j x(t)\Big),$$

i.e. the approximation error in the closed-loop nonlinear system is bounded by the specified structured bounding matrices A_p, A_{pd} and B_p.

Next, observe the closed-loop system (14) and let $x(t,\zeta)$ denote the state trajectory from the initial data $x(\theta) = \zeta(\theta)$ on $-h \leq \theta \leq 0$ in $L^2_{F_0}([-h,0];\, R^{2n})$. Clearly, the system (14) admits a trivial solution $x(t;0) \equiv 0$ corresponding to the initial data $\zeta = 0$. We introduce the following stability and stabilizability concepts.

Definition 1 [27] For the system (14) and every $\zeta \in L^2_{F_0}([-h,0]\,;\, R^{2n})$, the trivial solution is asymptotically stable in the mean square if

$$\lim_{t\to\infty} E\,|x(t;\,\zeta)|^2 = 0, \tag{25}$$

and is exponentially stable in the mean-square if there exist constants $\alpha > 0$ and $\beta > 0$ such that

$$E\,|x(t;\zeta)|^2 \leq \alpha e^{-\beta t} \sup_{-h\leq\theta\leq 0} E\,|\zeta(\theta)|^2\,. \tag{26}$$

Definition 2 [27] We say that the system (1)−(2) is exponentially stabilizable in mean-square if, for every $\zeta \in L^2_{F_0}([-h,0];\, R^{2n})$, there exists a fuzzy linear control law (13) such that the resulting closed-loop system is exponentially stable in mean-square.

The objective of this paper is to design a fuzzy linear control for the stochastic nonlinear time-delay system (1)−(2). More specifically, we are interested in seeking the control parameters K_j, for $j = 1, 2, \ldots, L$, such that the closed-loop system (14) is exponentially stable in mean-square, independent of the unknown time-delay h.

3 Main Results and Proofs

We first give the following lemma, which will be used in the proof of our main results.

Lemma 1 [31] *For any matrices X and Y with appropriate dimensions and for any constant $\eta > 0$, we have:*

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leq \eta X^{\mathrm{T}}X + \frac{1}{\eta}\, Y^{\mathrm{T}}Y. \tag{27}$$

3.1 Stochastic stability analysis

In this section, assuming that the fuzzy linear control is known and we will study the conditions under which the closed-loop system is stochastically exponentially stable in the mean-square. The following theorem will play a key role in the stability analysis of closed-loop system and design of the expected fuzzy linear control.

Theorem 1 *Let the control parameters K_j, for $j = 1, 2, \ldots, L$, be given. If the fuzzy controller (13) is employed in the nonlinear system (1)–(2) and there exists positive scalars ε_1, ε_2, ε_3, ε_4 and a positive definite matrix $P = P^{\mathrm{T}}$ such that the following matrix inequalities*

$$\begin{aligned}&(A_i + B_iK_j)^{\mathrm{T}}P + P(A_i + B_iK_j) + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)P^2\\ &+ \varepsilon_1^{-1}A_{id}^{\mathrm{T}}A_{id} + \varepsilon_2^{-1}A_p^{\mathrm{T}}A_p + \varepsilon_3^{-1}A_{pd}^{\mathrm{T}}A_{pd} + \varepsilon_4^{-1}(B_pK_j)^{\mathrm{T}}(B_pK_j) < 0\end{aligned} \tag{28}$$

are satisfied for all i, $j = 1, 2, ..., L$, then the closed-loop nonlinear system (14) is exponentially stable in the mean-square and independent of the unknown time-delay h.

Proof Fix $\zeta \in L^2_{F_0}([-h, 0]; R^{2n})$ arbitrarily, and write $x(t,\zeta) = x(t)$. We define the Lyapunov function candidate

$$\Upsilon(x(t),t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-h}^{t} x^{\mathrm{T}}(s)Qx(s)\,ds \tag{29}$$

where $P = P^{\mathrm{T}}$ is the positive definite solution to the matrix inequality (28) and $Q = Q^{\mathrm{T}} > 0$ is defined by

$$Q = \varepsilon_1^{-1}\Bigg(\sum_{i=1}^{L} h_i(z(t))A_{id}\Bigg)^{\mathrm{T}}\Bigg(\sum_{i=1}^{L} h_i(z(t))A_{id}\Bigg) + \varepsilon_3^{-1}A_{pd}^{\mathrm{T}}A_{pd}. \tag{30}$$

The stochastic differential of Υ along a given trajectory is obtained as

$$\begin{aligned}d\Upsilon(x(t),t) = \Bigg\{&x^{\mathrm{T}}(t)\Bigg(\Bigg\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))(A_i + B_iK_j)\Bigg\}^{\mathrm{T}}P + Q\Bigg)x(t)\\ &+ x^{\mathrm{T}}(t-h)\Bigg(\sum_{i=1}^{L} h_i(z(t))A_{id}\Bigg)^{\mathrm{T}}Px(t) + x^{\mathrm{T}}(t)P\Bigg(\sum_{i=1}^{L} h_i(z(t))A_{id}\Bigg)x(t-h)\\ &+ x^{\mathrm{T}}(t)P\Bigg(\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))(A_i + B_iK_j)\Bigg)x(t)\\ &+ (\Delta A + \Delta A_d + \Delta B)^{\mathrm{T}}Px(t) + x^{\mathrm{T}}(t)P(\Delta A + \Delta A_d + \Delta B)\\ &- x^{\mathrm{T}}(t-h)Qx(t-h)\Bigg\}dt + 2x^{\mathrm{T}}(t)PE_1\,dw(t).\end{aligned} \tag{31}$$

Now, by Lemma 1, it is trivial to show that for any positive scalars of ε_1, ε_2, ε_3, ε_4 the following matrix inequalities hold:

$$\begin{aligned}&\Bigg(\Bigg(\sum_{i=1}^{L} h_i(z(t))A_{id}\Bigg)x(t-h)\Bigg)^{\mathrm{T}}Px(t) + x^{\mathrm{T}}(t)P\Bigg(\Bigg(\sum_{i=1}^{L} h_i(z(t))A_{id}\Bigg)x(t-h)\Bigg)\\ &\le \varepsilon_1 x^{\mathrm{T}}(t)P^2x(t) + \varepsilon_1^{-1}x^{\mathrm{T}}(t-h)\Bigg(\sum_{i=1}^{L} h_i(z(t))A_{id}\Bigg)^{\mathrm{T}}\Bigg(\sum_{i=1}^{L} h_i(z(t))A_{id}\Bigg)x(t-h),\end{aligned} \tag{32}$$

$$
\begin{aligned}
(\Delta A)^{\mathrm{T}} P x(t) + x^{\mathrm{T}}(t) P(\Delta A) &\le \varepsilon_2 x^{\mathrm{T}}(t) P^2 x(t) + \varepsilon_2^{-1} (\Delta A)^{\mathrm{T}} (\Delta A) \\
&\le \varepsilon_2 x^{\mathrm{T}}(t) P^2 x(t) + \varepsilon_2^{-1} (A_p x(t))^{\mathrm{T}} (A_p x(t)) \\
&= x^{\mathrm{T}}(t) (\varepsilon_2 P^2 + \varepsilon_2^{-1} A_p^{\mathrm{T}} A_p) x(t),
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
(\Delta A_d)^{\mathrm{T}} P x(t) + x^{\mathrm{T}}(t) P(\Delta A_d) &\le \varepsilon_3 x^{\mathrm{T}}(t) P^2 x(t) + \varepsilon_3^{-1} (\Delta A_d)^{\mathrm{T}} (\Delta A_d) \\
&\le \varepsilon_3 x^{\mathrm{T}}(t) P^2 x(t) + \varepsilon_3^{-1} (A_{pd} x(t-h))^{\mathrm{T}} (A_{pd} x(t-h)) \\
&= \varepsilon_3 x^{\mathrm{T}}(t) P^2 x(t) + \varepsilon_3^{-1} x(t-h)^{\mathrm{T}} A_{pd}^{\mathrm{T}} A_{pd} x(t-h)
\end{aligned} \tag{34}
$$

and

$$
\begin{aligned}
&(\Delta B)^{\mathrm{T}} P x(t) + x^{\mathrm{T}}(t) P(\Delta B) \le \varepsilon_4 x^{\mathrm{T}}(t) P^2 x(t) + \varepsilon_4^{-1} (\Delta B)^{\mathrm{T}} (\Delta B) \\
&\le \varepsilon_4 x^{\mathrm{T}}(t) P^2 x(t) + \varepsilon_4^{-1} \bigg(\sum_{j=1}^{L} h_j(z(t)) B_p K_j x(t) \bigg)^{\mathrm{T}} \bigg(\sum_{j=1}^{L} h_j(z(t)) B_p K_j x(t) \bigg) \\
&= x^{\mathrm{T}}(t) \bigg(\varepsilon_4 P^2 + \varepsilon_4^{-1} \bigg(\sum_{j=1}^{L} h_j(z(t)) B_p K_j \bigg)^{\mathrm{T}} \bigg(\sum_{j=1}^{L} h_j(z(t)) B_p K_j \bigg) \bigg) x(t).
\end{aligned} \tag{35}
$$

Then, noticing the definition (30), substituting (32)–(35) into (31) result in

$$
\begin{aligned}
d\Upsilon(x(t),t) &\le x^{\mathrm{T}}(t) \bigg\{ \bigg(\sum_{i=1}^{L} \sum_{j=1}^{L} h_i(z(t)) h_j(z(t)) (A_i + B_i K_j) \bigg)^{\mathrm{T}} P \\
&+ P \bigg(\sum_{i=1}^{L} \sum_{j=1}^{L} h_i(z(t)) h_j(z(t)) (A_i + B_i K_j) \bigg) + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4) P^2 \\
&+ \varepsilon_1^{-1} \bigg(\sum_{i=1}^{L} h_i(z(t)) A_{id} \bigg)^{\mathrm{T}} \bigg(\sum_{i=1}^{L} h_i(z(t)) A_{id} \bigg) + \varepsilon_2^{-1} A_p^{\mathrm{T}} A_p + \varepsilon_3^{-1} A_{pd}^{\mathrm{T}} A_{pd} \\
&+ \varepsilon_4^{-1} \bigg(\sum_{j=1}^{L} h_j(z(t)) B_p K_j \bigg)^{\mathrm{T}} \bigg(\sum_{j=1}^{L} h_j(z(t)) B_p K_j \bigg) \bigg\} x(t) dt + 2x^{\mathrm{T}}(t) P E_1 \, dw(t) \\
&\le \sum_{i=1}^{L} \sum_{j=1}^{L} h_i(z(t)) h_j(z(t)) \{ x^{\mathrm{T}}(t) [(A_i + B_i K_j)^{\mathrm{T}} P + P(A_i + B_i K_j) \\
&+ (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4) P^2 + \varepsilon_1^{-1} A_{id}^{\mathrm{T}} A_{id} + \varepsilon_2^{-1} A_p^{\mathrm{T}} A_p + \varepsilon_3^{-1} A_{pd}^{\mathrm{T}} A_{pd} \\
&+ \varepsilon_4^{-1} (B_p K_j)^{\mathrm{T}} (B_p K_j)] x(t) \} \, dt + 2x^{\mathrm{T}}(t) P E_1 \, dw(t) \\
&\le - \sum_{i=1}^{L} \sum_{j=1}^{L} \lambda_{\min}(-\Pi_{ij}) x^{\mathrm{T}}(t) x(t) \, dt + 2x^{\mathrm{T}}(t) P E_1 \, dw(t),
\end{aligned} \tag{36}
$$

where

$$
\begin{aligned}
\Pi_{ij} &= (A_i + B_i K_j)^{\mathrm{T}} P + P(A_i + B_i K_j) + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4) P^2 \\
&+ \varepsilon_1^{-1} A_{id}^{\mathrm{T}} A_{id} + \varepsilon_2^{-1} A_p^{\mathrm{T}} A_p + \varepsilon_3^{-1} A_{pd}^{\mathrm{T}} A_{pd} + \varepsilon_4^{-1} (B_p K_j)^{\mathrm{T}} (B_p K_j).
\end{aligned} \tag{37}
$$

Then, according to the inequality (28), we find

$$\Pi_{ij} < 0, \quad \text{for} \quad i, j = 1, 2, \ldots, L. \tag{38}$$

Consequently, the inequalities (36) and (38) mean that the nonlinear stochastic time-delay closed-loop system (14) is asymptotically stable (in the mean-square) by the fuzzy control law (13).

The expected exponential stability (in the mean-square) of the closed-loop system (14) can be proved by making some standard manipulation on (36), see [16]. Let β_{ij} be the unique root of the equation

$$\lambda_{\min}(-\Pi_{ij}) - \beta_{ij}\lambda_{\max}(P) - \beta_{ij}h\lambda_{\max}(Q)e^{\beta_{ij}h} = 0, \tag{39}$$

where Π_{ij} and Q are defined, respectively, in (37) and (30) and P is the positive definite solution to (28) and h is the unknown time-delay. Then, by [26], we have

$$\begin{aligned} E|x(t)|^2 \le \lambda_{\min}^{-1}(P)\big([\lambda_{\max}(P) + h\lambda_{\max}(Q)] \\ + \beta_{ij}\lambda_{\max}(q)h^2 e^{\beta_{ij}h}\big) \sup_{-h\le\theta\le 0} E|\zeta(\theta)|^2 e^{-\beta_{ij}t}. \end{aligned} \tag{40}$$

Notice that, according to (40), the definition of exponential stable in Definition 1 is satisfied and this complete the proof of Theorem 1.

The result of Theorem 1 may be conservative due to the use of inequalities (32)–(35). However, such conservativeness can be significantly reduced by appropriate choices of the parameters ε_1 ε_2, ε_3, ε_4 in a matrix norm sense.

Remark 2 The result of Theorem 1 can be easily extended to the multiple state time-delay case. Consider the following nonlinear continuous-time multidelay stochastic system

$$\begin{gathered} dx(t) = \left[A(x(t))x(t) + \sum_{i=1}^{r} A_d(x(t))x(t-h_i) + B(x(t))u(t)\right]dt + \sum_{i=1}^{r} E_i\, dw_i(t), \\ x(t) = \varphi(t), \quad t \in [-h, 0], \quad 0 < h = \max_i(h_i), \end{gathered} \tag{41}$$

where $(w_1, w_2, \ldots, w_m)$ is an m-dimensional Brownian motion, instead of a scalar one in system (1)–(2). Also, instead of (29), we define the Lyapunov function

$$\Upsilon(x(t), t) = x^{\mathrm{T}}(t)Px(t) + \sum_{i=1}^{r} \int_{t-h_i}^{t} x^{\mathrm{T}}(s)Q_i\, x(s)\, ds. \tag{42}$$

Remark 3 We can conclude the following matrix inequality, similar to matrix inequality (28) in Theorem 1, for the T-S fuzzy systems with norm-bounded and structured parametric uncertainties introduced in Remark 1 as

$$\begin{gathered} (A_i + B_iK_j)^{\mathrm{T}}P + P(A_i + B_iK_j) + P\big((\eta_1 + \eta_2)D_iD_i^{\mathrm{T}} + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)I\big)P \\ + \varepsilon_1^{-1}A_{id}^{\mathrm{T}}A_{id} + \varepsilon_2^{-1}A_p^{\mathrm{T}}A_p + \varepsilon_3^{-1}A_{pd}^{\mathrm{T}}A_{pd} \\ + \eta_1^{-1}F_{1i}^{\mathrm{T}}F_{1i} + \varepsilon_4^{-1}(B_pK_j)^{\mathrm{T}}(B_pK_j) + \eta_2^{-1}(F_{2i}K_j)^{\mathrm{T}}F_{2i}K_j < 0, \end{gathered}$$

where according to Lemma 1 the following matrix inequalities are satisfied for $\forall\, \eta_1, \eta_2 > 0$

$$(\Delta A_i^p)^{\mathrm{T}} P + P\Delta A_i^p \le \eta_1 P D_i D_i^{\mathrm{T}} P + \eta_1^{-1} F_{1i}^{\mathrm{T}} F_{1i},$$

$$(\Delta B_i^p K_j)^{\mathrm{T}} P + P\Delta B_i^p K_j \le \eta_2 P D_i D_i^{\mathrm{T}} P + \eta_2^{-1} (F_{2i}K_j)^{\mathrm{T}} F_{2i} K_j.$$

3.2 Fuzzy control design

This subsection is devoted to the design of control parameters K_j, for $j = 1, 2, \dots, L$, by using the result in Theorem 1. We will show that the design of control parameters problem can be solved via the resolution of matrix inequalities. Our approach follows the one developed by Gahinet for the deterministic case [6]. The key tool, which makes this possible, is the stochastic version of the Bounded Real Lemma. From deterministic H_∞ control theory we will need the following lemma, so-called, *Projection Lemma*.

Lemma 2 [29] *Given a symmetric matrix $H \in R^{m\times m}$ and two matrices $N \in R^{l\times m}$ and $M \in R^{n\times m}$, consider the problem of finding some matrix X such that*

$$H + N^{\mathrm{T}} X^{\mathrm{T}} M + M^{\mathrm{T}} X N < 0. \tag{43}$$

Then, (43) is solvable for X if and only if

$$N^{T\perp} H\, N^{T\perp T} < 0, \quad M^{T\perp} H\, M^{T\perp T} < 0. \tag{44}$$

Here, if $\Sigma \in R^{n\times m}$ and $\operatorname{rank}\Sigma = r$, the orthogonal complement $\Sigma^\perp$ is defined as a possibly nonunique $(n-r)\times n$ matrix with rank $n-r$, such that $\Sigma^\perp \Sigma = 0$.

By using the *Schur complement formula*, inequality (28) is equivalent to

$$\begin{bmatrix} (A_i + B_iK_j)^{\mathrm{T}} P + P(A_i + B_iK_j) + \Psi_i^{\mathrm{T}}\Psi_i & (B_pK_j)^{\mathrm{T}} & P \\ B_pK_j & -\varepsilon_4 I & 0 \\ P & 0 & -(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4)^{-1} I \end{bmatrix} < 0, \tag{45}$$

where

$$\Psi_i = \begin{bmatrix} \varepsilon_1^{-1/2} A_{id} \\ \varepsilon_2^{-1/2} A_p \\ \varepsilon_3^{-1/2} A_{pd} \end{bmatrix}. \tag{46}$$

The inequality (45) has the form

$$\Gamma_i + N_i^{\mathrm{T}} \Omega M + M^{\mathrm{T}} \Omega^{\mathrm{T}} N_i < 0, \tag{47}$$

where

$$\begin{aligned} &\Omega = K_j, \quad M = [\,I \;\; 0 \;\; 0\,], \quad N_i^{\mathrm{T}} = \begin{bmatrix} PB_i \\ B_p \\ 0 \end{bmatrix} = \begin{bmatrix} P & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} B_i \\ B_p \\ 0 \end{bmatrix}, \\ &\Gamma_i = \begin{bmatrix} A_i^{\mathrm{T}} P + PA_i + \Psi_i^{\mathrm{T}}\Psi_i & 0 & P \\ 0 & -\varepsilon_4 I & 0 \\ P & 0 & -(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4)^{-1} I \end{bmatrix}. \end{aligned} \tag{48}$$

Then, we have the following result.

Theorem 2 *The closed-loop fuzzy system (14) is exponentially stable in the mean-square and independent of the unknown time-delay h, if the following conditions are satisfied, for* $i = 1, 2, \dots, L$,

$$\begin{aligned} N_i^{T\perp}\Gamma_i N_i^{T\perp T} &< 0, \\ M^{T\perp}\Gamma_i M^{T\perp T} &< 0, \\ P = P^{\mathrm{T}} &> 0, \end{aligned} \tag{49}$$

where M, N_i *and* Γ_i *are defined in (48).*

Proof The proof follows directly from Theorem 1 and Projection lemma.

Let $[\,V_{1i} \;\; V_2\,] = [\,B_i \;\; B_p\,]^{T\perp}$ and, by some calculation, we have

$$N_i^{T\perp} = \begin{bmatrix} V_{1i} & V_2 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} P^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \tag{50}$$

and

$$M^{T\perp} = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}. \tag{51}$$

Then, it follows from (49) that we have:

$$M^{T\perp}\Gamma_i M^{T\perp T} = \begin{bmatrix} -\varepsilon_4 I & 0 \\ 0 & -(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)^{-1}\, I \end{bmatrix} < 0. \tag{52}$$

This further implies that $M^{T\perp}\Gamma_i M^{T\perp T} < 0$ is satisfied for $i = 1, 2, \dots, L$ and

$$N_i^{T\perp}\Gamma_i N_i^{T\perp T} = \begin{bmatrix} W & [\,V_{1i} \quad V_2\,]\begin{bmatrix} I \\ 0 \end{bmatrix} \\ [\,I \quad 0\,]\begin{bmatrix} V_{1i}^{\mathrm{T}} \\ V_2^{\mathrm{T}} \end{bmatrix} & -(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)^{-1} I \end{bmatrix} < 0, \tag{53}$$

where

$$W = [\,V_{1i} \quad V_2\,] \begin{bmatrix} P^{-1}(A_i^{\mathrm{T}} P + P A_i + \Psi_i^{\mathrm{T}}\Psi_i)P^{-1} & 0 \\ 0 & -\varepsilon_4 I \end{bmatrix} \begin{bmatrix} V_{1i}^{\mathrm{T}} \\ V_2^{\mathrm{T}} \end{bmatrix}.$$

Using the Schur complement formula, it is easy to see that (53) is equivalent to

$$A_i^{\mathrm{T}} P + P A_i + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4) P^2 + \Psi_i^{\mathrm{T}}\Psi_i < 0. \tag{54}$$

If the LMI in (54) have a positive-definite solution for P, then the closed-loop system (14) is exponentially stable in the mean-square and independent of the unknown time-delay h. Moreover, in this case, a set of particular solutions of control parameters K_j, for $j = 1, 2, \dots, L$, corresponding to a feasible solution P can be obtained by using the result of matrix inequality (54). Then, we obtain the following result.

Theorem 3 *If there exist positive scalars ε_1, ε_2, ε_3, ε_4 such that the linear matrix inequality (54) has positive definite solution P, then, the fuzzy control with parameters $\Omega = K_j$ for $j = 1, 2, \dots, L$ can be easily obtained by solving (47) and will be such that the closed-loop system (14) is exponentially stable in the mean-square and independent of the unknown time-delay h.*

Remark 4 In the case when $E_1 = 0$, that is, the stochastic system (1) − (2) is specialized to a deterministic system. Therefore, Theorems 1, 2 and 3 can be viewed as extensions of existing results from deterministic systems to stochastic systems.

4 Simulation Results

In this section, to illustrate the effectiveness of the proposed method, we will design a fuzzy linear controller for the following stochastic nonlinear time-delay system

$$dx(t) = [-0.06\,x(t)^3 + x(t-h) + u(t)]\,dt + dw(t) \tag{55}$$

$$x(t) = 1, \qquad t \in [-h,\, 0]. \tag{56}$$

Consider $h = 1$ second as the time-delay parameter. To use the fuzzy linear controller design, we consider a fuzzy model, which represents the dynamics of the nonlinear plant. Therefore, we represent the system (55) − (56) by the following T-S fuzzy model

Plant Rule 1:

$$\begin{aligned} &\textit{If } x(t) \textit{ is } F_{11},\\ &\textit{then} \quad dx(t) = [-3\,x(t) + 0.5\,x(t-h) + 2\,u(t)]\,dt + dw(t). \end{aligned}$$

Plant Rule 2:

$$\begin{aligned} &\textit{If } x(t) \textit{ is } F_{21},\\ &\textit{then} \quad dx(t) = [-2\,x(t) + 0.1\,x(t-h) + u(t)]\,dt + dw(t). \end{aligned}$$

where the membership functions of F_{11} and F_{21} are given as follows:

$$F_{11} = 1 - \frac{1}{1 + e^{-x^2}}, \quad F_{21} = 1 - F_{11} = \frac{1}{1 + e^{-x^2}},$$

and the bounding matrices are chosen as $A_p = 0.5$, $A_{pd} = 0.5$ and $B_p = 1$.

Substituting the above parameters into Theorem 3, using the LMI toolbox in MATLAB the solutions of (47), i.e., state feedback gains, can be obtained as $K_1 = 0.1$ and $K_2 = 0.1709$ and the positive scalars ε_1, ε_2, ε_3, ε_4 found as $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.1$.

Robust stability of the state of system (55) in the presence of disturbance, i.e. Brownian motions has been depicted in Figure 4.1 and it is seen that due to Brownian motion as

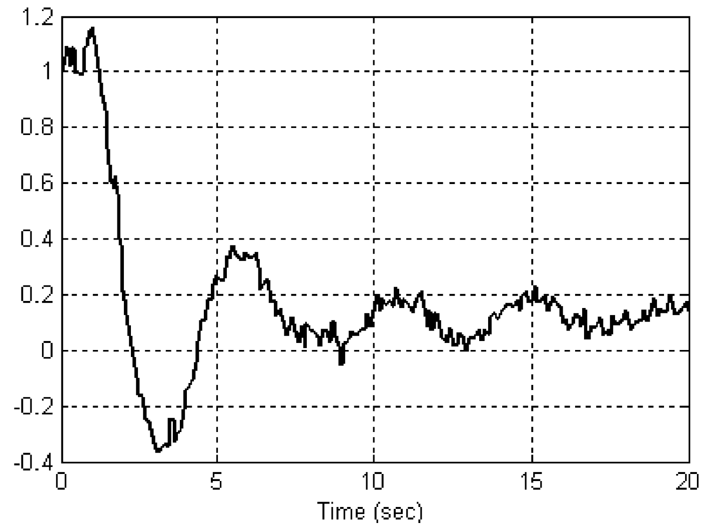

Figure 4.1. Time behavior of the state of system.

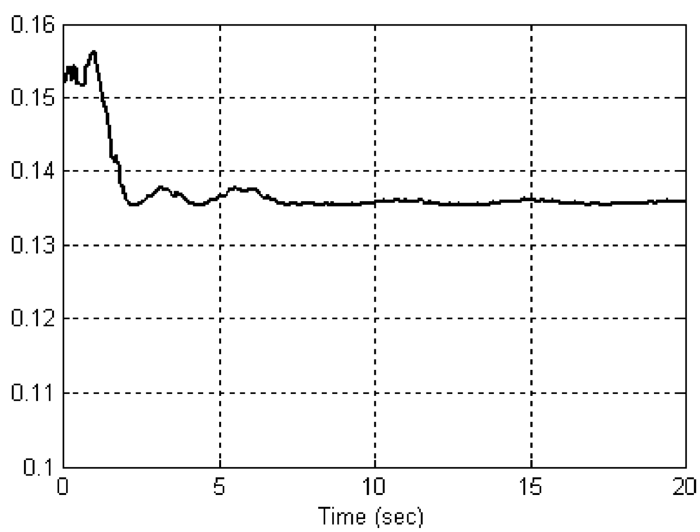

Figure 4.2. Control input.

the external disturbance, state still is bounded. The overall fuzzy controller is shown in Figure 4.2.

5 Conclusions

In this paper, the fuzzy linear control design method for a class of stochastic nonlinear time-delay systems with state feedback was developed. First, the Takagi and Sugeno fuzzy linear model was employed to approximate a nonlinear system. Next, based on the fuzzy linear model, a fuzzy linear controller was developed to stabilize the non-linear system. The control law has been obtained to ensure stochastical exponential stability in the mean-square, independent of the time-delay and the sufficient conditions for the existence of such a control were proposed in terms of certain linear matrix inequality. A simulation example was given to illustrate the applicability of the proposed design method.

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