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New Stability Conditions for TS Fuzzy Continuous Nonlinear Models

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Abstract: Several linear and nonlinear fuzzy models stability conditions are developed in the literature. Some of them concern the linear fuzzy Takagi–Sugeno (TS) model and are based on the determination of a common positive definite matrix, solution of linear matrix inequalities.

A new explicit formulation of stability conditions and an extension to the case of nonlinear TS fuzzy continuous models are given in this paper.

The proposed criteria are based on the use of the vector norm approach associated, in the state space description, to a specific characteristic matrix form, called arrow form matrix. This representation is such that only the elements of the diagonal, those of the last row and those of the last column can be different from zero.

The obtained stability conditions, explicitly expressed by the studied models and fuzzification parameters, applicable for TS fuzzy models in particular, make the approach useful for the synthesis of stabilizing fuzzy control law.

For a class of considered Lur'e–Postnikov continuous case, the stability criterion corresponds to a simple condition on the instantaneous characteristic polynomial of the nonlinear studied system.

Keywords: *Nonlinear continuous system; TS fuzzy model; stability; arrow form matrix; vector norm; Lur'e–Postnikov system.*

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1 Introduction

Takagi-Sugeno (TS) fuzzy models, proposed by Takagi and Sugeno [16] and further developed by Sugeno and Kang [15], are nonlinear systems described by a set of IF–THEN rules which gives a local linear representation of an underlining system. It is well known that such models can describe or approximate a wide class of nonlinear systems. Hence, it is important to study their stability or to synthesize their stabilizing controllers.

In fact, the stability study constitutes an important phase in the synthesis of a control law, as well as in the analysis of the dynamic behavior of a closed loop system. It has been one of the central issues concerning fuzzy control, refer to the brief survey on the stability issues given in [14].

Based on the stability conditions, model-based control of such systems has been developed for the continuous case in [5 – 7, 13, 19, 20] by using state-space models.

In recent literature, Tanaka and Sugeno [17], have provided a sufficient condition for the asymptotic stability of a fuzzy system in the sense of Lyapunov through the existence of a common Lyapunov function for all the subsystems.

This kind of design methods suffer mainly from a few limitations:

(1) one can construct a TS model if local description of the dynamical system to be controlled is available in terms of local linear models;

(2) a common positive definite matrix must be found to satisfy a matrix Lyapunov equation, which can be difficult especially when the number of fuzzy rules required to give a good plant model is large so that the dimension of the matrix equation is high;

(3) it appears that a necessary condition, for the existence of this common positive definite matrix, is that all subsystems must be asymptotically stable.

To overcome those difficulties, we propose, in this paper, to study the stability of TS fuzzy nonlinear model through the study of the convergence of a regular vector norm.

If the vector norm is of dimension one, then this is like the second Lyapunov method approach; therefore, if it is of higher dimension, then we deal with a vector-Lyapunov function [9 – 12].

The vector norm approach, based on the comparison/overvaluing principle, has a major advantage: it deals with a very large class of systems, since no restrictive assumption is made on the matrices of state equations, except that they are bounded for bounded states, in such a way that a unique continuous solution exists.

Nevertheless, although the overvaluing principle allows the simplification of the study, it also presents the corresponding drawback: overvaluation means losing information on the real behavior of the process. Thus, the cases of state equations which are the most resistant of this type of method are the ones in which replacement of coefficients by their absolute values leads to an overvaluing system which is far from reality, for instance an unstable one, whereas the initial system was stable. In many cases, this type of drawback can be bypassed by using changes of state variables leading to a good performance of the representation [2 – 4]. For instance, for continuous control, a particularly interesting case is the one in which the off-diagonal elements are naturally positive or equal to zero; in this case, the overvaluing is carried out without loss of information.

This paper is organized as follows: TS fuzzy nonlinear continuous model description is presented in Section 2. Section 3 reviews some existing stability conditions of such system. In Section 4, the vector norm approach combined with the arrow form matrix are employed to give the new stability criterion for TS fuzzy nonlinear continuous models. The case of Lur'e–Postnikov continuous system is studied in Section 5. Finally, conclusions are drawn in Section 6.

2 TS Fuzzy Nonlinear Continuous Model Description

Consider a TS fuzzy model when local description of the plant to be controlled is available in terms of nonlinear autonomous models

$$\dot{X}(t) = A_i(X)X(t) \tag{1}$$

where $X \in R^n$ describes the state vector, $A_i(\cdot)$ are matrices of appropriate dimensions, $A_i(\cdot) = \{a_{ij}(\cdot)\}$ and $a_{ij}(\cdot)\colon R^n \to R$, are nonlinear elements.

It is assumed that $X = 0$ is the unique equilibrium state of the studied system.

The above information is then fused with the available IF–THEN rules, where the i-th rule, $i = 1, \dots, r$, can have the form:

Rule i: IF $\{X(t)$ is $H_i(X)\}$ THEN $\Big\{\dot{X}(t) = A_i(\cdot)X(t)\Big\}$,

where $H_i(X)$ is the grade of the membership of the state $X(t)$.

The final output of the fuzzy system is inferred as follows:

$$\dot{X}(t) = \sum_{i=1}^{r} h_i(X)A_i(\cdot)X(t) \tag{2}$$

with, for $i = 1, \dots, r, \quad 0 \le h_i(X) \le 1$ and $\sum\limits_{i=1}^{r} h_i(t) = 1$.

3 Stability Conditions — Problem Statement

It is straightforward to show that a sufficient condition for asymptotic stability in the large of the equilibrium state $X = 0$ of the unforced fuzzy model, obtained by linearization of (2),

$$\dot{X}(t) = \sum_{i=1}^{r} h_i A_i X(t) \tag{3}$$

is that there exists a common symmetric positive definite matrix P such that, for $i = 1, 2, \dots, r$

$$A_i^{\mathrm{T}} P + P A_i < 0. \tag{4}$$

The necessary condition for the existence of matrix P is that each matrix must be asymptotically stable [17], i.e. all the subsystems are stable, or that matrices:

$$\sum_{j=1}^{k} A_{i_j} \tag{5}$$

where $i_j \in \{1, 2, \dots, r\}$ and $k = 2, 3, \dots, r$, are asymptotically stable [18].

The linear matrix inequality (LMI) based approaches have been used to determine the existence of a common symmetric positive definite matrix [20]. Their computation can be expensive in the case of high number of rules.

As it was shown, the stability study of the nonlinear model (2) requires the linearization of the nonlinear subsystems described by the instantaneous characteristic matrices A_i. If those matrices are in arrow form [2], stability conditions of the nonlinear system (2), as we will see in the next section, can be formulated easily.

4 New TS Fuzzy Nonlinear Model Stability Criterion

Let us consider the continuous process whose model is in the controllable form, that matrices $A_i(\cdot)$, of equation (2), are written as

$$A_i(\cdot) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ & & & & 0 \\ 0 & & \dots & 0 & 1 \\ -a_{i,0}(\cdot) & & \dots & & -a_{i,n-1}(\cdot) \end{bmatrix}. \tag{6}$$

A change of base under the form:

$$T = \begin{bmatrix} 1 & 1 & \cdots & 1 & 0 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_1^{n-2} & \alpha_2^{n-2} & \cdots & \alpha_{n-1}^{n-2} & 0 \\ \alpha_1^{n-1} & \alpha_2^{n-1} & \cdots & \alpha_{n-1}^{n-1} & 1 \end{bmatrix} \tag{7}$$

allows the new state matrices, denoted by $F_i(\cdot)$, to be in arrow form (2)

$$F_i(\cdot) = T^{-1}A_i(\cdot)T = \begin{bmatrix} \alpha_1 & & & \beta_1 \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & \beta_{n-1} \\ \gamma_{i,1}(\cdot) & \cdots & \gamma_{i,n-1}(\cdot) & \gamma_{i,n}(\cdot) \end{bmatrix}, \tag{8}$$

where

$$\beta_j = \prod_{\substack{k=1 \\ k\neq j}}^{n-1} (\alpha_j - \alpha_k)^{-1} \quad \forall\, j = 1, 2, \dots, n-1, \tag{8a}$$

$$\gamma_{i,j}(\cdot) = -P_{A_i}(\cdot, \alpha_j) \quad \forall\, j = 1, 2, \dots, n-1, \tag{8b}$$

$$\gamma_{i,n}(\cdot) = -a_{i,\,n-1}(\cdot) - \sum_{i=1}^{n-1} \alpha_i. \tag{8c}$$

$P_{A_i}(\cdot, \lambda)$ is the $A_i(\cdot)$ instantaneous characteristic polynomial such that

$$P_{A_i}(\cdot, \lambda) = \lambda^n + \sum_{l=0}^{n-1} a_{i,l}(\cdot)\lambda^l \tag{9}$$

and $\alpha_j,\ j=1,2,\dots,n-1,$ are distinct arbitrary parameters.

Let us note that the determinant of the arrow form matrix $F_i(\cdot)$ is computed as [2]

$$|F_i(\cdot)| = \left[\gamma_{i,n}(\cdot) - \sum_{j=1}^{n-1} \alpha_j^{-1}\gamma_{i,j}(\cdot)\,\beta_j\right] \prod_{k=1}^{n-1} \alpha_k. \tag{10}$$

The final output of the fuzzy system is then inferred as follows

$$\dot{Y}(t) = Q(\cdot)Y(t) \tag{11}$$

where $Y(t)$ is the new state vector such that $X(t)=TY(t)$,

$$Q(\cdot) = \sum_{i=1}^{r} h_i F_i(\cdot), \tag{11a}$$

$$Q(\cdot) = \begin{bmatrix} \alpha_1 & & & \beta_1 \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & \beta_{n-1} \\ \sum\limits_{i=1}^{r} h_i\gamma_{i,1}(\cdot) & \cdots & \sum\limits_{i=1}^{r} h_i\gamma_{i,n-1}(\cdot) & \sum\limits_{i=1}^{r} h_i\gamma_{i,n}(\cdot) \end{bmatrix}. \tag{11b}$$

In such conditions, if $p(Y)$ denotes a vector norm of Y, satisfying component to component the equality

$$p(Y) = |Y| \tag{12}$$

it is possible, by the use of the aggregation techniques [2, 9], to define a comparison system (13), $Z \in R^n$, of (11)

$$\dot{Z} = M(\cdot)Z. \tag{13}$$

In this expression, the matrix $M(\cdot)$ is deduced from the matrix $Q(\cdot)$ by substituting only the off-diagonal elements by their absolute values; it can be written as

$$M(\cdot) = \begin{bmatrix} \alpha_1 & & & |\beta_1| \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & |\beta_{n-1}| \\ \left|\sum\limits_{i=1}^{r} h_i\gamma_{i,1}(\cdot)\right| & \cdots & \left|\sum\limits_{i=1}^{r} h_i\gamma_{i,n-1}(\cdot)\right| & \sum\limits_{i=1}^{r} h_i\gamma_{i,n}(\cdot) \end{bmatrix}. \tag{14}$$

Noting that the non-constant elements are isolated in the last row of matrix $M(\cdot)$, then the stability condition of the continuous nonlinear system (2) can be easily deduced from the Borne and Gentina criterion [8, 11]. It comes

$$(-1)^i \Delta_i > 0, \quad i = 1,2,\dots,n, \tag{15}$$

with Δ_i the i-th $M(\cdot)$ principal minor.

It is clear that, for $i=1,2,\dots,n-1,$ the condition (15) is verified for $\alpha_i \in R_-$, therefore, for $i=n$ and using the relation (10), it leads to the stability condition (16).

Then, the TS fuzzy nonlinear model stability, in the continuous case, can be studied by the following proposed theorem.

Theorem 4.1 *If there exist* $\alpha_i \in R_-$, $i = 1, 2, \ldots, n-1$, $\alpha_i \neq \alpha_j$ *for all* $i \neq j$ *and* $\varepsilon \in R_+$ *such that the inequality*

$$-\sum_{i=1}^{r} h_i \gamma_{i,n}(\cdot) + \sum_{j=1}^{n-1} \left| \sum_{i=1}^{r} h_i \gamma_{i,j}(\cdot) \beta_j \right| \alpha_j^{-1} \geq \varepsilon \quad \forall X \in R^n \tag{16}$$

is satisfied, the equilibrium state of the studied continuous nonlinear system (3) and (7) is asymptotically globally stable.

If there exist α_j, $j = 1, 2, \ldots, n-1$, such that

$$\sum_{i=1}^{r} h_i \gamma_{i,j}(\cdot) \beta_j > 0 \quad j = 1, 2, \ldots, n-1, \tag{17}$$

the Theorem 4.1 can be simplified and the comparison system (13) can be chosen identically to (11).

Since for $Q(\cdot)$

$$\Delta_n = \sum_{i=1}^{r} h_i P_{A_i}(\cdot, 0), \tag{18}$$

$$-\sum_{i=1}^{r} h_i \gamma_{i,n}(\cdot) + \sum_{j=1}^{n-1} \alpha_j^{-1} \sum_{i=1}^{r} h_i \gamma_{i,j}(\cdot) \beta_j = \prod_{j=1}^{n-1} (-\alpha_j)^{-1} \sum_{i=1}^{r} h_i P_{A_i}(\cdot, 0). \tag{19}$$

Hence to Corollary 4.1.

Corollary 4.1 *If there exist* $\alpha_j \in R_-$, $\alpha_j \neq \alpha_k$ *for all* $j \neq k$ *and* $\varepsilon \in R_+$ *such that:*

(i) *the inequalities (17) are satisfied for all* $X \in R^n$*,*

(ii) $\sum_{i=1}^{r} h_i(t)\, P_{A_i}(\cdot, 0) \geq \varepsilon$ *for all* $X \in R^n$*,* *(20)*

the equilibrium state of the continuous system described by (2) and (6) is globally asymptotically stable.

Example 4.1. Unstable TS fuzzy model case

Given the unforced fuzzy linear system model described by (3), where $r = 2$ and

$$A_1 = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}.$$

Obviously, the first subsystem is unstable whereas the second one is stable. However, there is no common positive definite matrix P to verify the stability condition (4).

The matrices A_1 and A_2 can be transformed to arrow form matrices F_1 and F_2, by the same change of base under the form

$$T = \begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix}, \qquad F_1 = T^{-1} A_1 T = \begin{bmatrix} \alpha & 1 \\ -(\alpha^2 + \alpha - 2) & -1 - \alpha \end{bmatrix},$$

$$F_2 = T^{-1} A_2 T = \begin{bmatrix} \alpha & 1 \\ -(\alpha^2 + \alpha + 1) & -1 - \alpha \end{bmatrix}$$

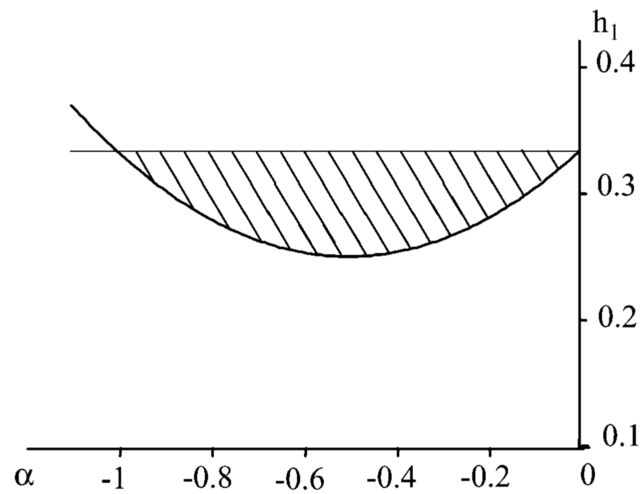

Figure 4.1. Stability domain for example 1.

where α is an arbitrary non zero parameter.

Since $h_1 + h_2 = 1$, the global fuzzy system is then described by

$$\dot{y}(t) = (h_1 F_1 + h_2 F_2) Y(t) = \begin{bmatrix} \alpha & 1 \\ -\alpha^2 - \alpha + 2h_1 - h_2 & -1-\alpha \end{bmatrix} Y(t).$$

The application of Theorem 4.1 leads to the following stability conditions

(i) $\alpha < 0$,

(ii) $1 + \alpha + \dfrac{|-\alpha^2 - \alpha + 2h_1 - h_2|}{\alpha} > 0$,

since $h_2 = 1 - h_1$, $h_1 \in [0;1]$, the corresponding stability domain is represented by the hatching domain in Figure 4.1.

5 Lur'e–Postnikov System Case

Consider the nonlinear system given in Figure 5.1, with $e = -C\sigma$, $\sigma = [y, y^{(1)}, \dots, y^{(n-1)}]^{\mathrm{T}}$, $\sigma \in R^n$, $C = [c_0, c_1, \dots, c_{n-2}, c_{n-1}]$ is a vector with constant elements, $c_{n-1} = 1$, $f \in \Lambda = \{f \colon R^n \to R,\ f(C\sigma)/C\sigma \equiv f^*(C\sigma),\ C\sigma \neq 0;\ f^*(\cdot) \in [\underline{f}^*, \overline{f}^*] \subset R,\ \forall \sigma \in R^n\}$ and

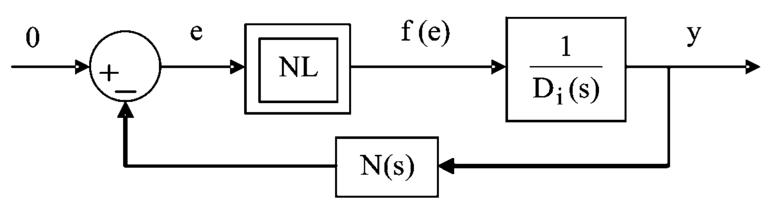

Figure 5.1. The i-th Lur'e–Postnikov model.

$$D_i(s) \;=\; s^n + \sum_{j=0}^{n-1} d_{i,j} s^j \equiv \prod_{j=1}^{n} (s - p_{i,j}), \tag{21}$$

$$N(s) = s^{n-1} + \sum_{j=0}^{n-2} c_j s^j \equiv \prod_{j=1}^{n-1} (s - z_j). \tag{22}$$

Thus, the i-th unforced Lur'e–Postnikov system can be described by

$$D_i(s)y(s) = f(e) = f^*(\cdot)e = -f^*(\cdot)N(s)y(s) \tag{23}$$

witch leads to the nonlinear differential equation

$$y^{(n)} + \sum_{j=0}^{n-1} (d_{i,j} + f^*(\cdot)c_j)y^{(j)} = 0. \tag{24}$$

With the choice of arbitrary parameters α'_j such that

$$\alpha_j = z_j, \quad j = 1, \dots, n-1, \tag{25}$$

this system can be described [1], in the state space arrow form, by

$$F'_i(\cdot) = \begin{bmatrix} z_1 & & & \beta'_1 \\ & \ddots & & \vdots \\ & & z_{n-1} & \beta'_{n-1} \\ \gamma'_{i,1}(\cdot) & \dots & \gamma'_{i,n-1}(\cdot) & \gamma'_{i,n}(\cdot) \end{bmatrix} \tag{26}$$

with

$$\beta'_j = \prod_{\substack{k=1 \\ k \neq j}}^{n-1} (z_j - z_k)^{-1}, \quad \forall\, j = 1, \dots, n-1, \tag{26a}$$

$$\gamma'_{i,n}(\cdot) = -(d_{i,n-1} + f^*(\cdot)) - \sum_{j=1}^{n-1} z_j, \tag{26b}$$

$$\gamma'_{i,j}(\cdot) = -P'_{A_i}(\cdot, z_j), \quad \forall\, j = 1, \dots, n-1, \tag{26c}$$

$$P'_{A_i}(\cdot, z_j) = D_i(z_j) + f^*(\cdot)\, N_i(z_j) \equiv D_i(z_j). \tag{26d}$$

The final output of the fuzzy Lur'e–Postnikov system is then inferred as (11)

$$\dot{Y}(t) = \sum_{i=1}^{r} h_i F'_i(\cdot) Y(t) \tag{27}$$

with

$$\sum_{i=1}^{r} h_i F'_i(\cdot) = \begin{bmatrix} z_1 & & & \beta'_1 \\ & \ddots & & \vdots \\ & & z_{n-1} & \beta'_{n-1} \\ \sum\limits_{i=1}^{r} h_i \gamma'_{i,1}(\cdot) & \dots & \sum\limits_{i=1}^{r} h_i \gamma'_{i,n-1}(\cdot) & \sum\limits_{i=1}^{r} h_i \gamma'_{i,n}(\cdot) \end{bmatrix}. \tag{27a}$$

The stability conditions of the studied Lur'e–Postnikov system can be deduced by using the following proposed theorem.

Theorem 5.1 *The Lur'e–Postnikov system described by (24) is globally asymptotically stable if there exist $\varepsilon \in R_+$ such that the following conditions are verified*

$$z_j \in R_-,\ j = 1, \dots, n-1, \quad z_i \neq z_j \quad \forall\, i \neq j; \tag{28}$$

$$-\sum_{i=1}^{r} h_i(t)\gamma'_{i,n}(\cdot) + \sum_{j=1}^{n-1} (z_j)^{-1} \left| \beta'_j \sum_{i=1}^{r} h_i(t)\gamma'_{i,j}(\cdot) \right| \geq \varepsilon > 0. \tag{29}$$

Proof The non-constant elements in (27a) are isolated in the last row. Hence, the stability conditions can be easily deduced from the Theorem 4.1.

If for parameters $z_j,\ j = 1, \dots, n,$ the following condition is verified

$$\beta'_j \sum_{i=1}^{r} h_i \gamma'_{i,j}(\cdot) > 0 \tag{30}$$

the inequality (29) can then be written:

$$\prod_{i=1}^{n-1} (-z_j)^{-1} \sum_{i=1}^{r} h_i P'_{A_i}(\cdot, 0) \geq \varepsilon > 0. \tag{31}$$

Hence to Corollary 5.1.

Corollary 5.1 *If $z_j \in R_-,\ j = 1, 2, \dots, n-1,\ z_j \neq z_k,\ \forall\, j \neq k,$ and $\varepsilon \in R_+$ such that $\forall\, X \in R^n$*

(i) *the inequality (30) is satisfied;* *(32)*

(ii) $\sum\limits_{i=1}^{r} h_i P'_{A_i}(\cdot, 0) \geq \varepsilon,$ *(33)*

the Lur'e–Postnikov continuous system described by (24), (26) and (27) is globally asymptotically stable.

Example 5.1 Consider the Lur'e–Postnikov system shown in Figure 5.1 with $n = 2$, $r = 2,\ f(e) = f^*(\cdot)e,\ p_{1,1} = -1,\ p_{1,2} = -3,\ p_{2,1} = -2,\ p_{2,2} = -4$ and $z_1 = -2.5$.
From (21) and (22), one can obtain then $d_{1,1} = 4,\ d_{1,0} = 3,\ d_{2,1} = 6,\ d_{2,0} = 8$, and $c_0 = 2.5$.
According to (26), the characteristic matrices, in the arrow form, are given by

$$F'_1(\cdot) = \begin{bmatrix} z_1 & 1 \\ -(z_1^2 + 4z_1 + 3) & -4 - f^*(\cdot) - z_1 \end{bmatrix} = \begin{bmatrix} -2.5 & 1 \\ 0.75 & -1.5 - f^*(\cdot) \end{bmatrix},$$
$$F'_2(\cdot) = \begin{bmatrix} z_1 & 1 \\ -(z_1^2 + 6z_1 + 8) & -6 - f^*(\cdot) - z_1 \end{bmatrix} = \begin{bmatrix} -2.5 & 1 \\ 0.75 & -3.5 - f^*(\cdot) \end{bmatrix}.$$

The global fuzzy system is then described by

$$F'(\cdot) = \sum_{i=1}^{2} h_i F'_i(\cdot)$$

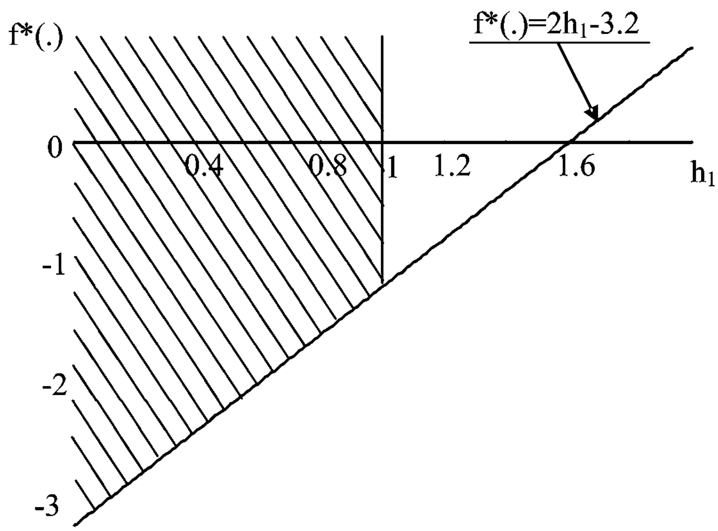

Figure 5.2. Stability domain of the studied Lur'e–Postnikov system.

such that

$$F'(\cdot) = \begin{bmatrix} z_1 & 1 \\ -(z_1^2 + (4h_1 + 6h_2)z_1 + 3h_1 + 8h_2) & -4h_1 - 6h_2 - f^*(\cdot) - z_1 \end{bmatrix}$$

or, for $z_1 = -2.5$

$$F'(\cdot) = \begin{bmatrix} -2.5 & 1 \\ 0.75 & -1.5h_1 - 3.5h_2 - f^*(\cdot) \end{bmatrix}$$

The stability condition of the global fuzzy system, using Theorem 5.1, is then given by

$$1.5h_1 + 3.5h_2 + f^*(\cdot) + \frac{0.75}{-2.5} > 0.$$

Since $h_2 = 1 - h_1$, $h_1 \in [0;\ 1]$, it can be written as

$$f^*(\cdot) > 2h_1 - 3.2$$

which is represented by the hatching domain in Figure 5.2.

6 Conclusions

The new stability conditions, formulated for nonlinear TS fuzzy continuous models case, are based on the use of the vector norm approach combined with an arrow form matrix description.

The obtained stability conditions, explicitly expressed by the studied models and fuzzification parameters, applicable for TS fuzzy models in particular, make the approach useful for the synthesis of stabilization fuzzy control law.

For an important class of Lur'e–Postnikov continuous system, the stability criterion corresponds to a simple instantaneous characteristic polynomial condition.

The considered illustrative examples showed the efficiency of the proposed new approaches.

Other similar results can be obtained easily for nonlinear TS fuzzy discrete systems.

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