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PERSONAGE IN SCIENCE

Professor A.A.Martynyuk

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On March 6, 2006, the Corresponding Member of the National Academy of Sciences of Ukraine, Habilitation Doctor and Ph.D. of physical and mathematical sciences, Professor Anatoly Andreevich Martynyuk turns 65. The Editorial Board of the International Scientific Journal "Nonlinear Dynamics and Systems Theory" congratulates him on the occasion of his 65th birthday and wishes him a great health and new significant achievements in his scientific endeavors. In this regard, the Editorial Board of "Nonlinear Dynamics and Systems Theory" publishes a biographical sketch highlighting Martynyuk's research and scholarly activities.

1 A Brief Survey of Martynyuk's Life

Anatoly A. Martynyuk was born in the family of a railwayman, Andrey Gerasimovich Martynyuk, who lived in Ukraine (Cherkassy region). In 1958, Martynyuk graduated from a high school and the same year he was admitted to the Department of Physics and Mathematics of the Cherkassy State Pedagogical Institute (now B. Khmelnitzky Cherkassy State University). Martynyuk graduated from the Institute with an honor Master of Science degree and for one year he was employed as an instructor of physics and mathematics at a Polessye high school.

In September 1964, Martynyuk was admitted to the post-graduate school of the Institute of Mechanics of Acad. of Sci. of Ukr. SSR (now the S.P.Timoshenko Institute of Mechanics Nat. Acad. of Sci. of Ukraine) chaired by Professor A.N. Golubentzev. Martynyuk's Master's dissertation was focused on the problems of finite stability (on a given time interval). This research was supported both by Professor A.N. Golubentzev and the Department of Differential Equations of the Institute of Mathematics (the Head of the Department the Corresponding Member of Ac. of Sci. of Ukr. SSR, Prof. Yu.D. Sokolov). Martynyuk successfully defended his Master's thesis in the Institute of Mathematics in 1967. In 1969-1973, he worked for his doctorate under Yu.A.Mitropolskii. In 1973 he

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defended his highest degree of Habilitation (with the Academician Yu.A. Mitropolskii serving on his committee). Shortly thereafter, Martynyuk was employed at the Institute of Mathematics, Department of the Presidium Acad. of Sci. of Ukr. SSR, Division of Mathematics, Mechanics and Cybernetics of Acad. of Sci. of Ukr. SSR. In 1978 he founded the Department of Stability of Processes at the Institute of Mechanics of Acad. of Sci. of Ukr. SSR and since then he has been the Head of this Department. In 1988 Martynyuk was elected a Corresponding Member of the Acad. of Sci. of Ukr. SSR, and in 1981 he was presented the prestigious N.M. Kryloff's award of the Acad. of Sci. of Ukr. SSR for his celebrated series of works on nonlinear mechanics.

2 Main Directions of the Scientific Investigations

Martynyuk represents the scientific school of Bogolyubov–Mitropolskii, going back to A.M. Liapunov. Thorough his first studies on the theory of the stability of motion, carried out under the influence of works by Ye.A. Barbashin, N.G. Chetaev, I.G. Malkin, N.N. Krasovskii, K.P. Persidskii, V.I. Zubov. The main directions of the scientific research by Martynyuk are:

* construction of approximate solutions of differential equations systems,
* nonclassical motion stability theories (technical, practical, stability in the whole),
* applications of integral inequalities in the stability theory,
* development of the comparison technique in nonlinear dynamics,
* stability analysis of large scale systems,
* topological dynamics (the method of limiting equations),
* creation of the method of the matrix-valued Liapunov functions,
* qualitative analysis of mathematical models in biology.

We shall outline in brief the development of the above directions in the works by Martynyuk.

2.1 Construction of approximate solutions of differential equations

In a series of his papers Martynyuk treats systems of ordinary differential equations (autonomous or nonautonomous) and proposes construction of solutions in the form of power series or series in Poincaré variable

$$w = \left(e^{\nu(t-t_0)} - 1\right)\left(e^{\nu(t-t_0)} + 1\right)^{-1}, \qquad \nu = \frac{\pi}{2h}. \tag{2.1}$$

In transformation (2.1) the interior of the strip of width $2h$ in the τ-plane is transformed conformally onto the interior of the unit circle $\{|w| = 1\}$ in the w-plane.

If the domain containing solution of the initial system is totaly embedded into the domain of asymptotic stability of the system under investigation, the corresponding series converge for all values of the variable t. He constructs new recurrent formulas for calculation of coefficients of these series and studies stability of approximate solutions obtained as a result of approximate solutions obtained as a result of application of a finite number of terms of the series.

Monograph [I] provides generalization of the results [5–7, 9, 16, 18, 21, 22, 25, 30, 34] and presents some applications in dynamics of mechanical systems.

2.2 Nonclassical theories of motion stability

Martynyuk's research of nonclassical stability theory is the tantamount of stability analysis of solutions to nonlinear systems, in which the domain interval of the independent (time) variable is a fixed interval. His results are about technical stability, stability on a finite interval and practical stability. Stability problems in the nonclassical sense arise in aviation, rocket building, robot technical systems construction and alike.

In the early sixties, different methods were adapted to the problems of nonclassical stability theory by many investigators. In particular, Martynyuk introduced the "locally large Liapunov function" and proved general theorems on technical stability of the continuous nonlinear systems and systems with delay in general.

Under various assumptions imposed on the system

$$\frac{dx}{dt} = X(t, x), \qquad x(t_0) = x_0, \tag{2.2}$$

where $t \in R_+$, $x \in R^n$ and $X\colon R_+ \times R^n \to R^n$, numerous sufficient conditions for the technical stability of motion were established by means of the Liapunov functions of the form

$$v(t, x) = e^{-Dt} x^{\mathrm{T}} K x, \tag{2.3}$$

where D is the diameter of the domain of admissible motion deviations and K is a $n \times n$-matrix of definite sign.

The development of some ideas and results of [1–4, 8, 17, 20, 27–29, 39, 47, 48] enabled Martynyuk to obtain new tests for practical stability of motion for some classes of systems of equations presented in [60, 68–70, 75, 86, 91]. An efficient application of the direct Liapunov method in the practical stability problems by Martynyuk yielded significant extensions of this method, which are as follows:

(i) an extension of the class of auxiliary functions suitable for the studying practical stability of motion;
(ii) elimination of the property of having a fixed sign of the total derivative of an auxiliary function along with solutions of the system under investigation;
(iii) establishing a relationship between the quantitative values of the auxiliary function in given (finite) domains of the phase space and decrement (increment) of this function, along with solutions of the system under investigation.

The generalized results on practical stability of non-linear systems are found in monographs [V, X]. The criteria of practical stability presented in [X] was obtained with V. Lakshmikantham and S. Leela and include discrete and impulsive systems, systems of integral differential and functional-differential equations, reaction-diffusion equations, controlled systems, and systems with multi-valued right-hand sides.

Major recent developments in this direction are summarized in monographs [XVIII] published in Chinese. For the application of the obtained results in the dynamics of wheeled transport vehicles and rocket dynamics, the reader is referred to monographs [IV] and [XXIII].

2.3 Applications of the integral inequalities in the stability theory

It is rather difficult to study the behavior of solutions, with unbounded jumps of systems of the type

$$\frac{dx}{dt} = f(t,x) + g(t,x), \qquad x(t_0) = x_0, \tag{2.4}$$

where $x \in R^n$, $f \in C(R_+ \times R^n, R^n)$, $g(t,x) \in C(R_+ \times R^n, R^n)$. In system (2.4), the vector function $g(t,x)$ can characterize the terms of higher order of smallness compared to the function $f(t,x)$ or to persistent perturbations. Boundedness, asymptotic behavior, oscillations and stability of solutions to system (2.4) are of great interest.

A utilization of integral inequalities is at the heart of a fundamental approach when analyzing the above mentioned properties of solutions to system (2.4) and its particular cases.

Furthermore, an application of integral inequalities for a rough estimation of the qualitative behavior of solutions to linear and nonlinear systems of differential equations represents an essential part in the theory of motion stability.

In the papers [46, 58, 63], the author applied integral inequalities to problems of qualitative analysis of motion in the theory of motion stability. The idea leads to

(i) the utilization of known techniques and the development of new ones in order to reduce system (2.4) to a form suitable for application of integral inequalities;

(ii) developing the method of estimating the nonlinear terms in system (2.4) corresponding to the structure of the employed integral inequalities;

(iii) the investigation of general properties of systems with lumped and distributed parameters, such as boundedness, continuous dependence on the initial values and parameters, stability via Liapunov and Lagrange and stability under persistent perturbation, as well as nonclassical problems of stability theory.

Nonlinear systems of (2.4) type are investigated in [III] under various assumptions imposed on dynamical properties of solutions of nonlinear (linear) approximation to system (2.4).

For further progress of integral inequalities techniques in qualitative analysis of solutions to nonlinear systems of differential equations the reader is referred to monograph [IX], while some applications are found in monograph [XIV].

2.4 Comparison technique and averaging method in nonlinear dynamics

Difficulties in analyzing nonlinear systems (2.2) or (2.4) under their high dimensions stipulate a new method of qualitative analysis referred to as the comparison method. As it is known, this method is based on the construction of the comparison equation (system)

$$\frac{du}{dt} = G(t,u), \quad u(t_0) = u_0 \geq 0, \tag{2.5}$$

where $u \in R^m_+$, $G \in C(R_+ \times R_+, R^m)$, $G(t,0) = 0$ for all $t \geq t_0$ whose maximal $u^+(t;t_0,u_0)$ (minimal $u^-(t;t_0,u_0)$) solution is correlated with the solution $x(t;t_0,x_0)$ of system (2.2) as

$$\begin{aligned} Q(t, x(t;t_0,x_0)) &\leq u^+(t;t_0,u_0), \\ Q(t, x(t;t_0,x_0)) &\geq u^-(t;t_0,u_0), \end{aligned} \tag{2.6}$$

where $Q \in C(R_+ \times R^n, R_+^m)$, $Q(t,0) = 0$ for all $t \in R_+$.

In many fundamental works, researchers suggested constructing comparison system (2.5) and comparison functions $Q(t,\cdot)$, which allow one to analyze stability of the state $x = 0$ of system (2.2) in terms of the solution $u = 0$ to the comparison system.

In Martynyuk's monograph [II], the development of the comparison technique is associated with the analysis of systems of the type

$$\frac{dx_s}{dt} = f_s(t, x_s) + g_s(t, x_1, \dots, x_m), \qquad s = 1, 2, \dots, m, \tag{2.7}$$

where $x_s \in R^{n_s}$, $f_s \in C(R_+ \times R^{n_s}, R^{n_s})$, $g_s \in C(R_+ \times R^{n_1} \times \dots \times R^{n_m}, R^{n_s})$, $f_s(t,0) = 0$, $g(t, 0, \dots, 0) = 0$. Here the comparison technique is based on the integral inequalities

$$\varphi_s(t) \le \psi_s(t) + \int\limits_{t_0}^{t} \sigma_s(\tau, \varphi_1(\tau), \dots, \varphi_m(\tau))\, d\tau, \quad s = 1, 2, \dots, m, \tag{2.8}$$

and the comparison system

$$\begin{gathered} \frac{du_s}{dt} = \sigma_s(t, \psi_1(t) + u_1, \dots, \psi_m(t) + u_m), \\ s = 1, 2, \dots, m, \end{gathered} \tag{2.9}$$

where $\sigma(t, \psi(t) + y)$ is continuous on the open domain $D = \{(t, y) \colon a < t < b,\ y \in R^m\}$ and satisfies the condition of quasimonotonicity.

For inequalities (2.8) and some additional conditions, the estimates

$$\varphi_s(t) \le \psi_s(t) + u_s^+(t, t_0, u_{0s}), \quad s = 1, 2, \dots, m, \tag{2.10}$$

are valid, where $u_s^+(t, t_0, u_{0s})$ is the maximal solution of system (2.9).

Using this type of the comparison technique, the problems on technical stability with respect to separate coordinates and technical stability of multidimensional system were solved in [27]. The applications of this type of the comparison technique to various problems of qualitative analysis of solutions to nonlinear equations are found in the papers [35–37, 50, 55, 57]. In particular, Martynyuk, in his monograph [XIV], studied many problems concerning the qualitative behavior of solutions to equations in the standard form, systems with quick and slow variables, systems with small persistent perturbations, and singularly-perturbed systems. For other results in the direction see [41, 43, 49, 52–54, 61, 62, 66, 74, 78, 81, 95, 93, 99, 100].

2.5 Stability analysis of large-scale dynamical systems

Stability of large-scale system of (2.2) type or more general systems modeled by equations in a Banach space, has been discussed in many well-known monographs. An application of vector Liapunov functions or vector norms leads to comparison systems of (3.1) type or other types with the common property of the right-hand side being quasimonotone.

This way, we arrive at a stability problem of a quasimonotone system in a cone. The other important stability problem of large scale systems was an efficient account of the influence of small interconnections between the subsystems in the case when the subsystems are not asymptotically stable.

In the papers [32, 33], for the large scale system

$$\frac{dx_s}{dt} = f_s(t, x_s) + \mu g_s(t, x_1, \ldots, x_m), \qquad s = 1, 2, \ldots, m, \tag{2.11}$$

it was proposed to apply the vector function

$$V(t, x) = (v_1(t, x_1), \ldots, v_m(t, x_m))^{\mathrm{T}},$$

whose components $v_s \in C(R_+ \times R^{n_s}, R_+)$ are constructed for the independent subsystems

$$\frac{dx_s}{dt} = f_s(t, x_s), \qquad x_s(t_0) = x_{s0}, \tag{2.12}$$

of system (2.11). The functions

$$\psi_s(t) = \int_{t_0}^{t} (\nabla v_s(t, x_s))^{\mathrm{T}} g_s(s, \overline{x}_1(s), \ldots, \overline{x}_m(s))\, ds, \qquad s = 1, 2, \ldots, m, \tag{2.13}$$

yield the solutions $\overline{x}_1(s), \ldots, \overline{x}_m(s)$ of subsystems (2.12) and they estimate the influence of interconnections $g_s(t, x_1, \ldots, x_m)$ on the dynamics of whole system (2.11).

Comparison system (2.9), with functions (2.13), lead to estimates (2.10) to be obtained. The latter are a source of various sufficient stability conditions for system of (2.11) type.

In monographs [XVII, XIX], Martynyuk finds estimates (2.8) – (2.12) and develops stability theory of large-scale systems (3.3) under various assumptions on the dynamical properties of subsystems (2.12) and he establishes interconnection functions between them.

In monographs [VI, VII], he developed new aggregation forms for large-scale systems under nonclassical structural perturbations. See also many results in [77, 80, 82, 85, 87, 110–113, 122, 128].

2.6 Limiting equations and stability theory

The Poincaré and Liapunov ideas on qualitative solutions to the systems of differential equations with no direct integration, combined with abstract theory of dynamical systems, gave rise to a new direction in the theory of equations, which is based on the notion of limiting equation (system).

Stability or other steady state dynamical properties of system (2.2) are associated with the limiting behavior of solutions as $t \to \infty$ and, therefore, are determined by the limiting characteristics of (2.2) for $t \to \infty$.

It appeared to be fruitful to consider the translations

$$\frac{dx}{dt} = f^{\tau}(x, t) \tag{2.15}$$

of equation (2.2), where $f^\tau(x,t) = f(x, t+\tau)$. These results in a family of equations (2.15) are parametrized by the shift τ. The translation convergence when $t \to +\infty$ under some topology, enables one to study the asymptotic behavior of solutions to initial system (2.2). The equation obtained as a result of this convergence is referred to as the limiting equation.

Monographs [XI, XV] generalize in this direction the theory of motion stability. The authors treat a stability problem of nonautonomous systems modelled by ordinary differential equations, integral equations, equations with infinite delay, systems with small forces, integro-differential systems, abstract compact and uniform dynamical processes, dynamical processes on the space of convergence, asymptotically autonomous evolutionary equations of parabolic and hyperbolic type in Banach spaces, etc. Moreover, the method of limiting equations is applied here to investigate large-scale systems with weakly interacting subsystems. Besides, both stability and instability of large-scale systems are studied. The topics include stability with respect to a subset of variables. See also [115, 119].

2.7 The Liapunov's matrix-valued functions method

At the end of the 1970s, Martynyuk began research in the field of matrix-valued Liapunov functions. He proposed an approach to problems of stability based on the two-index system of functions

$$U(t,x) = [u_{ij}(t,x)], \quad i,\, j = 1, 2, \dots, m, \tag{2.16}$$

where $u_{ii} \in C(R_+ \times R^n, R_+)$ and $u_{ij} \in C(R_+ \times R^n, R)$ for all $i \neq j$, which is suitable for the construction of Liapunov functions.

Both the scalar function

$$v(t,x,\eta) = \eta^{\mathrm{T}} U(t,x)\eta, \quad \eta \in R^m, \tag{2.17}$$

and the vector function

$$V(t,x,w) = AU(t,x)w, \quad w \in R^m, \tag{2.18}$$

with A being a constant matrix $m \times m$, can be constructed in terms of matrix-valued function (2.16). The function (2.16) together with (2.17) and (2.18) were put by Martynyuk in the basis of the direct Liapunov method and comparison principle with matrix-valued function (see [64, 71–73, 79, 83, 94–97] and the monographs [XIV, XVII, XIX, XX, XXIV]).

The application of function (2.16) in the direct Liapunov method is beneficial in studying stability of large-scale systems (3.3), with no use of the comparison systems of (3.1) or (3.5) type. This enables one to bypass the quasimonotonicity condition when studying stability of large-scale systems, and as a by-product, it preserves the vector function

$$V(t,x) = \operatorname{diag}\,[u_{11}(t,x), \dots, u_{mm}(t,x)], \tag{2.19}$$

which is the principle diagonal of matrix-valued function (2.16).

The non-diagonal elements $u_{ij}(t, x_i, x_j)$ are constructed for all $(i \neq j) \in [1, m]$ in light of the interconnection functions $g_s(t, x_1, \dots, x_m)$ acting between the subsystems.

In nutshell, the development of the Liapunov matrix functions method rendered by Martynyuk is as follows:

* the discovery of a two-index system of functions as a structure suitable for construction of Liapunov functions;
* the introduction of the formalism of matrix Liapunov functions with the property of having fixed sign of the matrix-valued functions and their derivatives by virtue of the system of motion equations;
* the formulation of the invariance principle in terms of the matrix-valued functions and stability of solutions to the autonomous systems;
* the analytical construction of the matrix and hierarchical matrix-valued Liapunov functions.

As a result of the development of these powerful techniques, Martynyuk and his students established a new efficient stability condition for some classes of systems of equations. Namely,

(a) systems with lumped parameters;
(b) singularly perturbed systems including Lur'e–Postnikov systems;
(c) system with random parameters including singularly perturbed stochastic systems;
(d) impulsive systems;
(e) large-scale discrete systems;
(f) hybrid systems;
(g) large-scale power systems modelled by ODE;
(h) uncertain systems;
(i) systems with delay;
(j) systems in Banach and metric spaces;
(k) systems modelling the population dynamics (generalization of Kolmogorov model);
(l) classes (a), (b), (d) and (e) under nonclassical structural perturbations.

Recently, Martynyuk developed the method of matrix Liapunov functions for the investigation of polystability of motion, stability with respect to two measures, stability analysis of discontinuous systems, and polydynamics of nonlinear system on time scales (see [78, 84, 88-90, 101–106, 108, 118, 123–129].

2.8 Analysis of mathematical models in biology

The work of Martynyuk in this direction deals with the analysis of qualitative properties of solutions to the Lotka–Volterra system of equations and its generalizations in the form of the Kolmogorov system of equations

$$\begin{aligned} \frac{dx_i}{dt} &= \beta_i(x_i)F_i(t, x_1, \dots, x_n, \mu), \\ x_i(t_0) &= x_0 \geq 0, \quad i = 1, 2, \dots, n. \end{aligned} \tag{2.20}$$

Here β_i are the functions infinitely many times differentiable on $R_+ = [0, \infty)$, $\beta_i(0) = 0$, $\beta_i'(x_i) > 0$ for $x_i > 0$, $\beta_i^j(x_i) \geq 0$, $j = 2, 3, \dots$ and $F_i \in C(R_+ \times R_+^n \times M^k, R)$, where $M^k = [0, 1] \times \cdots \times [0, 1]$, $i = 1, 2, \dots, n$. System (2.20) is a multiplicatively and additively perturbed Kolmogorov system of equations that models the population dynamics.

Also, Martynyuk established the boundedness conditions for the population growth with respect to two measures, as well as the stability conditions for the population quantity. See also [110, 116].

3 Organizing and Scholarly Activity

Alongside the intensive scientific research, Martynyuk carries out great organizing and scholarly activity. He initiated the publication of “The Lectures on Theoretical Mechanics” of A.M.Liapunov in 1982. He also performs a considerable work as an Editor of the International Series of Scientific Monographs “Stability and Control: Theory, Methods, and Applications” at the Taylor and Francis Publishers (Great Britain). Since 1992, they published 22 volumes in this Series which have gained a world-wide recognition. He is founder of a new International journal “Nonlinear Dynamics and Systems Theory” since 2001, published in English.

Martynyuk serves on editorial boards of six international academic journals: *International Applied Mechanics*, *Elektronnoe Modelirovanie*, and *Nonlinear Oscillations* published in Russian and *Journal of Applied Mathematics and Stochastic Analysis* (USA), *Differential Equations and Dynamical Systems* (India), and *International Journal of Innovative, Computing & Control* (Japan) published in English.

He supervised 21 doctoral and 2 habilitation theses in physical and mathematical sciences. All of his former students are presently employed in different countries of the former Soviet Union.

4 Sports and Hobbies

Besides doing an incredible amount of scientific work, Martynyuk takes time to enjoy being with his family, his children and his grandchildren. On weekends he leaves his work for a cycle ride or walking about the forest suburbs of Kiev. Contacts with a wildlife is a source of delight and inspiration for him. Home library of Martynyuk contains about 2000 volumes of scientific literature, fiction and poetry. The books on history, philosophy, natural sciences and art are of his particular interest. He also collects postage-stamps and a series of periodicals “The Great Painters”.

After the Chernobyl nuclear accident in 1986 Martynyuk actively opposed to unfounded decision of Political Bureau of Communist Party on construction of 28 nuclear power blocks in Ukraine. His article “A warning to the careless mankind” (see “Vecherniy Kiev”, No. 273, November 29, 1989) produced a perceptible effect on the scientific and public society.

Professor Martynyuk and his apprentices proceed with the investigations in chosen areas of applied mathematics and mechanics providing the world science with new interesting results.

The American Biographical Institute recognized Martynyuk as an “Outstanding Man of the 20th Century” and awarded him the “2000 Millennium Medal of Honor”.

List of Monographs and Books by A.A. Martynyuk

I. *Technical Stability in Dynamics.* Tekhnika, Kiev, 1973. [Russian]
II. *Motion Stability of Composite Systems.* Naukova Dumka, Kiev, 1975. [Russian]
III. *Integral Inequalities and Stability of Motion.* Naukova Dumka, Kiev, 1979. (with R. Gutowski). [Russian]
IV. *Dynamics and Motion Stability of Wheeled Transporting Vehicles.* Tekhnika, Kiev, 1981. (with L.G. Lobas and N.V. Nikitina). [Russian]

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X. *Practical Stability of Nonlinear Systems.* World Scientific, Singapore, 1990. (with V. Lakshmikantham and S. Leela).
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XIX. *Qualitative Methods in Nonlinear Dynamics: Novel Approaches to Liapunov's Matrix Function.* Marcel Dekker, New York, 2002.
XX. *Stability and Stabilization of Nonlinear Systems with Random Structures.* Taylor & Francis, London and New York, 2002. (with I.Ya. Kats).
XXI. *Advances in Stability Theory at the End of the 20th Century.* Taylor & Francis, London and New York, 2003. (Ed.: A.A. Martynyuk).
XXII. *Theory of Practical Stability with Applications.* Second Edition, Revised and Expanded. Chinese Academy of Sciences Publishing Company, Beijing, 2003. (with Sun Zhen qi). [Chinese]
XXIII *Qualitative Analysis of Nonlinear Systems with Small Parameter.* Chinese Academy of Sciences Publishing Company, Beijing, 2006 (with Sun Zhen qi). [Chinese]
XXIV *Stability of Motion: The Role of Multicomponent Liapunov's Functions*, Cambridge Scientific Publishers, London, 2006.

List of Personal Papers by A.A. Martynyuk*

1. To the stability of transient motion on a given interval of time. *Prikl. Mekh.* **3**(5) (1967) 121–125. [Russian]
2. Statistical estimate of stability probability of motion on a given interval of time. *Dokl. Akad. Nauk USSR, Series A*, No.5, (1967) 443–445. [Russian]
3. On the stability in finite interval of systems with delay. *Dokl. Akad. Nauk USSR, Series A.*, No.8, (1969) 165–167. [Russian]

* *This list is prepared and checked by L.N. Chernetskaja and Yu.A. Martynyuk-Chernienko via Zentralblatt MATH CD-ROM.*

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