



## Numerical Search of Bounded Relative Satellite Motion

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**Abstract:** Relative motion between two or more satellites has been studied for a long time, as the works of W.H. Clohessy and R.S. Wiltshire, dated 1960, or the studies of J. Tschauner, dated 1967, can testify. Not only these early works are milestones for the relative motion modelling, as they provide linear models whose accuracy in terms of motion prediction is granted in the simplified assumption of pure Keplerian motion, but they are also powerful tools to gain insight into the complex dynamical properties of this type of motion. These models supply conditions on the initial relative position and velocity that allow the relative orbits to be periodic, that is closed orbits. When perturbations, such as Earth oblateness and air drag effects, or even the simple nonlinearities of the keplerian gravitational attraction are taken into account in the model, an analytical solution appears more and more complicated to be derived, if not impossible. Simple relations on the initial conditions leading to periodic orbits, such as those that are well known when considering Hill-Clohessy-Wiltshire (HCW) equations, are not to be expected without introducing some simplifications. In these cases a numerical approach could still be able to locate the exact conditions that result in a minimum drift per orbit. This work investigates the possibility of using a global optimization technique to locate the initial conditions resulting into minimal drift per orbit. Before using this approach in the nonlinear problem, the methodology is tested on Hill's and Tschauner-Hempel's models, where an analytical solution is well known. The global optimizer is essentially a genetic algorithm that considers the initial relative velocities between the satellites as the chromosomes of the individuals of the population, the initial relative position is considered as given. This not only reduces the number of variables the GA has to optimize, but it also allows to search for closed relative orbits of a predefined dimension. Results show that the methodology is returning the analytical results with a satisfactorily precision and that is able to locate bounded motion also when nonlinearities become important.

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