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A Simple Nonlinear Adaptive-Fuzzy Passivity-Based Control of Power Systems

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Abstract: A new intelligent nonlinear control for power system stabilizers that improves the transient stability is proposed. To guarantee high performance with low complexity cost, new concepts on the passivity design under unknown disturbance inputs, as well as on the adaptive fuzzy logic rule extraction are introduced. This permits the most possible simple design implementation of an adaptive-fuzzy logic passivity-based controller which is developed on an equivalent model of the system obtained by a suitable use of the backstepping technique. The overall scheme is decentralized providing local output feedback controllers, supported by a very simple adaptive-fuzzy scheme of only three rules. A detailed analysis proves that the proposed control scheme ensures uniform ultimate boundedness of all the error variables in an arbitrarily small region around the origin. Extensive simulations on a two machine infinite bus power system on which a permanent serious fault occurs, confirm the theoretical results and verify an excellent system performance.

Keywords: *Adaptive control; fuzzy logic control; passivity; power system control.*

Mathematics Subject Classification (2000): 93D05, 93C42.

1 Introduction

Advanced intelligent control designs are increasingly used in high technology applications to solve practical problems in nonlinear systems. Among others, a characteristic example of a highly nonlinear system is the power system where these techniques are recently applied. Particularly, power systems are nonlinear, large scale, distributed systems that include a number of synchronous machines as producers. One of the main goals of the excitation control of each machine is the enhancement of power system stability especially

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after faults such as short-circuits or significant power disturbances. To this end, power system stabilizers (PSS) are widely used as supplementary excitation control devices.

Last decade nonlinear control theory has been extensively used to account for the nonlinearities of the controlled power systems. Early designs are based on the feedback linearization technique [1]. Alternatively, the sliding-mode control technique has been applied on power systems providing rather simple control schemes [2, 3]. Nonlinear control techniques have been crucially enhanced by using robust control designs such as H_∞ control and L_2 disturbance attenuation [4, 5, 6, 7]. In recent years new approaches have been proposed for power stability designs based on advanced nonlinear schemes such as adaptive control [8, 9, 10], neuro-control [11] and fuzzy logic [12].

Fuzzy logic designs have been employed as promising controllers, since they provide a convenient method in nonlinear design via the use of qualitative rules characterizing the power system performance. However, due to different operating conditions a large rule base is needed to ensure an acceptable performance. In order to obtain a better performance, as compared to the standard design, an adaptive fuzzy logic stabilizer has been proposed [13, 14]. Although this on-line adaptation mechanism overcomes many of the drawbacks, the whole control scheme of each machine cannot be considered as a simple one; at least 9 rules for the two-input single-output fuzzy system are required while a 9-order adaptive system is needed [14].

In this paper, a new approach to the design of decentralized adaptive fuzzy excitation control is proposed that acts in coordination with the automatic voltage regulator. The design is based on the nonlinear third-order model of each machine [5]. On this model a suitable backstepping technique is applied that modifies the original n-machine system into n separate systems that are interconnected through highly nonlinear links. Each of these systems is a single-input single-output (SISO) minimum phase system with relative degree one. However, due to the highly nonlinear interconnections, it is shown that the system can be passive by output feedback anywhere in $\mathbf{R}^n$ except for a compact region Ω containing the origin. To describe this property the concept of Ω^- -passivity is introduced. As a result a simple standard passivity-based output feedback control design with negative gain [15, 16] can be applied that provides uniform ultimate boundedness (UUB). The size of Ω depends on the unknown nonlinear interconnections. In order to avoid high gains and large regions Ω, a very simple SISO adaptive fuzzy logic scheme is included to approximate these nonlinearities. As shown in the paper, the qualitative principle that holds for the fuzzy logic rule extraction has a SISO linguistic form with input, a fixed linear combination of the power angle deviation, the nominal frequency deviation and the accelerating power. Including additionally an adaptation mechanism that provides on-line the fuzzy logic output parameters, i.e. the centers of gravity of the membership functions, a very simple rule base of only three IF-THEN SISO statements accompanied by a 3-order adaptation scheme is proposed.

Hence, by the proposed scheme, a completely decentralized excitation control is achieved. Furthermore, exploiting some inherent structure properties of power systems, a passivity-based control scheme is developed that in turn is combined with advanced control techniques in a manner that results in an extremely simple form. Extensive stability analysis proves that the system becomes UUB while the estimated parameter errors remain bounded. The region around the origin inside which the variables converge can be arbitrarily small by suitably tuning the passivity control gain and the design parameters. Finally, the effectiveness of the proposed controller is successfully verified by simulation tests on a two machine infinite bus power system.

2 Preliminaries and system formulation

Before proceeding with our approach we give some definitions assuming that the concepts of relative degree and normal form of a dynamical system are familiar to the reader (see [15] and the references therein for details).

Definition 2.1 *The zero dynamics of a dynamical system with output y, represent those internal dynamics which are consistent with the constraint that the output is identically equal to zero. If the zero dynamics of a dynamical system are asymptotically stable, then this system is called a minimum-phase system.*

Definition 2.2 *A dynamical system with state vector $x \in \mathbf{R}^n$, input $u \in \mathbf{R}^m$ and output $y \in \mathbf{R}^m$ is said to be passive if there exists a positive definite radially unbounded storage function $V(x)$ and a positive definite function $S(x)$ such that for all $u \in U$ where U is the set of all admissible inputs holds true that*

$$V(x(t)) - V(x(0)) = \int_0^t y^T(s)u(s)ds - \int_0^t S(x(s))ds \quad \textit{for all } t \geq 0 \textit{ and } x \in \mathbf{R}^n.$$

Obviously from Definition 2.2, the following proposition can be made.

Proposition 2.1 *A dynamical system with state vector $x \in \mathbf{R}^n$, input $u \in \mathbf{R}^m$ and output $y \in \mathbf{R}^m$ has the passivity property if there exists a positive definite radially unbounded function $V(x)$ and a positive scalar $c > 0$ such that*

$$\dot{V} < -cV + y^T u \quad \forall x \in \mathbf{R}^n.$$

2.1 Ω^--passivity

At this point, the following definition is introduced.

Definition 2.3 *A dynamical system with state vector $x \in \mathbf{R}^n$, input $u \in \mathbf{R}^m$ and output $y \in \mathbf{R}^m$ is said to be Ω^--passive (read:* Omega minus passive*) if there exists a positive definite radially unbounded storage function $V(x)$ and a positive definite function $S(x)$ such that for all $u \in U$ where U is the set of all admissible inputs, it holds true that*

$$V(x(t)) - V(x(0)) = \int_0^t y^T(s)u(s)ds - \int_0^t S(x(s))ds$$
$$\textit{whenever } x(\tau) \in \mathbf{R}^n \backslash \Omega, \forall\, \tau \in [0, t],$$

where Ω is a compact set, $\Omega \subset \mathbf{R}^n$, containing the origin.

Obviously from Definition 2.3, the following proposition (analogous to Proposition 2.1) is given.

Proposition 2.2 *A dynamical system with state vector $x \in \mathbf{R}^n$, input $u \in \mathbf{R}^m$ and output $y \in \mathbf{R}^m$ has the Ω^--passivity property if there exists a positive definite radially unbounded function $V(x)$ and a positive scalar $c > 0$ such that*

$$\dot{V} < -cV + y^T u \quad \forall x \in \mathbf{R}^n \backslash \Omega,$$

where Ω is a compact set, $\Omega \subset \mathbf{R}^n$, containing the origin.

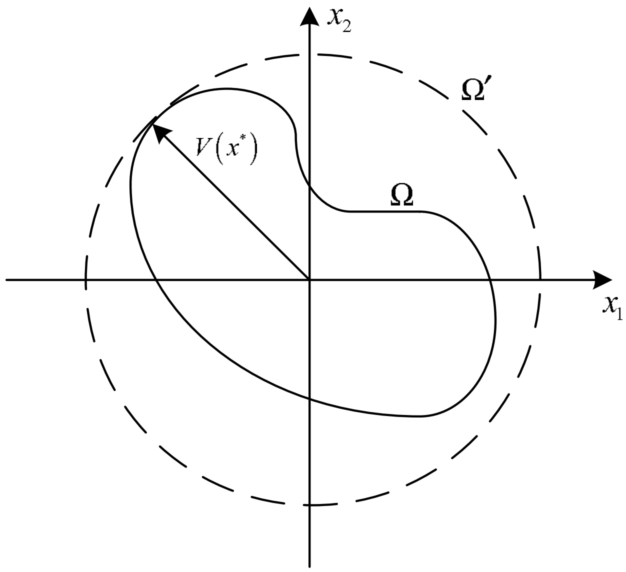

Figure 2.1: The regions Ω and Ω' for a Lyapunov function $V(x_1, x_2) = x_1^2 + x_2^2$.

From Definition 2.2 it is deduced that any passive unforced system ($u \equiv 0$) is asymptotically stable. On the other hand, one can easily see from Definition 2.3 that any unforced system that is Ω^--passive, ensures finite-time convergence of the state vector inside the region Ω. Particularly, in accordance to Definition 2.3, since S is positive definite, $\frac{\partial S}{\partial x} \neq 0 \quad \forall x \neq 0$ and therefore a local minimum of S does not exist in $\mathbf{R}^n \backslash \Omega$; hence the minimum of S in the closure of $\mathbf{R}^n \backslash \Omega$ is on $\partial\Omega$ where $\partial\Omega$ is the boundary surface of Ω. If we define $S_\ell := \min_{x \in \partial\Omega} S(x) \neq 0$ then the system trajectories starting from the region $\mathbf{R}^n \backslash \Omega$ will insert inside the region Ω in a finite-time less than $T = V_0/S_\ell$, since $0 - V_0 = -\int_0^t S(x(s))ds \leq -S_\ell \cdot T$ with $V_0 = V(x(0))$. Define now the point $x^* := \arg\max_{x \in \partial\Omega} V(x)$ and the compact set $\Omega' := \{x \in \mathbf{R}^n | V(x) \leq V(x^*)\}$. It is straightforward that the state trajectories remain in Ω' for all $t \geq T$ (obviously it is $\Omega \subset \Omega'$, see Figure 2.1).

The stability analysis based on the concept of Ω^--passivity generalizes the results of [17, 18] on quasi-dissipative systems and constitutes an effective tool in this field.

2.2 System Model

Now, we are ready to proceed with the system model. In the model used, the multimachine power system is reduced into a network with generator nodes only. For the design of the excitation controller the classical third-order single-axis dynamic generator model is used whereas differential equations that represent dynamics with very short time constants have been neglected. In general, for a n-generator power system, the dynamic model of the i-th generator is

$$\dot{\delta}_i(t) = \omega_i(t) - \omega_0, \tag{1}$$

$$\dot{\omega}_i(t) = -\frac{D_i}{M_i}(\omega_i(t) - \omega_0) + \frac{\omega_0}{M_i}(P_{mi} - P_{ei}(t)), \tag{2}$$

$$\dot{E}'_{qi}(t) = \frac{1}{T'_{d0i}}(E_{fi}(t) - E_{qi}(t)), \tag{3}$$

where

$$E_{qi}(t) = E'_{qi}(t) + (x_{di} - x'_{di})I_{di}(t), \tag{4}$$

$$E_{fi}(t) = k_{ci}u_{fi}(t), \tag{5}$$

$$I_{qi}(t) = \sum_{j=1}^{n} E'_{qj}\Big(B_{ij}\sin\delta_{ij}(t) + G_{ij}\cos\delta_{ij}(t)\Big), \tag{6}$$

$$I_{di}(t) = \sum_{j=1}^{n} E'_{qj}\Big(G_{ij}\sin\delta_{ij}(t) - B_{ij}\cos\delta_{ij}(t)\Big), \tag{7}$$

$$P_{ei}(t) = E'_{qi}(t)I_{qi}(t), \tag{8}$$

$$Q_{ei}(t) = E'_{qi}(t)I_{di}(t), \tag{9}$$

$$E_{qi}(t) = x_{adi}I_{fi}(t), \tag{10}$$

$$V_{tqi}(t) = E'_{qi}(t) - x'_{di}I_{di}(t), \tag{11}$$

$$V_{tdi}(t) = x'_{di}I_{qi}(t), \tag{12}$$

$$V_{ti}(t) = \sqrt{V^2_{tqi}(t) + V^2_{tdi}(t)}. \tag{13}$$

Applying the backstepping technique used in [9, 10], the following state transformation for the i-th machine is obtained

$$\begin{bmatrix} z_{i1} \\ z_{i2} \\ z_{i3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ c_{i1} & 1 & 0 \\ -\frac{M_i}{\omega_0}(1 + c_{i1}c_{i2}) & -\frac{M_i}{\omega_0}(c_{i1} + c_{i2} - \frac{D_i}{M_i}) & 1 \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \Delta\omega_i \\ \Delta P_{ei} \end{bmatrix}, \tag{14}$$

where $c_{i1} > 0$ and $c_{i2} > 0$.

Defining for each machine the excitation control law $E_{fi} = k_{ci}u_{fi}$ with $k_{ci} = 1$ and

$$u_{fi}(t) = \frac{T'_{d0i}}{I_{qi}}(k_{i1}\Delta\omega_i + k_{i2}\Delta P_{mi} + v_i) \tag{15}$$

with gains given by

$$k_{i1} = \frac{M_i}{\omega_0}[c_{i1}c_{i2} + 1 - \frac{D_i}{M_i}(c_{i1} + c_{i2} - \frac{D_i}{M_i})],$$
$$k_{i2} = c_{i1} + c_{i2} - \frac{D_i}{M_i}, \tag{16}$$

the dynamics of each machine with respect to the new z variables are given by

$$\begin{bmatrix} \dot{z}_{i1} \\ \dot{z}_{i2} \\ \dot{z}_{i3} \end{bmatrix} = \begin{bmatrix} -c_{i1} & 1 & 0 \\ -1 & -c_{i2} & -\frac{\omega_0}{M_i} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{i1} \\ z_{i2} \\ z_{i3} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_i - \begin{bmatrix} 0 \\ 0 \\ f_i \end{bmatrix}, \tag{17}$$

where

$$f_i(t) := \frac{1}{T'_{d0i}}\Big[E'_{qi}(t) + (x_{di} - x'_{di})I_{di}(t)\Big]I_{qi}(t) - E'_{qi}(t)\dot{I}_{qi}(t). \tag{18}$$

Thus, applying (14), the original system is transformed into (17), i.e. a system consisting of a linear part of the form $\dot{z}_i = A_i z_i + B_i v_i$ and a nonlinear term f_i that affects the state equations wherein the input v_i appears. As shown by (18), the nonlinear term cannot be reconstructed from the local i-th machines variables and therefore it can be considered as an unknown input function. Additionally, the unknown input $f_i(t)$ is considered to be bounded (this is always the case since the machine voltages and currents and their rates cannot take infinite values) , i.e.

$$|f_i(t)| \leq F_i < \infty. \tag{19}$$

If one considers the variable z_{i3} as the output of the i-th subsystem, i.e.

$$y_i = Cz_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_{i1} \\ z_{i2} \\ z_{i3} \end{bmatrix} = z_{i3},$$

then it is obvious that the system is minimum phase i.e. it holds true that $z_{i1}, z_{i2} \to 0$ as $t \to \infty$ for $z_{i3} \equiv 0$. Moreover the system has relative degree one since the input appears directly in the first derivative of the output.

In the case where $f_i \equiv 0$, system (17) becomes a purely linear system and in accordance to [15] it can be feedback equivalent to a passive system, since it is minimum phase with relative degree one. However, since in this case relative degree one is equivalent to the nonsingularity of the system high frequency gain CB and furthermore since CB is positive definite then as it has been shown in [15] and [16] an output feedback $v_i = -k_i y_i + v_{fi}$ can be determined with large enough gain $k_i > 0$ that ensures passivity of the closed-loop system with new input v_{fi} in accordance to Definition 2.2. In the case where $f_i \neq 0$ and since (19) holds true, we will prove in Section 4 that also for this case an output feedback

$$v_i = -k_i y_i + v_{fi} \tag{20}$$

can be determined with large enough gain $k_i > 0$ that ensures Ω^--passivity of the closed-loop system in accordance to Definition 2.3, where v_{fi} in (20) is an external input.

3 The proposed control scheme

Incorporating the passivity-based controller (20) into the control scheme given by (15), the excitation input takes a rather simple mathematical form

$$E_{fi}(t) = \frac{T'_{d0i}}{I_{qi}}(K_{i1}\Delta\delta_i + K_{i2}\Delta\omega_i + K_{i3}\Delta P_{mi} + v_{fi}), \tag{21}$$

where the constant gains are now given by

$$\begin{aligned} K_{i1} &= \frac{M_i}{\omega_0} k_i (1 + c_{i1}c_{i2}), \\ K_{i2} &= \frac{M_i}{\omega_0}\left[\left(k_i - \frac{D_i}{M_i}\right)\left(c_{i1} + c_{i2} - \frac{D_i}{M_i}\right) + c_{i1}c_{i2} + 1\right], \\ K_{i3} &= c_{i1} + c_{i2} + k_i - \frac{D_i}{M_i}, \end{aligned} \tag{22}$$

and the v_{fi} is an external input.

This control scheme requires only local measurements of P_{ei}, ω_i, δ_i and of the current I_{qi} that can be calculated from the measurements.

4 Ω^--passivity property of power systems

For the multimachine power system with the model of each machine described by the modified system (17), we consider the nonnegative candidate Lyapunov function

$$V_0 = \sum_{i=1}^{n} V_i \qquad \text{with} \qquad V_i = \frac{1}{2}\sum_{j=1}^{3} z_{ij}^2, \quad i = 1, 2, \dots, n. \tag{23}$$

The time derivative of V_i has the following form

$$\dot{V}_i = -c_{i1}z_{i1}^2 - c_{i2}z_{i2}^2 + z_{i3}\left[v_i(t) - f_i(t) - \frac{\omega_0}{M_i}z_{i2}\right]. \tag{24}$$

Then for the control law (21) we have

$$\dot{V}_i = -c_{i1}z_{i1}^2 - c_{i2}z_{i2}^2 - k_i z_{i3}^2 - \frac{\omega_0}{M_i}z_{i2}z_{i3} + z_{i3}(v_{fi} - f_i).$$

Using the inequality

$$F_i|z_{i3}| \le \rho_{fi}k_i z_{i3}^2 + \frac{F_i^2}{4\rho_{fi}k_i},$$

we arrive at

$$\dot{V}_i \le -c_{i1}z_{i1}^2 - \begin{bmatrix} z_{i2} & z_{i3} \end{bmatrix} \begin{bmatrix} c_{i2} & \frac{\omega_0}{2M_i} \\ \frac{\omega_0}{2M_i} & (1-\rho_{fi})k_i \end{bmatrix} \begin{bmatrix} z_{i2} \\ z_{i3} \end{bmatrix} + z_{i3}v_{fi} + \frac{F_i^2}{4\rho_{fi}k_i}$$

for arbitrary $\rho_{fi} :\ 0 < \rho_{fi} < 1$.

If the positive constants c_{i2}, k_i are selected so that

$$P_i := \begin{bmatrix} c_{i2} & \frac{\omega_0}{2M_i} \\ \frac{\omega_0}{2M_i} & (1-\rho_{fi})k_i \end{bmatrix} > 0,$$

i.e. if

$$c_{i1} > 0\,, \qquad c_{i2}k_i > \frac{1}{1-\rho_{fi}}\left(\frac{\omega_0}{2M_i}\right)^2, \tag{25}$$

we result in

$$\dot{V}_i \le -c_{i1}z_{i1}^2 - \lambda_{\min}(P_i)z_{i2}^2 - \lambda_{\min}(P_i)z_{i3}^2 + z_{i3}v_{fi} + \frac{F_i^2}{4\rho_{fi}k_i}.$$

Defining $m_{fi} := \min\{c_{i1}, \lambda_{\min}(P_i)\}$, $i = 1, \cdots, n$ and $m_f := min_{1\le i\le n}m_{fi}$, it is obvious that

$$\dot{V}_i \le -m_{fi}(z_{i1}^2 + z_{i2}^2 + z_{i3}^2) + z_{i3}v_{fi} + \frac{F_i^2}{4\rho_{fi}k_i}$$

or

$$\dot{V}_i \le -2m_{fi}V_i + z_{i3}v_{fi} + \frac{F_i^2}{4\rho_{fi}k_i}.$$

Let $m_f := min_{1\le i\le n}m_{fi}$, then for V_0 we have that

$$\dot{V}_0 \le -2m_f V_0 + \sum_{i=1}^{n}\left(z_{i3}v_{fi} + \frac{F_i^2}{4\rho_{fi}k_i}\right). \tag{26}$$

For every arbitrary parameter $\epsilon : 0 < \epsilon < 2$ and for $(z_{11}, z_{12}, z_{13}) \times \cdots \times (z_{n1}, z_{n2}, z_{n3}) \in \mathbf{R}^{3n}$ which do not belong to the compact set

$$\Omega_f := \left\{ (z_{11}, z_{12}, z_{13}) \times \cdots \times (z_{n1}, z_{n2}, z_{n3}) : \sum_{i=1}^{n} \sum_{j=1}^{3} z_{ij}^2 \le \sum_{i=1}^{n} \frac{F_i^2}{2(2-\epsilon) m_f \rho_{fi} k_i} \right\} \quad (27)$$

it is immediately deduced that

$$\sum_{i=1}^{n} \frac{F_i^2}{4\rho_{fi} k_i} = (2-\epsilon) \sum_{i=1}^{n} \frac{m_f}{2} \frac{F_i^2}{2(2-\epsilon) m_f \rho_{fi} k_i} \le (2-\epsilon) m_f \left(\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{3} z_{ij}^2 \right) = (2-\epsilon) m_f V_0.$$

Hence, (26) becomes

$$\dot{V}_0 \le -\epsilon\, m_f V_0 + \sum_{i=1}^{n} z_{i3} v_{fi}.$$

Thus, in accordance to Proposition 2.2 we have proven that the closed-loop system is Ω_f^--passive with constant $c = \epsilon\, m_f$, input and output vectors $\begin{bmatrix} v_{f1} & v_{f2} & \cdots & v_{fn} \end{bmatrix}^T$ and $\begin{bmatrix} z_{13} & z_{23} & \cdots & z_{n3} \end{bmatrix}^T$, respectively. Hence, for the unforced system ($v_{fi} \equiv 0\,, i = 1, 2, \cdots, n$), the region Ω_f that defines the Ω_f^--passivity property is identical to the UUB region, i.e. Ω_f^--passivity guarantees UUB in the region Ω_f. As can be easily seen from (27), as $\epsilon \to 0$ the region Ω_f decreases to its inferior limit. Also, as the feedback gain k_i takes larger values for a given F_i the region Ω_f becomes smaller. However, the unknown input f_i may have large values; this consequently may imply a large region Ω_f around the origin in which the states of the system converge making the output feedback controller performance inefficient. To reduce Ω_f, a high-gain controller is needed.

To avoid high-gain controls one can observe the following. The nonlinear term f_i appears in the 3rd equation of (17) that provides the z_{i3}-dynamics, i.e.

$$\dot{z}_{i3} = -k_i z_{i3} + (v_{fi} - f_i). \quad (28)$$

From (28) and (27) one can see that the region Ω_f around the origin can be closer to the origin if v_{fi} can effectively compensate f_i. In order to accommodate this requirement with a simple controller structure, we propose an adaptive fuzzy-logic controller for v_{fi} as it is explained in the following.

5 Adaptive fuzzy-logic controller

A general fuzzy system includes four basic parts. A fuzzifier and a defuzzifier are the interface between the fuzzy system and the crisp system. The rule base is a database of IF THEN statements extracted from qualitative rules characterizing the operation of the system. For each rule, the inference engine maps the input fuzzy set to an output fuzzy set according to the relation defined by the rule. All four parts of a fuzzy logic system (FLS) can be mathematically formulated [19].

By choosing product inference and employing the centre of gravity method for defuzzification, the output of the fuzzy system is written as

$$y = \frac{\sum_{\ell=1}^{M} \theta_\ell \prod_{i=1}^{n} \mu_{F_i^\ell}(x_i)}{\sum_{\ell=1}^{M} \prod_{i=1}^{n} \mu_{F_i^\ell}(x_i)}, \quad (29)$$

where M is the number of rules in the FLS, n is the number of inputs to the FLS, θ_ℓ is the center of gravity of the membership function corresponding to the ℓ-th rule and $\mu_{F_i^\ell}$ is the membership function. Defining the fuzzy basis functions (FBF) $\phi_\ell(x)$ as

$$\phi_\ell(x) = \frac{\prod_{i=1}^{n} \mu_{F_i^\ell}(x_i)}{\sum_{\ell=1}^{M} \prod_{i=1}^{n} \mu_{F_i^\ell}(x_i)}, \tag{30}$$

then equation (29) can be expressed as $y = \sum_{\ell=1}^{M} \theta_\ell \phi_\ell(x) = \theta^T \Phi(x)$ where $\Phi(x) = \begin{bmatrix} \phi_1(x) & \cdots & \phi_\ell(x) & \cdots & \phi_M(x) \end{bmatrix}^T$ is the vector of FBFs and $\theta = \begin{bmatrix} \theta_1 & \cdots & \theta_\ell & \cdots & \theta_M \end{bmatrix}^T$ is the center of gravity vector.
The choice of control law (21) results in the z_{i3}-dynamics given by (28) where v_{fi} is the output of a fuzzy logic controller.

Rule base extraction: From (28) one can immediately see that starting from $v_{fi} = f_i = 0$ then z_{i3} approaches the origin where it remains. Now, in the case where f_i takes a nonzero positive (negative) value and $v_{fi} = 0$, then z_{i3} also takes a nonzero negative (positive) value, i.e. the value of z_{i3} follows the value of $-f_i$. Therefore, in order to compensate the act of the unknown input f_i, so that z_{i3} to approach the origin, a suitable v_{fi} that follows $-z_{i3}$ can effectively compensate the action of f_i. This constitutes the basic qualitative principle for the rule base extraction of the fuzzy logic controller. Hence, a symmetrical fuzzy rule set can be implemented by a SISO fuzzy controller that needs only z_{i3} as an input and v_{fi} as an output, i.e. the linguistic rules can be of the simple form

$$\text{IF } z_{i3} \text{ is } F_i^\ell, \text{ THEN } v_{fi} \text{ is } G_i^\ell,$$

where F_i^ℓ and G_i^ℓ are suitable fuzzy sets selected in such a way that each of the input and output fuzzy variables assign linguistic values varying simultaneously from negative big to positive big values. Each linguistic value is associated with a normalized and symmetrical membership function.

The controller v_{fi} can then be written in accordance to the FBF expansion as

$$v_{fi} = \frac{\sum_{\ell=1}^{M} \theta_\ell^i \mu_{F^\ell}(z_{i3})}{\sum_{\ell=1}^{M} \mu_{F^\ell}(z_{i3})} = \theta_i^T \Phi_i(z_{i3}), \tag{31}$$

where $\Phi_i(z_{i3}) = \begin{bmatrix} \phi_1^i(z_{i3}) & \cdots & \phi_\ell^i(z_{i3}) & \cdots & \phi_M^i(z_{i3}) \end{bmatrix}^T$ is the vector of FBFs and $\theta_i = \begin{bmatrix} \theta_1^i & \cdots & \theta_\ell^i & \cdots & \theta_M^i \end{bmatrix}^T$ the center of gravity vector. The membership functions are generally defined to be Gaussian of the form

$$\mu_F(x_i) = \exp\left[-\left(\frac{x_i - a}{\sigma}\right)^2 \right],$$

where x_i represents z_{i3} or v_{fi} and a is the center and s is the width of the fuzzy set "F". We note that the first and last membership functions are of the sigmoid form:

$$\mu_F(x_i) = \exp\left[1 + exp\left[\pm \left(\frac{x_i - a}{\sigma}\right) \right] \right]^{-1}.$$

In accordance to the previous discussion, v_{fi} is designed to compensate f_i. However, depending on the conditions under which excitation control acts (after faults or large or small power disturbances), f_i and consequently v_{fi} may take values on a widely varying unknown range. Hence, in order to improve the performance of the fuzzy logic controller in such a way that the best possible approximation of f_i to be achieved, an increased number of rules is needed. At this point, we note that a fuzzy system implementation with the smallest rule base is the concern of any efficient design. To this end, we effectively reduce the rule base by updating on-line the parameters of the FLS output. As a consequence, the minimum possible rule base involving only the following three rules is used:

IF z_{i3} is N, THEN v_{fi} is P,

IF z_{i3} is ZE, THEN v_{fi} is ZE,

IF z_{i3} is P, THEN v_{fi} is N.

However, as it can be easily seen from (31), the FLS output parameters are determined through the centers of gravity θ_i of the membership functions, and therefore a suitable adaptation law is used to update on-line these parameters.
The adaptation law is chosen as

$$\dot{\theta}_i = \text{Proj}\{z_{i3}\Gamma_i\Phi_i(z_{i3})\} = z_{i3}\Gamma_i\Phi_i(z_{i3}) - \tau_i z_{i3}\frac{\theta_i\theta_i^T\Gamma i\Phi_i(z_{i3})}{\|\theta_i\|^2}, \tag{32}$$

where

$$\tau_i = \begin{cases} 0\,, & \text{if } \|\theta_i\| < M_\theta \text{ or } (\|\theta_i\| = M_\theta \text{ and } z_{i3}\theta_i^T\Gamma_i\Phi_i(z_{i3}) < 0) \\ 1\,, & \text{if } (\|\theta_i\| = M_\theta \text{ and } z_{i3}\theta_i^T\Gamma_i\Phi_i(z_{i3}) \geq 0) \end{cases} \tag{33}$$

and $\Gamma_i \in \mathbf{R}^{3\times 3}$ is a symmetric positive definite adaptation gain matrix.

This adaptation mechanism is a projection law which is commonly used in Lyapunov stability analysis.

6 Stability analysis

From the previous analysis, it is clear that without the FLS operation a particular region Ω_f is determined for a given f_i and a reasonable gain k_i $(i = 1, 2, \cdots, n\ )$. Taking into account the FLS operation, let θ_i^* be defined so that $v_{fi}^* = \theta_i^{*T}\Phi_i(z_{i3})$ is the optimal approximation of F_i [20], inside the compact subset Ω_f of $\mathbf{R}^{3n}$ (given by (27)) i.e.

$$\theta_i^* := \arg\min_{\theta_i}\left[\sup_{(z_{11},z_{12},z_{13})\times\cdots\times(z_{n1},z_{n2},z_{n3})\in\,\Omega_f} |f_i - \theta_i^T\Phi_i(z_{i3})|\right]. \tag{34}$$

Then there exists a $0 \leq \mu_i < 1$ such that

$$|f_i - \theta_i^{*T}\Phi_i(z_{i3})| \leq \mu_i F_i. \tag{35}$$

Let $\tilde{\theta}_i := \theta_i - \theta_i^*$, then from (31) it is

$$v_{fi} = \tilde{\theta}_i^T\Phi_i(z_{i3}) + \theta_i^{*T}\Phi_i(z_{i3}).$$

Choosing a Lyapunov function candidate as

$$V = V_0 + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i,$$

then for the control law (21), (31) we have

$$\dot{V} = -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} k_i z_{i3}^2 - \sum_{i=1}^{n} \frac{\omega_0}{M_i} z_{i2} z_{i3} + \sum_{i=1}^{n} z_{i3}[f_i - \theta_i^{*T} \Phi_i(z_{i3})] - \sum_{i=1}^{n} \tilde{\theta}_i^T [z_{i3}\Phi_i(z_{i3}) - \Gamma_i^{-1}\dot{\theta}_i]$$

and

$$\dot{V} \le -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} k_i z_{i3}^2 - \sum_{i=1}^{n} \frac{\omega_0}{M_i} z_{i2} z_{i3} + \sum_{i=1}^{n} \mu_i F_i |z_{i3}| - \sum_{i=1}^{n} \tilde{\theta}_i^T [z_{i3}\Phi_i(z_{i3}) - \Gamma_i^{-1}\dot{\theta}_i]. \tag{36}$$

Now, one can see from the above inequality that (32), (33) is a reasonable choice of the update law since it cancels the last term in the right-hand side of (36) when $\|\theta_i\| \le M_\theta$. Moreover, the boundedness of the parameter vectors θ_i is ensured from the projection law, in the sense that if $\theta_i(0) \in \Omega_\theta$ where $\Omega_\theta := \{\theta_i : \|\theta_i\| \le M_\theta\}$ then $\theta_i(t) \in \Omega_\theta$, $\forall\, t \ge 0$ [21]. This means that the parameter errors $\tilde{\theta}_i$ are also bounded i.e.

$$\|\tilde{\theta}_i(t)\| \le \varepsilon_\theta \ , \qquad \varepsilon_\theta = M_\theta + \max_{1 \le i \le n} \|\theta_i^*\|.$$

The basis functions are also bounded i.e. there exists a constant $\bar{\phi}_M$ such that

$$\|\Phi_i(z_{i3})\| \le \bar{\phi}_M.$$

Due to the boundedness of the parameter errors $\tilde{\theta}_i$ we can proceed with the stability analysis by using the non-negative function V_0 instead of V. In this case its derivative is

$$\dot{V}_0 \le -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} k_i z_{i3}^2 - \sum_{i=1}^{n} \frac{\omega_0}{M_i} z_{i2} z_{i3} + \sum_{i=1}^{n} z_{i3}[f_i - \theta_i^T \Phi_i(z_{i3})]$$

and since it holds true that

$$z_{i3}[f_i - {\theta_i}^T \Phi_i(z_{i3})] \le -\tilde{\theta}_i^T \Phi_i(z_{i3}) z_{i3} + \mu_i F_i |z_{i3}|,$$

we equivalently have

$$\dot{V}_0 \le -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} k_i z_{i3}^2 - \sum_{i=1}^{n} \frac{\omega_0}{M_i} z_{i2} z_{i3} + \sum_{i=1}^{n} |z_{i3}| \Big[\|\tilde{\theta}_i\| \|\Phi_i(z_{i3})\| + \mu_i F_i\Big].$$

Using the inequality

$$(\varepsilon_\theta \bar{\phi}_M + \mu_i F_i)|z_{i3}| \le \rho_{fi} k_i z_{i3}^2 + \frac{(\varepsilon_\theta \bar{\phi}_M + \mu_i F_i)^2}{4\rho_{fi} k_i},$$

we arrive at

$$\dot{V}_0 \le -2m_f V_0 + \sum_{i=1}^{n} \frac{(\varepsilon_\theta \bar{\phi}_M + \mu_i F_i)^2}{4\rho_{fi} k_i}. \tag{37}$$

Writing (37) as

$$\dot{V}_0 \leq -2m_f \left[V_0 - \sum_{i=1}^{n} \frac{(\varepsilon_\theta \bar{\phi}_M + \mu_i F_i)^2}{8 m_f \rho_{fi} k_i} \right]$$

and using the comparison principle, [22], we sequentially have

$$V_0(t) - \sum_{i=1}^{n} \frac{(\varepsilon_\theta \bar{\phi}_M + \mu_i F_i)^2}{8 m_f \rho_{fi} k_i} \leq \left[V_0(0) - \sum_{i=1}^{n} \frac{(\varepsilon_\theta \bar{\phi}_M + \mu_i F_i)^2}{8 m_f \rho_{fi} k_i} \right] e^{-2m_f t},$$

$$V_0(t) \leq \sum_{i=1}^{n} \frac{(\varepsilon_\theta \bar{\phi}_M + \mu_i F_i)^2}{8 m_f \rho_{fi} k_i} + V_0(0) e^{-2m_f t}. \quad (38)$$

From (38) one can see that for every $0 < \epsilon < 2$ there exists a $T = T(\epsilon) \geq 0$ such that

$$V_0(t) \leq \sum_{i=1}^{n} \frac{(\varepsilon_\theta \bar{\phi}_M + \mu_i F_i)^2}{4(2-\epsilon) m_f \rho_{fi} k_i} \qquad \forall\, t \geq T,$$

i.e. the state trajectories enter in finite-time in the compact set

$$\Omega_{fc} := \left\{ (z_{11}, z_{12}, z_{13}) \times \cdots \times (z_{n1}, z_{n2}, z_{n3}) : \sum_{i=1}^{n} \sum_{j=1}^{3} z_{ij}^2 \leq \sum_{i=1}^{n} \frac{(\varepsilon_\theta \bar{\phi}_M + \mu_i F_i)^2}{2(2-\epsilon) m_f \rho_{fi} k_i} \right\}, \quad (39)$$

wherein they remain thereafter. Thus, we have proven that the closed-loop system is UUB in the region Ω_{fc}.

As the FLS output approaches the optimal i.e. as $\varepsilon_\theta \to 0$ and $\mu_i \ll 1$, the region Ω_{fc} is significantly reduced with respect to the initial region Ω_f. We have therefore proven that as the FLS operates closer to its optimal, the error variables are UUB in a much smaller region.

7 Case study

A two-generator infinite bus power system is used to demonstrate the efficiency of the proposed controller. The power system is shown in Figure 7.1.
The system parameters are as follows:

$x_{T1} = 0.129$ p.u.,	$x_{T2} = 0.11$ p.u.,	$x_{12} = 0.55$ p.u.,
$x_{13} = 0.53$ p.u.,	$x_{23} = 0.6$ p.u.,	$T'_{d01} = 6.9$ sec,
$x_{d1} = 1.863$ p.u.,	$x'_{d1} = 0.257$ p.u.,	$D_1 = 5.0$ p.u.,
$M_1 = 8.0$ sec,	$M_2 = 10.2$ sec,	$D_2 = 3.0$ p.u.,
$x_{d2} = 2.36$ p.u.,	$x'_{d2} = 0.319$ p.u.,	$T'_{d02} = 7.96$ sec,
$k_{c1} = 1.0$ p.u.,	$k_{c2} = 1.0$ p.u..	

For a more accurate evaluation of the proposed controller, we take into account in the simulation the physical limits of the excitation voltage which are:

$$|k_{c1} u_{f1}| \leq 5.0\, p.u. \qquad |k_{c2} u_{f2}| \leq 5.0\, p.u.$$

A symmetrical three phase short circuit fault occurs on one of the two transmission lines

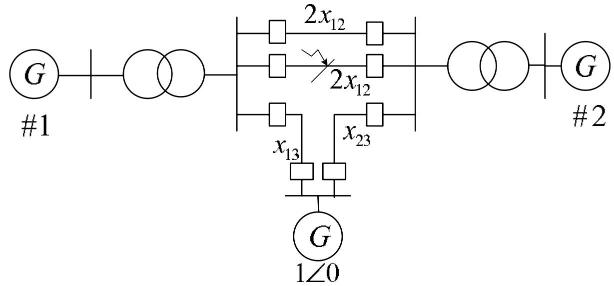

Figure 7.1: The two machine infinite bus test system.

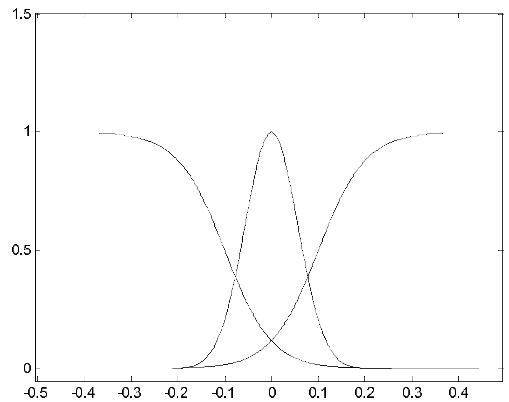

Figure 7.2: The input membership functions of FLS.

between Generator # 1 and Generator # 2 at $t = 20.1$ second. The fault is removed by opening the brakers of the faulted line at $t = 20.5$ second and the system is restored at $t = 21.5$ seconds. If we use λ to represent the fraction of the fault, simulations are made for $\lambda = 0.6$ i.e. for a fault near the middle of the line and towards Generator # 2. The operating point considered in the simulation is:

$$\delta_{10} = 40^o, \quad V_{t10} = 0.93, \quad P_{m10} = 0.95;$$
$$\delta_{20} = 35^o, \quad V_{t20} = 0.937, \quad P_{m20} = 0.8.$$

Most common used power system stabilizers are of fixed parameter lead-lag compensation type designed using linear control techniques. However, since power systems are extremely nonlinear and among the PSS tasks is to damp low frequency oscillations and to improve dynamic performance in a wide range of operating conditions, linear control schemes may be inefficient; this is clear especially in cases of large disturbances such as transmission line faults. Therefore, in order to better evaluate the performance of the proposed controller the case of a permanent serious fault is examined, since this can be considered as the worst case for the power system. The parameter *lambda* of the fault position is taken close to the center of the transmission line between generators #1 and #2, in order to create a balanced impact of the abnormal conditions on both the generators. Obviously, as λ becomes smaller the impact is larger for generator #1 while the opposite occurs as λ becomes larger.

Using the proposed method, the controllers parameters are selected as follows: As shown by the state transformation (14) c_{i1} and c_{i2} determine the coupling of the states of the equivalent system. Hence, suitable positive values that determine a reasonable coupling must be used. In this case we select $c_{11} = c_{21} = 3$, $c_{12} = c_{22} = 5$. For this parameter selection, the following gains $k_1 = k_2 = 100$ are selected that satisfy stability requirement (25) and avoid high-gain performance. Also the adaptation gains are $\Gamma_1 = \Gamma_2 = \mathrm{diag}\{10, 40, 10\}$. Figure 7.2 shows the input membership functions used in FLS.

The response of the system is shown in Figures 7.3 – 7.6. One can clearly see that the system maintains stability after the fault. Additionally, the excitation control input of the proposed Passivity-based Adaptive Fuzzy (PAF) controller effectively penalizes the angle and speed deviations to relatively limited values. As clearly shown in Figures 7.3 and 7.4, a significantly improved dynamic performance of the angle and speed deviations is achieved by the proposed method compared to the performance obtained by a conventional simple linear PSS controller with form and parameters taken from [23]. Finally, the adaptation mechanism suitably adjusts the FLS center of gravity parameters.

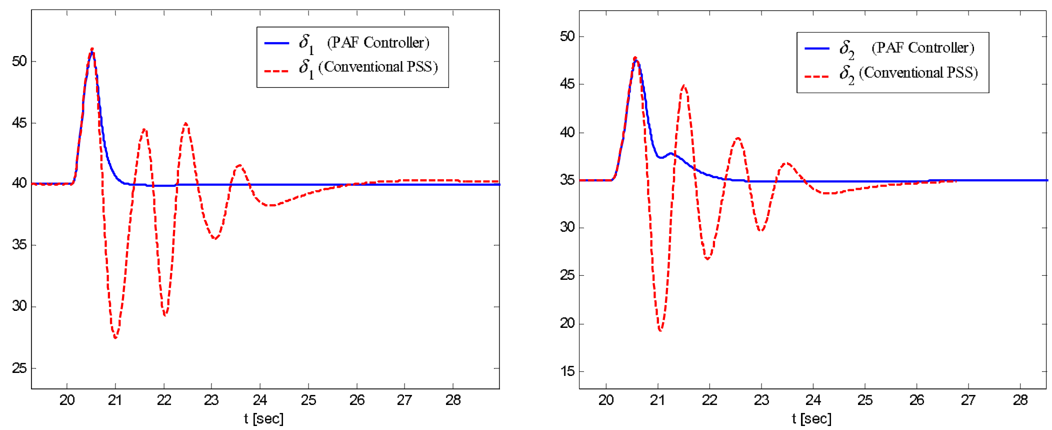

Figure 7.3: Power angle deviations for machines #1 and #2 (in deg).

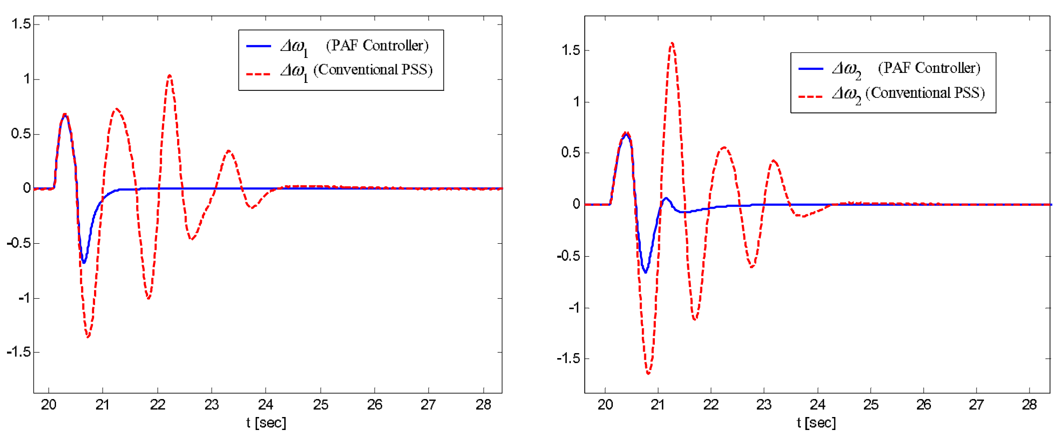

Figure 7.4: Speed deviations for machines #1 and #2 (in rad/sec) respectively.

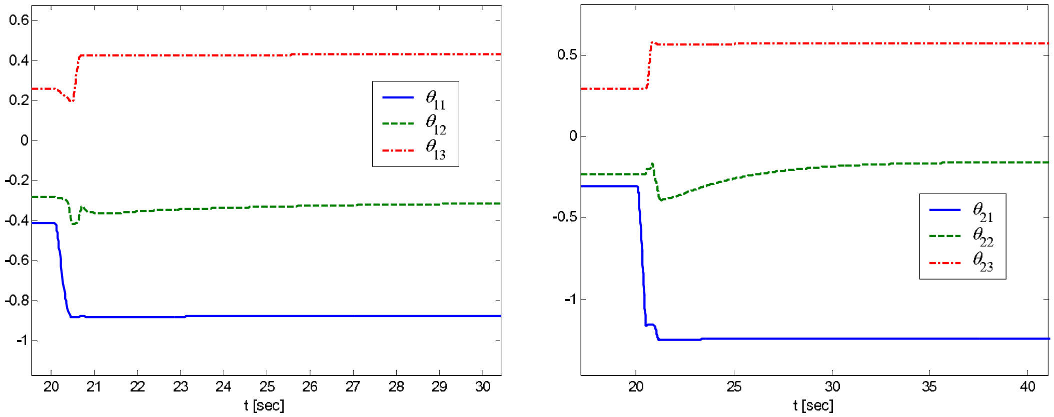

Figure 7.5: The centres of gravity for the FLS #1 and #2 respectively.

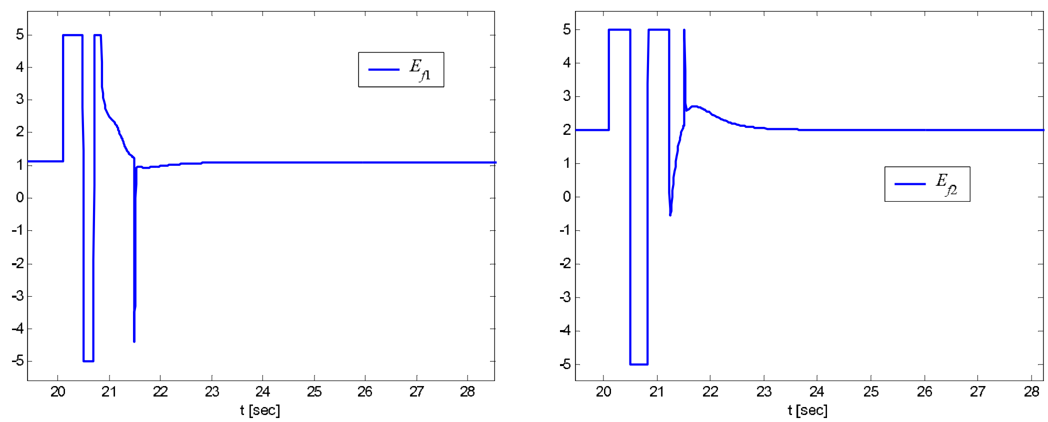

Figure 7.6: Excitation input for machines #1 and #2 respectively.

Comparing the proposed method with other similar advanced nonlinear control methods applied on power systems [9, 10, 13, 14] we can make the following remarks.

In [9, 10] the proposed adaptive scheme may result in high-gain controllers in order to obtain the bounds of the unknown nonlinearities included in f_i. In the present paper, we overcome this disadvantage by using a suitable FLS for the approximation of f_i without increasing the complexity. In [13, 14] a self-learning model reference adaptive fuzzy algorithm is proposed to approximate the system model which is considered to be totally unknown. Therefore, a more complex algorithm is needed with a lot of fuzzy rules and a lot of parameters that must be estimated by adaptation techniques. In our case a simple SISO fuzzy logic controller with only three rules and adaptation loops is needed. This significantly reduces the on-line computational effort. A disadvantage is the requirement of an extra state measurement which however contributes to a better system performance.

8 Conclusions

An intelligent-based simple nonlinear passive control suitable for power system applications is proposed. Stability analysis and simulation tests verify the effectiveness in a variety of operating conditions resulting from large unknown disturbances such as short-circuit faults.

9 Notation

$\delta_i(t)$	: power angle, in radian;
$\omega_i(t)$	: rotor speed, in rad/sec;
ω_0	: synchronous machine speed, in rad/sec;
P_{mi}	: mechanical input power, in p.u;
$P_{ei}(t)$	: active electrical power, in p.u.;
D_i	: damping constant, in p.u.;
M_i	: inertia coefficient, in seconds;
$E'_{qi}(t)$	: transient EMF in the q-axis in p.u.;
$E_{qi}(t)$	: EMF in the q-axis, in p.u.;
$E_{fi}(t)$	: equivalent EMF in excitation coil, in p.u.;
T'_{d0i}	: d-axis transient short circuit time constant, in sec;
$I_{fi}(t)$	: excitation current, in p.u.;
$I_{qi}(t)$	: q-axis current, in p.u.;
$I_{di}(t)$	: d-axis current, in p.u.;
$Q_{ei}(t)$	: reactive electrical power, in p.u.;
$V_{ti}(t)$	: generator terminal voltage, in p.u.;
k_{ci}	: gain of generator excitation amplifier, in p.u.;
$u_{fi}(t)$	: input of the SCR amplifier, in p.u.;
x'_{di}	: d-axis transient reactance, in p.u.;
x_{di}	: d-axis reactance, in p.u.;
x_{adi}	: mutual reactance between the excitation coil and the stator coil, in p.u.;
$Y_{ij} = G_{ij} + jB_{ij}$	: the i-th row and j-th column element of nodal admittance matrix, in p.u.;
$\Delta\delta_i(t) = \delta_i(t) - \delta_{i0}$	: nominal angle deviation, in deg;
$\Delta\omega_i(t) = \omega_i(t) - \omega_0$	: nominal speed deviation, in rad/sec;
$\Delta P_{ei}(t) = P_{ei}(t) - P_{mi}$ $:= -\Delta P_{mi}$	: where ΔP_{mi} the accelerating power, in p.u.
$\mathbf{R}^n \backslash \Omega$	: $\mathbf{R}^n$except a region defined by the compact set $\Omega \subset \mathbf{R}^n$ containing the origin.

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