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Near-Time-Optimal Path Planning of a Rigid Machine with Multiple Axes

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Abstract: A path planning method which is nearly time-optimal is designed for computer numerical control machines which must handle sharp corners. The nominal geometrical trajectory is modified in a way that limitations of the drives' accelerations are taken into account, which will avoid acceleration discontinuities at the cornering point. The method uses two consecutive optimization procedures based on the theory of time-optimal control of single axes while maximizing the travel length of the fastest axis. Simulation results show that the method, which can be generalized to a machine with several axes, is quite effective.

Keywords: *Computer numerical control machines; path-planning; contour error.*

Mathematics Subject Classification (2000): 49K15, 93C05, 93C95.

1 Introduction

The need for increased productivity leads computer numerical control (CNC) machine tools to be faster, i.e. to reduce cycle time, while keeping a good contouring quality (i.e. keeping the tool path within prescribed bounds). Whereas the main goal of trajectory planning is to ensure the following of a nominal geometrical path, smooth modifications of the path can be used as pre-filtering functions which act as a feedforward controller for each individual axis. Afterwards, a feedback control algorithm will be designed which will allow to maintain the positioning accuracy while taking the dynamics into account. However, high speed machines are generally flexible and have to bear vibrations which

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are harmful for the mechanical parts and deteriorate the accuracy, which excludes discontinuities in the drive speed and/or the acceleration. The path planning is thus an important stage of control because it has to take into account speed, acceleration or jerk limitations, which is necessary to obtain good overall performances [2].

Time-optimal path planning of a machine tool with one single rigid axis usually results in a bang-bang scheme [5]. The problem is more complex when considering a machine with multiple axes, because geometrical constraints generate coupling terms between the trajectories of the different axes. When considering limitations in acceleration and jerk, the optimal trajectory of each of the individual axes cannot generally be left to an optimal bang-bang scheme because the geometrical trajectory would be outside the prescribed error bounds. For example, when a discontinuity in curvature occurs, the speed of the axes before and after the cornering point have to be adapted, and, thus, a discontinuity in acceleration will appear, which is not acceptable in practice. Indeed, this would excite vibratory modes – that could be neglected or compensated when the acceleration is smooth [2]. A first way to keep close to the nominal trajectory without bearing discontinuities in accelerations consists of letting the manipulator come at a full stop at the corner, and then accelerate again, or gradually reduce the speed to zero (e.g. introducing jerk limitations in the individual axes) [2], [1], [4]. One can also design a feedback controller which will manage in a way such that the contouring accuracy keeps acceptable, e.g. by cross-coupling controllers [8]. This is partially achieved by the look-ahead function that is built into CNC machines which will ensure that acceleration commands in the interpolated trajectory never exceed their allowed limitations, or by low-pass filtering of acceleration commands [9], [10], [11].

Sharp corners can also be traveled by modifying the toolpath and adjusting the feedrate, which is classical in robotics applications where interpolation is only needed,[3] and related references. It is possible to replace sharp corner with a smooth curve, which can be, for example, a circular arc [6] or an under or over-corner quintic spline [4]. However, very few indications exist how to perform this smoothing in a way that the traveling time keeps close to the optimum, while respecting the geometrical error bandwidth.

This paper proposes a method to obtain a near-time-optimal path planning for machines with several axes, considering speed and acceleration limitations. This optimal trajectory will be given as a geometrical path where the time is not directly given, and will be a function of the allowed contouring error. For the sake of simplicity, the algorithm will be presented for only two axes. The trajectory will be divided into 3 parts, the first one consist of a sequence where the path follows the nominal trajectory which will be a straight line. Then, the modified geometrical path leaves the nominal trajectory before corner crossing and will reach the new direction after the cornering point. The second sequence consists of a point-to-point motion between the leaving and reaching points. The motion will be designed in a way that it is time-optimal for each part of the path taken separately, and, in a second time, that the resulting geometrical path uses the fastest axis at full speed during the maximal time, while staying within the contouring error bandwidth.

2 Point to Point Time-Optimal Trajectory Planning for One-Axis Rigid Machines

The rigid machine is supposed to exhibit an ideal dynamics :

$$\dot{X} = ku,$$

where X is the position, u is the driving force. For the sake of simplicity, k will be set to 1.

The limitations in speed and acceleration of the drive will be considered, i.e. there exist A, U such that $|\dot{X}| = u \le U$, $|\ddot{X}| = \dot{u} \le A$.

The general solution is given for example in [5]. A particular solution is recalled hereafter when the constraints are met (trajectory with full speed and maximum acceleration):

$$\exists\, (t_1, t_2)\colon\ |\dot{X}(t_1)| = U, \quad |\ddot{X}(t_2)| = A.$$

A "point to point" trajectory starts from $X_0(t_0)$, $\dot{X}_0(t_0)$, where $\ddot{X}_0(t_0), \dots, X_0^{(n)}(t_0) = 0$ and reaches $X_f(t_f)$, $\dot{X}_f(t_f)$, where $\ddot{X}_f(t_f), \dots, X_f^{(n)}(t_f) = 0$.

Particular cases include "rest-to-rest" motion ($\dot{X}_0(t_0) = \dot{X}_f(t_f) = 0$), "starting stage" ($\dot{X}_0(t_0) = 0,\ \dot{X}_f(t_f) \ne 0$) and "stop stage" ($\dot{X}_0(t_0) \ne 0,\ \dot{X}_f(t_f) = 0$).

Time minimal control $t = t_f$ leads to maximize the speed along the trajectory which increases from 0 to U, which yields a piecewise-polynomial curve,i.e, for a rest-to-rest motion from $X_0(t_0 = 0) = 0$ to X_f:

$$\begin{aligned} t \le \frac{U}{A}, &\quad \dot{X} = At, \quad X = At^2/2, \\ \frac{U}{A} \le t \le \frac{X_f}{U}, &\quad \dot{X} = U, \quad X = Ut - \frac{U^2}{2A}, \\ \frac{X_f}{U} \le t \le \frac{X_f}{U} + \frac{U}{A} = t_f, &\quad X = X_f - A(t - t_f)^2/2. \end{aligned}$$

3 Optimal Control of a 2-Axes Rigid Machine: Objectives And Constraints

The aim of time-optimal control is to minimize the final time t_f for a motion between (X_0, Y_0) and (X_f, Y_f) (where $\dot{X}_0 = \dot{Y}_0 = \dot{X}_f = \dot{Y}_f = 0$), when spatial and drive constraints are taken into account. This is a far most difficult problem than in Section 1, because, even when the axes are not coupled dynamically, they are made dependent by the geometric constraints imposed by the trajectory. This is particularly crucial when a change in angle occurs, because the speed both axes have to change "simultaneously". Without any constraints on speed and acceleration, it would be only necessary to follow the nominal trajectory and adapt the driving forces at the cornering point. In fact, this is not possible because of drive speed and acceleration limitations, and, in practice, abrupt changes are not desirable because they would excite oscillating modes that are present in mechanical structures. For high-speed machining which are lighter and thus very flexible, these oscillations enforce, in industrial drives, to decrease the speed to zero (or nearly zero) at the crossing point, thus generating an important loss of time.

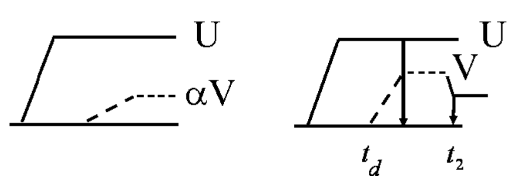

Figure 3.1: Corner crossing.

For the sake of simplicity, the case where one axis will move in the first straight line ($Y_0 = 0$), and the final direction has a slope α will be addressed. Both axes are supposed to exhibit rigid dynamics:

$$\dot{X} = u, \quad \dot{Y} = v.$$

Constraints are of three kinds:

– saturation on drive speed and acceleration,

– constraints on final states (zero derivative and acceleration), which are not considered in the present case;

– geometrical constraints;

The machine should follow the following contour:

$$|Y| \le \varepsilon, \quad X \le X_c, \quad |Y - \alpha(X - X_c)| \le \varepsilon, \quad X \ge X_c.$$

As shown in Figure 3.1, is is difficult to stick to the nominal trajectory when drive constraints exist, when the speed is changed and does not decrease to zero. Moreover, the trajectory is supposed to be modified as follows: the motion stays on the nominal trajectory until the point X_d (which is to be determined), and reaches the new direction at the point X_a (also to be determined), while staying in the error bounds.

The following additional hypotheses are taken:

– Straight lines before and after corner crossing are long enough to reach maximum speed and acceleration.

– Limitations in speed and acceleration occur, i.e. $|\dot{X}| = |u| \le U$, $|\ddot{X}| = |\dot{u}| \le A$, $|\dot{Y}| = |v| \le V$, $|\ddot{Y}| = |\dot{v}| \le B$.

The methodology will be presented with an illustrative case, but can be generalized to multiple axes and additional configurations (e.g. maximum speed is not reached, etc...).

4 Near time-Optimal Control

4.1 Basic algorithm

The time optimal criterion can be written as follows

$$J = \int_0^{t_f} dt, \tag{1}$$

and can be separated into three parts

$$J = \int_0^{t_d} dt + \int_{t_d}^{t_a} dt + \int_{t_a}^{t_d} dt = J_1 + J_2 + J_3. \tag{2}$$

where t_d is the time where the motion leaves the horizontal axis, t_a is the time where the new direction is reached t_f is the final time where the final position is reached.

The near-optimal trajectory planning consists of 3 steps.

Given the positions, X_0, X_d X_a, X_f the *first step* consists in minimizing the final time which leads to minimize the time of motion for each of the three parts

(a) minimize t_d for fixed X_0, X_d given initial conditions in X_0,

(b) minimize $t_f - t_a$ for fixed X_f, X_a given final conditions in X_f,

(c) minimized $t_a - t_d$ for a motion from fixed X_d to X_a starting from initial conditions in X_d given by the solution in (a) and final conditions X_a given by (b).

In the case (a), the time optimal control is a "start" from $X_0(t_0)$ to X_d, where $\dot{X}_0 = 0$ which yields $\dot{X} = U$.

In the case (b), geometric constraints imply that $Y = \alpha(X - X_c)$. The maximum speed of the axis Y is $\min(\alpha U, V)$ and the maximum speed of X is $\min\left(\frac{V}{\alpha}, U\right)$. If the maximum speed is reached at the point (X_a, Y_a), the minimum-time control $t_f - t_a$ is of the "stop" type.

The time-optimal control (c) will be a "point-to-point" strategy for both axes, where the speed in Y increases from 0 to $\min(\alpha U, V)$ between Y_d and Y_a and the speed of axis X stays equal to U or decreases from U to $\frac{V}{\alpha}$ between X_d and Y_a.

In summary, the problem is simplified by solving three time-optimal control problems for one-axis machines where the solutions are given in Section 2. These solutions are parametrized by the points X_d, X_a. This is of course a near-optimal control because it is well known that the sum of optima is not necessarily the optimal solution. However, the solution is quite simple to obtain and can be expected to be close to the true optimal control.

The *second step* consists of minimizing t_f, by the optimization of the location of X_d (and thus of X_a) which will consists of keeping the longest possible trajectory on the axis which exhibits the higher velocity. Two cases arise based on the comparative values of αU and V. The strategy will be different whether the axis X is faster or if the motion is faster along the slope.

In fact, one now tries to keep the maximum speed on the fastest axis, and thus try to adapt the trajectory and point X_d. Only the first case $\alpha U \le V$ will be considered for illustration of the methodology, as the other case can be considered as "dual".

4.2 Illustrative example

Let us suppose that $\alpha U \le V$ and $\alpha A \le B$. In this case, the velocity of the axis X is kept to U, from $t = 0$ to $t = t_f$ (while respecting acceleration constraints). Since the axis Y is faster, it has to adapt and to be bounded.

Applying point (a), the axis Y starts to move at time t_d, until $Y = Y_a$, the velocity of axis Y will increase from 0 to αU.

The near optimal control consists of minimizing t_d and thus X_d while respecting drive constraints and geometric constraints i.e. $|Y| \le \varepsilon$, $X \le X_c$, $|Y - \alpha(X - X_c)| \le \varepsilon$, $X \ge X_c$.

The configuration (1) does not answer the problem correctly since t_d is imposed by geometric considerations which does not leave any degree of freedom for optimization. Configuration (2) will allow to maximize the speed on the fastest (slope) axis: one supposed, for the sake of simplicity, that the maximum speed V is reached by axis Y and decreases again to reach the nominal speed.

On the X axis, the motion will be:

$$X = Ut - \frac{U^2}{2A}, \quad t \ge \frac{U}{A}. \tag{3}$$

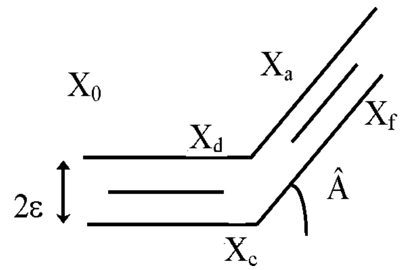

Figure 4.1: Speed profile for X and Y axes.

On the axis Y, time-optimal motion is:

$$
\begin{gathered}
Y = B(t-t_d)^2/2, \quad t - t_d \leq \frac{V}{B}, \\
Y = V(t-t_d) - \frac{(V)^2}{2B}, \ \text{ if } \ t_1 \geq t - t_d \geq \frac{V}{B}, \\
Y = V(t-t_1) - B(t-t_1)^2/2 + Y_1, \ \text{ where } \ t_2 \geq t - t_d \geq t_1, \\
Y = U\alpha(t-t_2) + Y_2, \ \text{ where } \ t - t_d \geq t_2
\end{gathered}
$$

with $Y_1 = Y(t_1)$, $Y_2 = Y(t_2)$.

Continuity considerations (same derivative and position at breaking points) for axis Y yield:

$$
U\alpha = V - B(t_2 - t_1), \quad Y_2 = V(t_2-t_1) - \frac{B(t_2-t_1)^2}{2} + Y_1, \quad Y_1 = V(t_1 - t_d) - \frac{(V)^2}{2B}.
$$

Condition $Y = \alpha(X - X_c),\ t \geq t_2,$ yields $U\alpha(t-t_2) + Y_2 = \alpha\left(Ut - \frac{U^2}{2A} - X_c\right),$ i.e.

$$
U\alpha t_2 - Y_2 = \alpha\left(\frac{U^2}{2A} + X_c\right).
$$

All variables can be expressed as a function of one degree of freedom (i.e. t_d or t_2 can be "freely" chosen), one obtains:

$$
U\alpha = V - B(t_2 - t_1), \tag{4}
$$

$$
Y_2 = V(t_2 - t_1) - B(t_2-t_1)^2/2 + Y_1, \tag{5}
$$

$$
Y_1 = V(t_1 - t_d) - (V)^2/2B, \tag{6}
$$

$$
U\alpha t_2 - Y_2 = \alpha\left(\frac{U^2}{2A} + X_c\right), \tag{7}
$$

where

$$
t_1 = \frac{AU^2\alpha^2 - 2AUV\alpha + B\alpha U^2 + 2AB\alpha X_c - 2ABVt_d}{2AB(U\alpha - V)}.
$$

One can eliminate the expression of time within the equations. Since $X - X_d = U(t - t_d)$ one obtains a piecewise polynomial curve:

$$
Y = \frac{B}{2}\left(\frac{X - X_d}{U}\right)^2, \quad X \geq X_d, \quad Y \leq \frac{V^2}{2B}, \tag{8}
$$

$$Y = V\frac{X - X_d}{U} - \frac{V^2}{2B}, \quad \frac{V^2}{2B} \le Y \le Y_1 \tag{9}$$

as $X - X_1 = U(t - t_1)$,

$$Y = V\frac{X - X_1}{U} - \frac{B}{2}\left(\frac{X - X_1}{U}\right)^2 + Y_1, \quad Y_1 \le Y \le Y_2, \tag{10}$$

$$Y = \alpha(X - X_c), \quad Y_2 \le Y, \tag{11}$$

where X_d, Y_1, Y_2 are given above.

This solution is particularly interesting because it is a geometric path which is supposed to yield the time-optimal controller. One can use it as a geometrical reference trajectory which the axes should follow using feedback control to eliminate the effect of disturbances.

Now, one should determine t_d (or X_d) such that $|Y| \le \varepsilon,\ X \le X_c,\ |Y - \alpha(X - X_c)| \le \varepsilon,\ X \ge X_c$. Since corner crossing X_c can be met in the piecewise parts (8,9,10), the resulting constraints will be different, and several cases can be considered.

Lets now write the constraints on the trajectory as a function of t_d:

The profile in Figure 4.1 can exist if the nominal speed is reached after the full acceleration stage:

$$t_d + V/B < t_1 \tag{12}$$

and the leaving time should occur after the acceleration step has been completed:

$$\begin{gathered} \frac{U}{A} \le t_d, \\ t_d + \frac{V}{B} \le \frac{AU^2\alpha^2 - 2AUV\alpha + B\alpha U^2 + 2AB\alpha X_c - 2ABVt_d}{2AB(U\alpha - V)}, \end{gathered} \tag{13}$$

i.e.

$$\frac{AU^2\alpha^2 - 4AUV\alpha + B\alpha U^2 + 2AB\alpha X_c + 2V^2A}{2ABU\alpha} \le t_d.$$

Suppose that the cornering point X_c is met during the motion (8). This implies that the leaving point lies before the corner and $t_d \le t_c = \dfrac{X_c}{U} + \dfrac{U}{2A}$, which can be turned, eliminating the time, $\dfrac{B}{2}\left(\dfrac{X_c - X_d}{U}\right)^2 \le \dfrac{V^2}{2B}$ and thus

$$\frac{X_c}{U} - \frac{V}{B} + \frac{U}{2A} \le t_d \le \frac{X_c}{U} + \frac{U}{2A}\,. \tag{14}$$

Once the path has left the nominal trajectory, it should stay nevertheless between prescribed error bounds, e.g. for the section described by equation (8), when the trajectory stays ahead of the corner: $\dfrac{B}{2}\left(\dfrac{X_c - X_d}{U}\right)^2 < \varepsilon$ and thus

$$\frac{X_c}{U} + \frac{U}{2A} - U\sqrt{\frac{2\varepsilon}{B}} < t_d. \tag{15}$$

Since when $X \ge X_c,\ Y \le \frac{V^2}{2B}$ one must have for equation (8), when the path travels the corner and the trajectory should be close to the new direction $-\varepsilon < Y - \alpha(X - X_c) < \varepsilon$, and, replacing:

$$-\varepsilon < \frac{B}{2}\left(\frac{X - X_d}{U}\right)^2 - \alpha(X - X_c) < \varepsilon. \tag{16}$$

The maximum of this function is given for $X = X_d + \frac{\alpha U}{B}^2$ which yields:

$$-\frac{\varepsilon}{\alpha U} + \frac{\alpha U}{2B} \le \frac{X_c}{U} + \frac{U}{2A} - t_d \le \frac{\varepsilon}{\alpha U} + \frac{\alpha U}{2B},$$

i.e.

$$\frac{X_c}{U} + \frac{U}{2A} - \frac{\varepsilon}{\alpha U} - \frac{\alpha U}{2B} \le t_d \le \frac{X_c}{U} + \frac{U}{2A} + \frac{\varepsilon}{\alpha U} - \frac{\alpha U}{2B}\,. \tag{17}$$

When, in a second time, the trajectory is given by equation (9), it should also stay within prescribed error bounds

$$-\varepsilon \le V\frac{X - X_d}{U} - \frac{V^2}{2B} - \alpha(X - X_c) \le \varepsilon,$$

where

$$\frac{V^2}{2B} \le V\frac{X - X_d}{U} - \frac{V^2}{2B} \le Y_1.$$

Since the function is increasing, one has only to verify, that $-\varepsilon \le Y_1 - \alpha(X_1 - X_c) \le \varepsilon$, where $Y_1 = V(t_1 - t_d) - (V)^2/2B$ which yields:

$$-\varepsilon \le V(t_1 - t_d) - \frac{V^2}{2B} - \alpha\left(Ut_1 - \frac{U^2}{2A} - X_c\right) \le \varepsilon$$

and one obtains

$$\begin{gathered} -\varepsilon \le V\frac{X - X_1}{U} - \frac{B}{2}\left(\frac{X - X_1}{U}\right)^2 + Y_1 - \alpha(X - X_c) \le \varepsilon, \\ \text{if } \; Y_1 \le V\frac{X - X_1}{U} - \frac{B}{2}\left(\frac{X - X_1}{U}\right)^2 + Y_1 \le Y_2. \end{gathered} \tag{18}$$

Last, when the trajectory is described by equation (10), one has to check that

$$-\varepsilon \le V\frac{X_2 - X_1}{U} - \frac{B}{2}\left(\frac{X_2 - X_1}{U}\right)^2 + Y_1 - \alpha(X_2 - X_c)) \le \varepsilon$$

which leads to $-\varepsilon \le U\alpha t_2 - \alpha\left(\frac{U^2}{2A} + X_c\right) - \alpha\left(Ut_2 + \frac{U^2}{2A} - X_c\right) \le \varepsilon$, i.e.

$$\alpha\left(\frac{U^2}{A}\right) \le \varepsilon. \tag{19}$$

In summary, one obtains easily a set of inequality constraints (13)–(19) which should in a first time be all compatible in a way such that the profile (2) in Figure 4.1 is really feasible. This gives upper and lower bounds on the value of t_d, and, since the motion on the fastest axis (the vertical one) should be preferred, the value of t_d will be the minimum one.

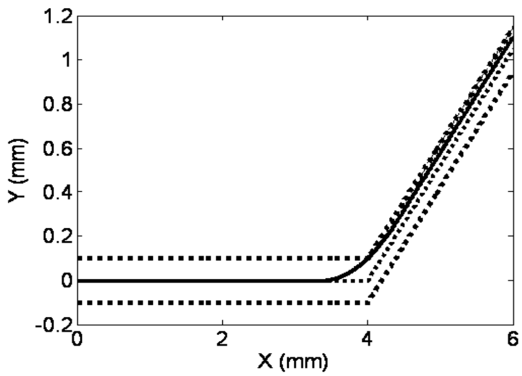

Figure 4.2: Near-time-optimal trajectory.

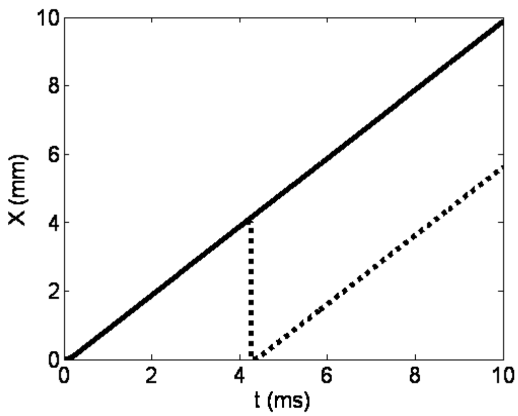

Figure 4.3: Near-optimal versus full stop at corner trajectory.

Example 4.1 Taking numerical values as

$$U = 1, \quad A = 4, \quad B = 0.4, \quad V = 1, \quad X_c = 4, \quad \varepsilon = 0.2, \quad \alpha = 0.8.$$

The maximum constraints are given by (13) and (18) which yields $3.25 \le t_d \le 3.37$.

The optimal path is given in Figure 4.2. The "nominal" path (a stop of the axis X at point X_c, and a "start" from point X_c of axes X and Y, considering speed and acceleration limitations) is also represented in Figure 4.2. One sees that the result is an "under corner" trajectory smoothing [4]. In Figure 4.3, the time history of axis X is represented; for the nominal trajectory following, ones sees that a stop is needed for $X = X_c$. Modified trajectory (solid), nominal trajectory and error bounds (dotted) X position as a function of time, near-optimal trajectory in solid. Classical (with full stop and restart at the corner) dotted. In the case of the modified trajectory, the axis X stays at full speed. The saved time exceeds that which would have been saved by canceling the start and stop procedures, i.e. $\frac{2U}{A}$. The time for which the modified trajectory reaches $X = 8$ equals 8.14 s compared to 12.38 s for the traditional algorithm. One also can verify that the modified trajectory does not reach the upper breaking point $(X = X_c, Y = \varepsilon)$

since, in this case, other limitations (in speed, acceleration, or geometrical) would not be respected. This demonstrates that the optimum path planning does not reduce to taking the chord.

5 Conclusion

A near-time-optimal path planning method for traveling sharp corners has been designed for a machine with multiple axes. Its main originality consists of modifying, on purpose, the geometric path in order to smooth the nominal trajectory and to respect the drives' capabilities in term of acceleration and speed. The time-dependent trajectory is bang-bang when traveling straight lines and is a point-to-point optimal trajectory between the two points where the trajectory deviates from the geometrical discontinuity. The second step of the algorithm consists of maximizing the travelling time of the fastest axis, by moving forward or backward the point where the modified trajectory leaves the nominal path, while staying within the prescribed contouring accuracy.

This method proves to be quite effective and can be generalized to a machine with more than two axes. In a next work, this algorithm will be tested on a real-time cartesian machine tool.

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