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Rendezvous Maneuvers under Thrust Deviations and Mass Variation

A.D.C. Jesus * and T.N. Teles

Universidade Estadual de Feira de Santana (UEFS), Departamento de Física
Caixa Postal 252-294, BR116-Norte, Km 3, 44.031-460, Feira de Santana, BA, Brazil

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Abstract: The Rendezvous maneuvers are used in many important technological space missions. Today, the interception between space bodies (vehicles, stations, debris, etc.) is far from negligible, due the large number of such bodies in Earth orbit and the growth of the current rate space activities. The Rendezvous are realized during many satellites special formations, interception between space stations and satellites or spacecrafts, interception between this bodies and space debris, runaway maneuvers, Formation Flying, etc. In this paper, we study the Rendezvous maneuvers between one satellite and other space vehicle, considering the thrust direction deviations and the mass variation in the satellite, due to the non-ideal propulsion system. We found to the noncoplanar maneuvers, one nonlinear cause/effect relations between the position coordinates uncertainess of the vehicle-interceptor and the "pitch" and "yaw" deviations. Besides, this relation is weighed by time penalty functions, due the variation mass effect. This model is very close to the realistic case and can be implemented inside the technological missions range to the thrust deviations.

Keywords: *Rendezvous; thrust deviations; mass variation.*

Mathematics Subject Classification (2000): 70M20.

* Corresponding author: adj@uefs.br

1 Introduction

The Rendezvous is one completely constrained, with several applications, mainly for the space station and space debris. In this maneuver the orbit parameters and the distance between the two space objects can be divergent. The encounter between the two vehicles must occur without collisions between them, that is, the relative velocity must be null in same time. The Rendezvous coplanar solution, given the impulses and fixed time, was found by Clohessy-Wiltshire [1] in 1960. After him, many authors studied this maneuvers under many conditions and constraints. Stern [2] in 1984 approximated the Clohessy-Wiltshire equations to the small time transfer and obtained non-jouned rectilinear trajectories. Examples of applications of this result are the terminal and extra-vehicle Rendezvous and satellite operation service. The generalization to planar, minimum consumption Rendezvous case, in the general central force field was done by Humi [3]in 1993. In this year Abramovitz and Grunwald [4] developed an iterative graphical method to the optimal and planar Rendezvous inside many spacecraft of one space station environment, under several operational constraints saving more than 30 per cent fuel. Also in 1993, Lutze and Lawton [5] investigated optimal Rendezvous with free time, using regularized variables of the true anomaly, obtaining a simple form for the co-states equations during coast arcs. They established a new optimal necessary condition to the optimality problem. In 1994 Yuan and Hsu [6] proposed a new direction scheme to use the terminal Rendezvous phase. The solution related the fuel consumption to the new direction guidance law with propellant mass. They used the spacecraft variation mass and non-variation mass approach. Shaohua et all [7] also in 1994 applied a transverse propulsion to the Rendezvous trajectory, transforming it in an omni-direction and more fuel-economic trajectory with respect to the conventional cases and Jones and Bishop [8] developed one law target for the Rendezvous terminal phase, using a small Halo translunar orbit ratio with 3 bodies approach. They found 3D Rendezvous in terminal phase and a total minimum cost function for the transfer time, inclination angle and inital condition angle. Pardis and Carter [9] in 1995 considered the impulse saturation effects in optimal Rendezvous with limited power propulsion system and found that the saturation pointed to a degradation of the consumption performance index, which could be improved if the fly time was increased or if additional impulses were applied. In this year, Yu [10] showed that an stable equilibrium state can occur in the relative motion between two close spacecrafts to Rendezvous inside a local coordinate system. Prado [11] also in 1995, derived an algorithm to solve optimal Rendezvous maneuvers with two impulses for a mono-revolution transfer or a multi-revolutions transfer, coplanar or non-coplanar. He found fits of the fuel consumption as function of transfer time. In 2001 Prado and Felipe [12] used impulsive control to study the Rendezvous maneuvers ... . All this results were obtained to impulsive maneuvers and ideal propulsion system and the most with non-variable mass. Our approach is non-impulsive continuous Rendezvous maneuvers under thrust directions deviations and mass variations. We applied Rendezvous maneuvers between the control satellite and the interceptor satellite in one "Formation Flying".

2 Mathematical Model and Preliminaries

The mathematical model considers one control satellite in R ratio initial orbit with velocity $v = (GM_T/R)^{1/2}$ and one satellite interceptor in a transfer orbit with apogee close to R, conform Figure 2.1.

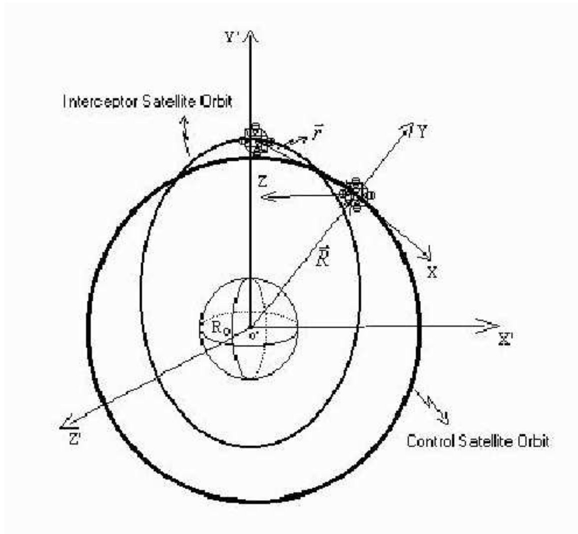

Figure 2.1: Space Satellites in Rendezvous.

In this Figure we show two reference systems: $\{X', Y', Z'\}$ - inertial, Earth-centered, ratio R_0 and $\{X, Y, Z\}$ - rotational, satellite control-centered. For our Rendezvous maneuver the condition that the distance r between the satellites compared with the distance R between the control satellite and the Earth is small must be satisfied. This condition can be satisfied, to technological purposes, with $\{(x(t)^2 + y(t)^2 + z(t)^2)\}^{1/2} \leq 200mi$. This condition allows us to neglect terms in higher order of the gravitational force expansion in serie. The movie equations for the satellite interceptor with respect the rotational system are

$$\ddot{x}(t) - 2W\dot{y}(t) = -v_{ex}\frac{d\{\ln[M(t)]\}}{dt}, \tag{1}$$

$$\ddot{y}(t) - 3W^2y(t) + 2W\dot{x}(t) = -v_{ey}\frac{d\{\ln[M(t)]\}}{dt}, \tag{2}$$

$$\ddot{z}(t) + W^2z(t) = -v_{ez}\frac{d\{\ln[M(t)]\}}{dt}. \tag{3}$$

These equations determine the Rendezvous dynamics between two satellites under thrusters and gravitational forces. In the right size of these equations are the propulsion force components, modeled as

$$\vec{f} = \left\{-\vec{v}_e\frac{dm}{dt}\right\}\frac{1}{M(t)}, \tag{4}$$

where $\vec{v}_e$ is the escape velocity vector of the fuel. The total satellite mass can be modeled as the sum of the satellite constant mass M, and the fuel variable mass m(t), that is,

$$M(t) = M + m(t). \tag{5}$$

Besides this, we consider that the satellite mass is proportional to the initial fuel mass So,

$$\chi \equiv \frac{M}{m(0)} = \frac{M}{m_0}. \tag{6}$$

The solution of the differential equations $(1), (2), (3)$ depend of the satellite time variation mass model.We consider in this paper the exponential model, that is,

$$M(t) = m_0(\chi + 1) + \dot{m}t. \tag{7}$$

In this equation $\dot{m}$ = constant < 0. If we suppose that $\chi \geq 1$ (technological approximation), we can expand the logarithms function. In this way, the solution of those equations are, after many algebraic manipulations,

$$x(t) = 2A\sin(Wt) - 2B\cos(Wt) + Et + \sum_{n=1}^{\infty} F_n e^{-n\gamma t} + G, \tag{8}$$

$$y(t) = A\cos(Wt) + B\sin(Wt) + \sum_{n=1}^{\infty} C_n e^{-n\gamma t} + D, \tag{9}$$

$$z(t) = H\cos(Wt) + I\sin(Wt) - \sum_{n=1}^{\infty} J_n e^{-n\gamma t}. \tag{10}$$

The constants A,B,D,E,G,H,I depend of the initial conditions and of the χ, γ, W. The constants C_n, F_n and J_n are sum in n.

For introduce the thrust direction "pitch", $\Delta\alpha(t)$, and "yaw", $\Delta\beta(t)$, deviations, we write the $\vec{v}_e$ components and the solutions $x(t)$, $y(t)$ and $z(t)$ with symbol $(*)$ and without it for these variables without deviations. So,

$$v_{ex}(t) = v\sin\alpha(t)\cos\beta(t), \tag{11}$$

$$v_{ey}(t) = v\cos\alpha(t)\cos\beta(t), \tag{12}$$

$$v_{ez}(t) = v\sin\beta(t). \tag{13}$$

And these variables with direction deviations,

$$v^*_{ex}(t) = v\sin[\alpha(t) + \Delta\alpha(t)]\cos[\beta(t) + \Delta\beta(t)], \tag{14}$$

$$v^*_{ey}(t) = v\cos[\alpha(t) + \Delta\alpha(t)]\cos[\beta(t) + \Delta\beta(t)], \tag{15}$$

$$v^*_{ez}(t) = v\sin[\beta(t) + \Delta\beta(t)]. \tag{16}$$

We define the difference between the both values, to coordinate y(t), for example,

$$y^*(t) - y(t) = \Delta y(t) = \frac{1}{W}\int_0^t [G^*(\tau) - G(\tau)]\sin[W(t-\tau)]d\tau, \tag{17}$$

where

$$G^*(\tau) = 2Wv^*_{ex}\ln[M(\tau)] - v^*_{ey}\frac{d[\ln M(\tau)]}{d\tau} - 2WC_1 \qquad (18)$$

or, considering this result, we have

$$\Delta y(t) = \frac{1}{W}\int_0^t [2W(v^*_{ex} - v_{ex})\ln M(\tau) - (v^*_{ey} - v_{ey})\frac{d[\ln M(\tau)]}{d\tau}]\sin[W(t-\tau)]d\tau. \qquad (19)$$

We adopted probabilistic approach, that is, we adopted the mean variables values, because we do not know about the final variables values. The thrust direction deviations were modeled through one uniform or gaussian probability distribution function. The expectation operator $\mathcal{E}$ is the mean in the assemble values. We consider that the stochastic processes are ergodic, so, the expectation operator commutes with the integral operator (in time). We consider too that the $\ln[M(\tau)]$ and $\sin[W(t-\tau)]$ functions are deterministic in time. So,

$$\mathcal{E}\{\Delta y(t)\} = \frac{1}{W}\int_0^t [2W\mathcal{E}\{(v^*_{ex} - v_{ex})\}\ln M(\tau) - \mathcal{E}\{(v^*_{ey} - v_{ey})\}\frac{d[\ln M(\tau)]}{d\tau}]\sin[W(t-\tau)]d\tau. \qquad (20)$$

Equation (20) is general for any probability distribution deviations. We considered the uniform probability distribution.

3 Rendezvous under Direction "pitch" Deviations

To compute the means in Equation (20) in the fixed time and considering the random-bias deviations,that is, ($\Delta\alpha(t) = \Delta\alpha = constant$), we have

$$\mathcal{E}\{\Delta y(t)\} = K_1(t)\{\frac{\sin\Delta\alpha_{max}}{\Delta\alpha_{max}} - 1\}, \qquad (21)$$

where

$$K_1(t) = \{2v_{ex}(t_f)\int_0^t \ln M(\tau)\sin W(t-\tau)d\tau - \frac{v_{ef}(t_f)}{W}\int_0^t \frac{d\{\ln M(\tau)\}}{d\tau}\sin W(t-\tau)d\tau\}. \qquad (22)$$

Equation (21) is the cause/effect relation very important between the thrust deviations through the "pitch" direction and the position satellite deviation y(t) coordinate. We observe too that in this relation there is one penalty time-function $K_1(t)$. This function depends of the mass parameters χ,γ and the control satellite angular velocity W. After the computation,

$$\mathcal{E}\{\Delta y(t)\} = \sum_{j=2}^{\infty}\frac{(-1)^{j+1}\{\Delta\alpha_{max}\}^{2(j-1)}}{(2j-1)!}[A'\cos(Wt) + B'\sin(Wt) + \sum_{n=1}^{\infty}C_n e^{-n\gamma t} + D'], \qquad (23)$$

where

$$A' = \{-\frac{2v_{ex}(t_f)}{W}\ln(m_o\chi) - \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\chi^n n}\{\frac{2v_{ex}(t_f)}{W} + \frac{n\gamma v_{ex}(t_f)}{W^2}\}\frac{1}{\{1+(\frac{n\gamma}{W})^2\}}, \quad (24)$$

and

$$B' = \{\frac{v_{ey}(t_f)}{W}\ln(\frac{\chi+1}{\chi}) + \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\chi^n n}\{-\frac{v_{ey}(t_f)}{W} + \frac{2n\gamma v_{ex}(t_f)}{W^2}\}\frac{1}{\{1+\{\frac{n\gamma}{W}\}^2\}}, \quad (25)$$

and

$$C_n = \frac{(-1)^{n+1}}{\chi^n n}\{\frac{2v_{ex}(t_f)}{W} + \frac{n\gamma v_{ey}(t_f)}{W^2}\}\frac{1}{\{1+\{\frac{n\gamma}{W}\}^2\}}, \quad (26)$$

and

$$D' = \{\frac{2v_{ex}(t_f)}{W}\ln(m_o\chi)\}. \quad (27)$$

The penalty function $K_1(t)$ weighted the cause/effect relation in time, besides the thrust deviations effects. Its effect is oscillate in the increasing time and the orbit will be damaged. But, the Rendezvous maneuvers under the realistic conditions are wanted realized in minimum time.

The similar mathematical proceedings to the x(t) coordinate, integrating the Equation (1), give

$$\dot{x}(t) = 2Wy(t) - v_{ex}\ln[M(t)] + C_1, \quad (28)$$

and with the thrust deviations

$$\dot{x}^*(t) = 2Wy^*(t) - v_{ex}^*\ln[M(t)] + C_1, \quad (29)$$

and

$$\Delta x(t) = 2W\int_0^t \Delta y(t')dt' - \int_0^t (v_{ex}^* - v_{ex})\ln[M(t')dt']. \quad (30)$$

Applying the expectation operator $\mathcal{E}$,

$$\mathcal{E}\{\Delta x(t)\} = 2W\int_0^t \mathcal{E}\{\Delta y(t')\}dt' - \int_0^t \mathcal{E}\{(v_{ex}^* - v_{ex})\}\ln[M(t')]dt'. \quad (31)$$

Taking deviations only in "pitch" direction,

$$\mathcal{E}\{\Delta x(t)\} = K_2(t)\{\frac{\sin\Delta\alpha_{max}}{\Delta\alpha_{max}} - 1\}, \quad (32)$$

where

$$K_2(t) = 2W\int_0^t K_1(t')dt' - v_{ex}(t_f)\int_0^t \ln[M(t')]dt'. \quad (33)$$

Expanding in $\Delta\alpha_{max}$ power serie, and integrating,

$$\mathcal{E}\{\Delta x(t)\} = \sum_{j=2}^{\infty} \frac{(-1)^{j+1}\{\Delta\alpha_{max}\}^{2(j-1)}}{(2j-1)!}[2A' \sin(Wt) -$$
$$2B' \cos(Wt) - 2\sum_{n=1}^{\infty} C'_n e^{-n\gamma t} + D''t - L], \tag{34}$$

where

$$C'_n = \frac{(-1)^{n+1}}{\chi^n n^2 \gamma}\{\{2v_{ex}(t_f) + \frac{n\gamma v_{ey}(t_f)}{W}\}\frac{1}{\{1+(\frac{n\gamma}{W})^2\}} - v_{ex}(t_f)\}, \tag{35}$$

and

$$D'' = \frac{3WD'}{2}, \tag{36}$$

and

$$L = v_{ex}(t_f)\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\chi^n n^2 \gamma}. \tag{37}$$

We observe, again, the nonlinear cause/effect relation in the "pitch" deviations and too the time penalty function K_2. This function presents a growing linear time term.

4 Rendezvous under Direction "yaw" Deviations

We consider the random-bias deviations in "yaw" direction, that is, ($\Delta\beta(t) = \Delta\beta =$ constant). With similar steps used previously, we obtain the results to the $\Delta y(t)$ and $\Delta x(t)$, that is,

$$\mathcal{E}\{\Delta y(t)\} = \sum_{j=2}^{\infty} \frac{(-1)^{j+1}\{\Delta\beta_{max}\}^{2(j-1)}}{(2j-1)!}[A' \cos(Wt) +$$
$$B' \sin(Wt) + \sum_{n=1}^{\infty} C_n e^{-n\gamma t} + D'] \tag{38}$$

and

$$\mathcal{E}\{\Delta x(t)\} = \sum_{j=2}^{\infty} \frac{(-1)^{j+1}\{\Delta\beta_{max}\}^{2(j-1)}}{(2j-1)!}[2A' \sin(Wt) -$$
$$2B' \cos(Wt) - 2\sum_{n=1}^{\infty} C'_n e^{-n\gamma t} + D''t - L]. \tag{39}$$

But, in this case, we must consider the $z(t)$ coordinate, because the "yaw" deviation affects the movie in this direction. The velocity component in this direction depends only this angle. So, the solution for this coordinate with "yaw" deviation is

$$z^*(t) = C_1 \cos(Wt) + C_2 \sin(Wt) - \frac{1}{W}\int_0^t v^*_{ez} \frac{d\{\ln[M(\tau)]\}}{d\tau} \sin[W(t-\tau)]d\tau \tag{40}$$

and without this deviations is

$$z(t) = C_1 \cos(Wt) + C_2 \sin(Wt) - \frac{1}{W} \int_0^t v_{ez} \frac{d\{\ln[M(\tau)]\}}{d\tau} \sin[W(t-\tau)]d\tau. \tag{41}$$

Through the similar way, we can compute the difference between these function

$$\Delta z(t) = \frac{1}{W} \int_0^t [G^*(\tau) - G(\tau)] \sin[W(t-\tau)]d\tau, \tag{42}$$

where

$$G^*(\tau) = -v_{ez}^* \frac{d\{\ln[M(\tau)]\}}{d\tau}. \tag{43}$$

So,

$$\Delta z(t) = \frac{1}{W} \int_0^t \{(v_{ez} - v_{ez}^*) \frac{d\{\ln[M(\tau)]\}}{d\tau}\} \sin[W(t-\tau)]d\tau. \tag{44}$$

Applying the expectation operator $\mathcal{E}$,

$$\mathcal{E}\{\Delta z(t)\} = \frac{1}{W} \int_0^t \{\mathcal{E}\{(v_{ez} - v_{ez}^*)\} \frac{d\{\ln[M(\tau)]\}}{d\tau}\} \sin[W(t-\tau)]d\tau \tag{45}$$

and

$$\mathcal{E}\{\Delta z(t)\} = K_3(t)\{\frac{\sin \Delta\beta_{max}}{\Delta\beta_{max}} - 1\}, \tag{46}$$

where

$$\mathcal{E}\{\Delta z(t)\} = \sum_{j=2}^{\infty} \frac{(-1)^{j+1}\{\Delta\beta_{max}\}^{2(j-1)}}{(2j-1)!}[H' \cos(Wt) + I' \sin(Wt) - \sum_{n=1}^{\infty} J'_n e^{-n\gamma t}], \tag{47}$$

and

$$H' = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} v_{ez}(t_f)\gamma}{\chi^n W^2\{1 + (\frac{n\gamma}{W})^2\}}, \tag{48}$$

and

$$I' = -\frac{v_{ez}(t_f)}{W} \ln\{\frac{\chi+1}{\chi}\} + J'_n, \tag{49}$$

and

$$J'_n = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} v_{ez}(t_f)}{n\chi^n W\{1 + (\frac{n\gamma}{W})^2\}}. \tag{50}$$

These results show that the the ”yaw” deviations affect all the velocity components, that is, these deviations affects all the movie of the satellite. We observe too the presence of the time penalty function K_3 in this nonlinear relation.

5 Rendezvous under Superposed Direction "pitch" and "yaw" Deviations

Satellite trajectories under superposed direction "pitch" and "yaw" deviations is the general and realistic case, because, during the thrusters burns these deviations occur simultaneously. In this approach we consider deviations non-correlated, that is, they occur non affecting each in other. With the same approach and mathematical proceedings used previously,

$$\mathcal{E}\{\Delta y(t)\} = K_1(t)\{\frac{\sin\Delta\alpha_{max}}{\Delta\alpha_{max}}\frac{\sin\Delta\beta_{max}}{\Delta\beta_{max}} - 1\} \tag{51}$$

or

$$\mathcal{E}\{\Delta y(t)\} = \sum_{j=2}^{\infty}\sum_{s=2}^{\infty}\frac{(-1)^{j+s+2}\{\Delta\beta_{max}\}^{2(j-1)}\{\Delta\alpha_{max}\}^{2(s-1)}}{(2j-1)!(2s-1)!}[A'\cos(Wt) + B'\sin(Wt) + \sum_{n=1}^{\infty} C_n e^{-n\gamma t} + D'], \tag{52}$$

and

$$\mathcal{E}\{\Delta x(t)\} = K_2(t)\{\frac{\sin\Delta\alpha_{max}}{\Delta\alpha_{max}}\frac{\sin\Delta\beta_{max}}{\Delta\beta_{max}} - 1\} \tag{53}$$

or

$$\mathcal{E}\{\Delta x(t)\} = \sum_{j=2}^{\infty}\sum_{s=2}^{\infty}\frac{(-1)^{j+s+2}\{\Delta\beta_{max}\}^{2(j-1)}\{\Delta\alpha_{max}\}^{2(s-1)}}{(2j-1)!(2s-1)!}[2A'\sin(Wt) - 2B'\cos(Wt) - 2\sum_{n=1}^{\infty} C'_n e^{-n\gamma t} + D''t - L], \tag{54}$$

and

$$\mathcal{E}\{\Delta z(t)\} = K_3(t)\{\frac{\sin\Delta\alpha_{max}}{\Delta\alpha_{max}}\frac{\sin\Delta\beta_{max}}{\Delta\beta_{max}} - 1\} \tag{55}$$

or

$$\mathcal{E}\{\Delta z(t)\} = \sum_{j=2}^{\infty}\sum_{s=2}^{\infty}\frac{(-1)^{j+s+2}\{\Delta\beta_{max}\}^{2(j-1)}\{\Delta\alpha_{max}\}^{2(s-1)}}{(2j-1)!(2s-1)!}[H'\cos(Wt) + I'\sin(Wt) - \sum_{n=1}^{\infty} J'_n e^{-n\gamma t}]. \tag{56}$$

So, we obtain the nonlinear cause/effect relations for the superposed "pitch" and "yaw" direction deviations between these thrust deviations and the satellite position coordinates uncertainess. These relations are more realistic and are pondered under penalty time-functions $K_1(t)$, $K_2(t)$, $K_3(t)$ due the mass variation.These results are obtained when we considered the eject velocity components constants.

6 Conclusions

The results obtained in this study showed nonlinear cause/effect relations between the thrust direction deviations, "pitch" and "yaw", and the Rendezvous satellite position coordinates uncertainess. When the eject velocity components are constants, these relations are averaged by penalty time-functions, due the effects of the mass variation. This functions are like weight-functions over the nonlinear relations and the Rendezvous conditions are affected due two reasons: the thrust deviations and the mass variation. The time dependence in these functions shows that the wanted Rendezvous maneuvers are the minimum time maneuvers, because these dependence is linear. For the long time Rendezvous maneuver the penalty functions are time oscillate functions in the $y(t)$ and $z(t)$ coordinates and linear in the $x(t)$ coordinate. It means larger uncertainess in this coordinate in this time regime. Besides this, in general, the results showed that there is a satellite position probability region where it occurs the Rendezvous maneuvers. These uncertainess are due the deviations influence during the thrusters burns. If the eject velocity components were not constants in the time, the penalty functions would be quadratures.

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